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Navigating the Crossroads of Rhetoric and Science: Towards an Informed Public in an Era of Pandemics and Pandemonium

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As the world grapples with the unprecedented challenges of pandemics and pandemonium, the significance of addressing the intersection of rhetoric and science cannot be overstated. The COVID-19 pandemic has spotlighted the limitations of public understanding of science and mathematics and exposed the widespread resistance to scientific facts and the prevalence of mathematical misconceptions. The need for an informed public capable of discerning and evaluating complex scientific information is urgent. In response, this special issue of the *Electronic Journal for Research in Science & Mathematics Education* seeks to spark conversations at the crossroads of rhetoric and science, exploring the role of public education, public rhetorics, and scientific argumentation in fostering a scientifically literate and engaged citizenry.

Historically, the sociology of scientific knowledge has been fraught with instances of skepticism and resistance to scientific claims. Gaonkar's (1993) critique of the rhetoric of science, for example, sparked an ongoing conversation that underscored the importance of examining the discursive debris that envelops our collective understanding of scientific knowledge. In the current era of pandemics and pandemonium, this conversation must not only continue but evolve, as we face the necessity of reconciling public perception and scientific reality. The intersection of rhetoric and science raises a multitude of complex questions about the relationship between scientific facts, public discourse, and public understanding. How can the rhetoric of science be better employed to engage the public in informed and nuanced discussions about critical scientific issues? How might educational and communication strategies be developed to counteract misinformation and dispel misconceptions, thereby cultivating an informed and science-savvy citizenry? These are among the many questions this special issue seeks to explore and catalyze, drawing on disciplinary perspectives from education, rhetoric, science, mathematics, and critical theory.

Addressing the rhetoric of science is particularly crucial in light of the pressing global challenges we face today, such as climate change (IPCC, 2017), clean water scarcity (UNICEF, 2022), food safety (WHO, 2018), and the ethics of artificial intelligence (Bostrom & Yudkowsky, 2014) or CRISPR-driven genetic enhancement (Brokowski & Adli, 2019). With lives at stake, the importance of ensuring that the public is equipped with the requisite knowledge and critical thinking skills to engage in well-informed discussions and decision-making processes cannot be underestimated. In addition to the rhetoric of science, the rhetoric of mathematics also warrants attention in our pursuit of an informed and scientifically literate public. As a discipline, mathematics often appears inaccessible and abstract to many, fostering a divide between those proficient in its language and the general public. The manner in which mathematical knowledge is communicated can either bridge or exacerbate this divide. Consequently, understanding and navigating the rhetoric of mathematics is crucial for facilitating public comprehension of and engagement with quantitative information.

The first article, *"Simply a matter of numbers": Public Commentators' Construction of a Mathematical Model of Equality Perpetuating the Myth of Mathematics as Objective and Neutral* (Gómez Marchant et al., 2023), critiques the commonly held perception that mathematics is an objective and neutral subject, arguing that it is frequently used to perpetuate white supremacy in public political spaces. The authors draw on critical race spatial theory to demonstrate how white parents used a mathematical model of equality to maintain and perpetuate white supremacy in a school board meeting on redrawing the attendance zone of an elementary school. Through a discourse analysis of public comments, the authors highlight how the mathematical model was co-constructed through public comments and how the variables included and excluded perpetuated injustice. The authors argue that the use of mathematics in political spaces cloaks individuals in a guise of neutrality, obscuring human decision making and diverting responsibility.

The second article, *More Complexity, Less Uncertainty: Changing How We Talk (and Think) about Science* (Mays, 2023), argues that the problem with science communication, particularly around complex topics like the COVID-19 pandemic, is not uncertainty but rather complexity. While the framing of uncertainty has been useful in scientific communication discourse, it can have deleterious effects on public discourse by leading to reductive rhetorical treatments of scientific concepts. The simplification of scientific topics can cause anemic science communication that promotes political division and rhetorical disengagement. Mays suggests that communicators should shift their focus from uncertainty to complexity to better communicate scientific concepts. He concludes that communicators cannot ignore certain stases of argument in their rhetorical approaches and emphasizes the importance of robust communication and understanding of complex scientific subjects to public knowledge and a healthy political sphere.

This special issue features contributions that span theoretical, empirical, and practitioner-oriented domains, investigating the relationship between rhetoric, science, and public education, as well as exploring pedagogical approaches and communication strategies that promote scientific literacy in both formal and informal learning contexts. By offering a platform for diverse and interdisciplinary insights, the two articles included here begin a conversation about the challenges and opportunities presented by the intersection of rhetoric and science, and ultimately, contribute to the cultivation of an informed public capable of navigating the complexities of our rapidly evolving world. As the world contends with pandemics and pandemonium, the role of rhetoric and science in shaping public understanding and fostering informed engagement with scientific issues becomes ever more critical. It is my hope that this special issue will not only spur insightful discussions and further research, but also serve as a catalyst for collective efforts to advance the cause of science education, mathematics literacy, and public engagement in this era of uncertainty and turbulence.

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"Simply a matter of numbers": Public Commentators' Construction of a Mathematical Model of Equality Perpetuating the Myth of Mathematics as Objective and Neutral

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ABSTRACT

Within the larger narrative of mathematics as the key to both individuals' and society's economic prosperity (Jones, 2022; Shah, 2019), lies the commonly held perception that mathematics is an emotionless and objective subject (Goldin & DeBellis, 2006; Taylor, 1996). In the public political sphere quantitative measures have long been used to provide a mirage of logic and objectivity to arguments, and end conversations because one can only argue numbers with other numbers (see e.g., Ewing, 2018, Mudry, 2009). Additionally, the use of mathematics in political spaces cloaks the individual in a guise of neutrality because the numbers suggest a nonpartisan perspective of phenomena. These myths of mathematics as objective and neutral (i.e., acultural, ahistorical) are weaponized to divert responsibility such that the perpetuation of injustice goes unremedied and irremediable (see e.g., Bonilla-Silva, 2010). In this paper, we use a critical race spatial perspective (Morrison et al. 2017; Solórzano & Vélez, 2016; Vélez & Solórzano, 2017) to demonstrate how the myth of mathematics as objective and neutral provides opportunities to use those narratives to maintain and perpetuate white supremacy. We reveal this by focusing on the discourse of public comments given during a series of school board meetings on the redrawing of Wilhelm elementary school's attendance zone (all names are pseudonyms). Through the public comments, mathematics was evoked by those advocating for the proposed attendance zone to move 311 students, the majority of which are South Asian and Latinx, as a way to position themselves as neutral. Understanding how mathematics is used in public spheres, particularly in local political spaces like school board meetings, can provide insight into how racism is present in these conversations, yet not explicitly discussed.

Keywords: critical race spatial analysis, whiteness, civic engagement, school board, discourse analysis

Introduction

The issue here is not racism, classism, or fighting against diversity, you know, some people like to use these social shaming strategies to try to shut down discussions. To try to get their way, but this is not what this is about. This is only about enrollment data and geography. (Mason, Jan. 27 Boundary Hearing)

Within the larger narrative of mathematics as the key to both individuals' and society's economic prosperity (Jones, 2022; Shah, 2019), lies the commonly held perception that mathematics is an objective and neutral subject (e.g., acultural, ahistorical, emotionless; DeBellis & Goldin, 2007; Taylor, 1996). In the public political sphere, quantitative measures have long been used to provide a mirage of logic and objectivity to mathematical models and characterizations of a phenomenon; rather than supporting public discourse, numbers often end conversations because one can only argue numbers with other numbers (see e.g., Ewing, 2018, Mudry, 2009). Espeland and Sauder (2016) emphasize the perception of objectivity quantitative measures carry:

> [Quantitative measures] have the patina of objectivity: stripped of rhetoric and emotion, they show what is 'really going on.' Even more, they can reduce vast amounts of information to a figure that is easy to understand, a simplicity that intimates that there is nothing to hide, and indeed that nothing can be hidden. (p. 1)

Quantitative measures become normalized even when their construction and continued perpetuation is violent. We use the language of *flatten* to describe how the processes through which complex phenomena of human behavior and reality in a 3-dimensional world become a 2-dimensional mathematical model (see Tate et al., 1993). The construction of mathematical models and quantification of human phenomena is an important part of the maintenance and perpetuation of white supremacy (see Ewing, 2018; Harrison, 2021; Mudry, 2009; Zuberi, 2001). All mathematical models require human decision making about the inclusion and exclusion of particular variables. The longstanding practice of constructing mathematical models that exclude variables related to race, ethnicity, gender, nationality, sexuality, etc. serves to frame a phenomenon, and thereby a space, in such a way to maintain the comfort of white people (see Brunsma et al. 2020).

Additionally, the use of mathematics in political spaces cloaks the individual in a guise of neutrality because the numbers suggest a nonpartisan perspective of phenomena. For example, Ewing (2018) described a school representative at a school board meeting bombarding the public with quantitative measures (e.g., enrollment efficiency ranges; space utilization standards; value-added scores) to justify school closures. The school board representative did not adequately explain how the measures were determined and their connection to the school closures. Thereby, the guise of objectivity and neutrality obscured the human decision making which dictated the school closures; relieving the district personnel of responsibility. According to Ewing (2018), "[The school representative] is absolved of any personal responsibility for this decision. She is merely the messenger, delivering facts and numbers that can't be denied" (p. 101). This representative used mathematics to divert anger from the leaderships' decision-making processes to these presumed objective truths the quantitative measures captured. The district constructed a mathematical model explaining their reality of the situation through their own quantifiable measures, presenting them as settled and not up for discussion (Ewing, 2018). This mathematical model was constructed intentionally for serving the goals of those in power.

Building on Ewing's (2018) and Castro et al.'s (2022) work, we show how it is not just official authorities (e.g., school board trustees) who invoke the myth of mathematics as objective and neutral to maintain whiteness, but in addition white caregivers do so when participating in local political discourses. These myths of mathematics as objective and neutral are weaponized through the construction of mathematical models of equality—a mathematical solution to a social problem (Tate et al., 1993)—which divert responsibility and perpetuate injustice that normally goes unremedied and is irremediable (see Bonilla-Silva, 2010). To emphasize the roles of these myths in perpetuating white supremacy in the public political sphere, we focus on the public comments revolving around the redistricting of the attendance zone of an elementary school. We use a critical race spatial perspective

(Morrison et al. 2017; Solórzano & Vélez, 2016) to demonstrate how the myth of mathematics as objective and neutral provided opportunities for a group of parents read as white[1] to maintain and perpetuate white supremacy through the collaborative construction of a mathematical model for equality (Tate et al., 1993). We reveal this through a discourse analysis of public comments given during a series of school board and community meetings on the redrawing of Wilhelm Elementary school's attendance zone (all names are pseudonyms). Through the public comments, those advocating the proposed attendance zone changes evoked mathematics to appear neutral to redistricting 311 students; the majority being South Asian and Latinx. Understanding how mathematics is used in public spheres, particularly in local political spaces like school board meetings, can provide insight into how racism is present in these conversations, yet not explicitly discussed (see also Bonilla-Silva, 2010; Castro et al., 2022).

We begin with a summary of critical race theory and our application of a critical race spatial perspective to understand how the white parents worked towards shifting the color-line (Du Bois, 1903/1994; Solórzano & Vélez, 2016) of Wilhelm Elementary (i.e., the attendance zone). We continue by providing more context about Creator Independent School District (ISD) and Wilhelm Elementary. Thereafter, we describe our methodology analyzing the discourse of the white parents evoking a mathematical model of equality (Tate et al., 1993) co-constructed through their public comments. In the results, we explicate the white parents' mathematical model of equality by examining the variables they included and excluded as evidenced in their public comments. We concluded with a discussion and call for future projects on the relationship between white supremacy and the perpetuation of the myth of mathematics as objective and neutral during civic engagement.

Critical Race Theory and a Critical Race Spatial Perspective

Critical race theory (CRT) is a movement started in response to the inability of critical legal studies scholars to recognize "how race is a central component to the very systems of law being challenged" (Martinez, 2014, p. 17). Derrick Bell and several colleagues including Mari Matsuda, Richard Delgado, and Kimberlé Williams Crenshaw saw reforms since the civil rights movement as moving too slowly and being insufficient in disrupting systemic racism (Delgado & Stefncic, 2016). CRT emphasizes the endemic nature of racism in our everyday ways of being and acting in the world. Race matters, as West (2001) argued, and exploring race is fundamental to our democratic engagement, requiring action and accountability to be taken in political spheres.

> Race is the most explosive issue in American life precisely because it forces us to confront the tragic facts of poverty and paranoia, despair and distrust. In short, a candid examination of *race* matters takes us to the core of the crisis of American democracy. And the degree to which race *matters* in the plight and predicament of fellow citizens is a crucial measure of whether we can keep alive the best of this democratic experiment we call America. (West, 2001, p. 107, emphasis in original)

CRT brings to the forefront how racial (in)justice is embedded in our everyday discourses and white supremacy is within the entrails of our society.

Race continues to be a significant factor in education (e.g., Ladson-Billings & Tate, 1995; Tate et al., 1993; Miller et al., 2020; Solórzano, 1997; Solórzano & Yosso, 2002) and mathematics education

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[1] We use "read as white" to recognize the authors are projecting a racial categorization to the individuals based on name and other physical features (e.g., hair, skin tone). This also emphasizes the white privilege they benefited from to be heard as they would be read as white by the audience, administrators, and school board members until they chose to provide evidence otherwise. For brevity future notations will be white parents.

(e.g., Battey & Leyva, 2016; Gutiérrez, 2014; Martin, 2009). Solórzano (1998) described at least five tenets of CRT in education (pp. 122–123):

- The centrality and intersectionality of race and racism
- The challenge to dominant ideology
- The commitment to social justice
- The centrality of experiential knowledge
- The interdisciplinary perspective

These tenets guide our larger project of exploring how whiteness was maintained in the political discourses of Wilhelm Elementary's color-line. As Leonardo (2004) wrote, "The hidden curriculum of whiteness saturates everyday school life and one of the first steps to articulating its features is coming to terms with its specific modes of discourse" (p. 144). In this paper, we use CRT to help in understanding the mathematical model of equality (Tate et al., 1993) constructed through the ideal (white) reality reliant on the myths of mathematics as objective and neutral.

Mathematical Model of Equality

Mathematical modeling requires individuals to "simplify the realistic situation by making justified assumptions and by identifying those variables they consider essential, leading to an idealized version of the reality" (Anhalt et al., 2018, p. 204). Modeling with mathematics is lauded as providing learners with rich and rigorous mathematical experiences (NCTM, 2016; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Through a CRT lens, we need to consider how mathematical models are constructed in political discourses and applied towards the maintenance of an idealized version of (white) reality. Moore (2005, 2020) describes this as a white institutional space: "a theoretical explication of organizations and institutions focusing on how advantage and disadvantage, exploitation and control, action and emotion, and meaning and identity get patterned in terms of a distinction between Whiteness and non-Whiteness" (Embrick & Moore, 2020, p. 1940). As we demonstrate, mathematical models can be co-constructed through civic engagement in socio-political contexts to preserve whiteness, maintain white institutional spaces, and nourish oppressive systems.

Our work is guided by Tate et al.'s (1993) application of the tenets of critical race theory to (re)read and (re)tell the story of *Brown v Board of Education* to demonstrate how the supreme court's decision pushed the mathematizing of a social problem (desegregation of public schools). A mathematical solution to a social problem flattens the complicated lived experiences of People of Color to an overly simplistic mathematical model centering equality over equity by focusing on purely quantitative measures and ignoring socio-political factors. As Tate et al. (1993) argued, desegregation became squarely about the number of Black bodies moved to violent white spaces (predominantly white schools) rather than the flourishing of Black learners. School districts responsible for desegregation did not need to report on the number of Black teachers, resources provided to Black learners, personal safety and well-being, nor the achievement of minoritized learners. Thereby, the mathematical model of equality constructed after *Brown v Board of Education* continued to provide opportunities to maintain idealized (white) realities.

Critical Race Spatial Perspective

A critical race spatial perspective combines critical race theory (Delgado & Stefancic, 2017) with a spatial justice consciousness (Soja, 2010). A critical race spatial perspective is "an explanatory framework and methodological approach that accounts for the role of race, racism and white supremacy in examining geographic and social spaces" (Vélez & Solórzano, 2017, p. 20). Critical race theory emphasizes "the lived experience of the law" (Miller et al., 2020) and a critical race spatial perspective stresses the lived experiences within constructed racialized spaces of (white) realities. Therefore, we see the attendance zone as a racialized space delineating Wilhelm Elementary's student racial makeup and defining "privilege and opportunity, as well as subordination and marginality" (Solórzano & Vélez, 2016, p. 429). The power of the school board to change the attendance zone, and thereby the racial makeup of the student body, means that community conversations about these demarcations must be seen as racialized discourses. Fitting into Soja's (2010) description of "thoughts about space" or "how materialized space is conceptualized, imagined, or represented in various ways" (p. 101).

Building on Du Bois' (1903/1994) conceptualization of the color-line, Vélez and Solórzano (2016) describe the importance of the color-line in using critical race theory to understand the "way *space* comes to be defined and experienced as the conceived and constructed reality of a racist society" (p. 14). A critical race spatial perspective highlights how the color-line constructed maintains white supremacy in spaces and places like law schools (Moore, 2007), museums (Domínguez et al., 2020), and academia (Bracey & McIntosh, 2020; Martin, 2015). Thereby, an investigation into the discourse of attendance zones can be used to better understand the color-line controlling access to Wilhelm Elementary. In this paper, we emphasize the strategic uses of a mathematical model of equality invoking the myth of mathematics as objective and neutral by those white parents wanting to shift the color-line of Wilhelm Elementary.

Methodology

Our methods follow a critical race spatial analysis by focusing on how the public comments provide insight on the thoughts about the racialized space of Wilhelm Elementary. In this paper, we followed a grounded theory approach to conduct a discourse analysis of the white parents' public comments. Our goal was to ground in the data our mathematical model of equality constructed by the white parents and the included and excluded variables of the model. We begin by describing the context of the study including a timeline of the opportunities for public comments. This is followed by the data we collected and how it was analyzed.

Context of Study

Creator ISD is located in central Texas and is a suburb of a large metropolitan area. Wilhelm is one of 35 elementary schools (see Table 1 for demographics). The district school board is made up of seven individuals (no education/policy experience necessary) elected for a four-year term to determine policy alongside district administration (typically with relevant degrees in education). At the November 21, 2019, Creator ISD school board meeting, the administration presented a plan for rezoning four elementary schools to help with overcrowding and align feeder patterns. One of the proposals was for the rezoning of Wilhelm Elementary. The proposal had approximately 311 students from the Figure Eight luxury apartments rezoned to attend Nuno Pereira Elementary. The majority of those who reside at Figure Eight are identified as South Asian and Latinx.

Table 1

Wilhelm Elementary and District Demographics (2019-2020 TEA School Report card)

	Wilhelm Elementary (874 students enrolled)	Nuno Pereira Elementary (461 students enrolled)	Creator ISD
African American	3.3%	13.9%	8.9%
Latinx	13.3%	29.6%	30.4%
White	34.1%	33.0%	37.4%
American Indian	0.3%	0.6%	0.4%
Asian	44.6%	16.2%	18.7%
Pacific Islander	0.1%	0.4%	0.2%
Two or more races	4.3%	6.3%	4.0%
Economically Disadvantaged	7.8%	40.8%	26.6%
Special Education	6.4%	18.1%	10.3%
English Learners	17.3%	19.1%	10.7%

As part of the plan, the district administration recommended providing the varying school communities opportunities to provide public comments beyond the scheduled school board meetings between November 21 and February 20, 2020. The administration recommended scheduling community hearings at the schools whose attendance zones would be changed. Public comments about the plan would also be taken at the regular school board meetings (December 19, January 16, and February 20). In addition, to make sure enough conversation and consideration was given to the boundary changes, the school board members called a meeting—a boundary workshop (Feb. 13th)—specifically to get a summary of the community discussions and provide another opportunity for public comments. On January 27th, a boundary hearing was held at Wilhelm Elementary. By district policy, boundary changes need to be voted on and determined by February 20th to take place the next school year. Table 2 provides a detailed timeline of the meetings where public comments were made.

Data Collected

Video and audio recordings of the meetings are available online on the Creator ISD website. Public comments were heard at each meeting with community members having up to 3 minutes to speak to administrators and/or the school board members. There were a total of 81 public comments given across the five meetings by 45 individuals. Myths of mathematics as objective and neutral were evoked by those in favor of changing the color-line of Wilhelm Elementary (i.e., rezoning 311 mostly minoritized learners to Nuno Pereira Elementary). These 16 parents made 34 public comments and all these speakers read as white.

Table 2

Timeline of School Board Meetings and Public Comments

Date of meeting	Meeting type	Number of public comments regarding Wilhelm Elementary	Total number of public comments
Nov. 21, 2019	Regular	4	10
Dec. 19, 2019	Regular	4	9
Jan. 16, 2020	Regular	0	9
Jan. 27, 2020	Boundary Hearing at Wilhelm Elementary	28	31
Feb. 13, 2020	Boundary Workshop (Called meeting)	22	44
Feb. 20, 2020	Regular	23	35
TOTAL		81	138

Note: Three public comments made during the boundary hearing are not included in this analysis. A student and her father spoke as representatives of Nuno Pereira Elementary and the third speaker did not state a stance on the issue but asked for clarification about the 2018 bond.

Data Analysis

For this paper, we focus specifically on the 34 public comments made by white parents in favor of shifting the color-line of Wilhelm Elementary. Each of the public comments were transcribed for analysis. Critical race scholars recommend a grounded theory approach (Solórzano & Yosso, 2001, 2002). Malagon et al., (2009) argued, "A CRT framework may influence what is observed, how discussion topics arise, and so forth, but the emerging theory is driven by the data, not by a theoretical framework" (p. 263). Therefore, we sought to follow a grounded theory approach to conduct a discourse analysis of the white parents' public comments. An initial round of open coding (Glaser & Strauss, 1969) called our attention to how the proponents of the shifted color-line would invoke mathematical computations and ideas to warrant their claims. Through iterative rounds of coding and discussion amongst the research team, the white parents' specific ways of weaponizing mathematics' myth of objectivity and neutrality became the central concern of our analysis. As we returned to coding for these specific instances, we referred to these discursive moves as strategies used by the individuals during their public comments. But, this did not capture the preservation of whiteness occurring across the meetings nor how collectively the white parents learned the genre of speaking to the school board (see Tracey & Durfy, 2007). It also failed, in our opinion, to strongly ground a theory in the data.

We recognized the actions of the white parents as a byproduct of systemic white supremacy ideals in laws, policy, and other dominant narratives. Our objective through this analysis was to better understand the white parents' perpetuation of mathematical solutions to social problems that provided them an opportunity to redraw the color-line of Wilhelm Elementary. It was decided to return to the literature to help us in determining how to move forward with our discourse analysis. Tate et al.'s (1993) uncovering of the mathematical model of equality constructed after *Brown v Board of Education* gave us the needed discourse and framing to demonstrate how the white parents' were accruing whiteness. We returned to the data to capture moments where the myths were evoked and also coded them for the included and excluded variables described. This provided us a way to (re)construct the white parents' mathematical model of equality.

A Mathematical Model of Equality For Shifting the Color-Line of Wilhelm Elementary

Those seeking to maintain the whiteness of Wilhelm Elementary applied a mathematical model of equality (Tate et al., 1993) to flatten complicated sociocultural issues and cloak white supremacist ideals in a guise of neutrality by offering a mathematical solution to overcrowding at Wilhelm Elementary. To argue shifting the color-line of Wilhelm Elementary, the parents' applied mathematical model of equality included school population variables and explicitly excluded sociocultural identities and affective variables. Thereby, their mathematical model of equality cyclically relied on and perpetuated the narrative of mathematics as an objective and neutral tool to warrant the political actions of the board. This narrative succeeded in shifting the color-line to construct a whiter space.

Balancing school populations

The white parents' mathematical model of equality is constructed to include and exclude particular variables for consideration when making meaning of Wilhelm Elementary's overcrowding. The model helps determine the criteria for an appropriate solution to the problem. The parents' inclusion variables focused on the goal of *balancing* the aggregate student populations at Wilhelm and Nuno Pereira Elementary. Throughout their public comments, the white proponents of shifting the color-line prioritized matching each school's student enrollment to school capacity. Thereby, the model simplifies the overcrowding problem to one of moving bodies from Wilhelm to Nuno Pereira Elementary. In this section, we discuss two of the inclusion variables used to emphasize how balancing the schools would resolve the crisis of overcrowding. The first variable is focused on the notions of fairness (equal distribution), and the second, on the quantification of capacity. Together, these variables, in combination with the excluded variables of race and emotion, perpetuate the myth of mathematics as objective and neutral. The mathematical model of equality provided the white parents a way to promote a mathematical solution to a social problem; consequently, dehumanizing the learners and community from the Figure Eight Apartments.

Fairness for all? Martin (2003) asserted educational policies or public conversations that purport to be focused on providing *Mathematic for All* arguments are vague and nonspecific; providing an illusion of care for equity and social justice. As part of the parents' mathematical model for equality stressing balance, the notions of "best for all", "for all students", and "best interest of all students" were used as justification for their solution to the overcrowding problem. The *for all* rhetoric was combined with the idea for fairness or equality. These parents argued that it was important for the solution to be one that was fair for all students; but in arguing this, they did not acknowledge that fairness would be achieved by shifting the color-line. Kayla spoke at the December 19th regular board meeting and during her public comment she described the mathematical model for equality's objective or goal. "Our goal is to bring our whole community together to work with the board to *ensure fair and equal solutions*, while considering *the best interest of all of Wilhelm students*" (Kayla, Dec. 19 Regular Board Meeting, emphasis added). Once the goal of the mathematical model had been determined, the parents' could continue to emphasize how to define fairness mathematically through a balance of the student populations.

The discourse around fairness was entangled with descriptions about space and the facilities of the schools. Working within the constructed model of mathematical equality, Mason adds to his argument how moving the Figure Eight Apartment learners to Nuno Pereira Elementary will be what is really fair for all because it alleviates the overcrowding.

> You might hear about *fairness—people complain about fairness, but what's unfair is all these kids in one one place overcrowding a school.* It's much better to have two schools that are at equal capacity to support the needs of all the students. *Everyone's going to be in great schools.* We are still in Creator ISD. We're still in the same feeder patterns. So *really rezoning is what's fair to all these students.* It's giving them the facilities that they need. (Mason, Jan. 27 Boundary Meeting, emphasis added)

While not acknowledging the resultant shift in the color-line, Mason emphasizes how rezoning is the only fair act because leaving the schools imbalanced will hurt the learners in the long run, not the act of moving them to another school. It assumes the resources and facilities available to each school are equivalent. But Wilhelm Elementary does not serve the same percentage of students living in poverty nor those receiving special education services. Nuno Pereira Elementary had 40.8% and 18.1% of students considered economically disadvantaged and receiving special education services respectively versus the 7.8% economically disadvantaged and 6.4% receiving special education services at Wilhelm Elementary. These differences in context were flattened in the white parents' model. Fairness was advantageous to their whiteness.

School capacity

A second related discourse used by the white parents advocating for shifting the color-line involved the capacity of Wilhelm Elementary. The quantification of the schools' capacity, usually discussed as a percentage, was weaponized by the white parents to demonstrate the urgency of rezoning. This included the usage of both the number of students attending the school and the consequences of being over capacity. It was important for the advocates to be explicit in their comparison to the under enrollment at Nuno Pereira Elementary. Mason used the percentage of capacity to justify the shift of the color-line: "Wilhelm is at nearly 140% capacity. Nuno Pereira Elementary is the number one most unenrolled school right now at 64% capacity" (Mason, Jan. 27 Boundary Hearing). Mason justifies the included variable and argues why it meets proponents' criteria of balancing the schools. If the attendance zones are redrawn, then the two schools would be used appropriately according to their capacity. Like Ewing's (2018) example previously discussed, there is no discussion by the district administration or the white parents of how capacity percentage is determined nor its relation to the phenomenon. Wyatt's comment at the February 20th school board meeting provides another example; he stated why capacity was consequential to the learners of Wilhelm Elementary and the importance for both schools to be at capacity.

> We've seen the data, *heard the stories of the overcrowding problems experienced at Wilhelm.* I know many of us have emailed each and every one of you. You've seen the data. You've seen the stories. *That's why it's imperative to make the decision now to provide relief to the almost 900 students currently at Wilhelm.* I'd like to point out that this is—doing so is in complete alignment with [the] Creator ISD strategic plan goals, the first of which states, *we will ensure that all facilities are safe* and advanced learning for every student while planning with our community for sustainable growth. I urge you to carry out this plan. *After the change both schools will then be operating within their design capacity, which reduces the strain on Wilhelm's inadequate bathrooms, its tiny gym space, undersized cafeteria, and many other problems.* From the data from the Creator ISD website, it shows that both schools *would be within capacity*, this will *improve safety and advance the learning of every student.* (Wyatt, Feb. 20 Regular Board Meeting, emphasis added)

Wyatt is demonstrating how focusing on capacity will alleviate the other issues of overcrowding at the school and align with Creator ISD's strategic plan. Quantifying capacity provided one way for the mathematical model of equality to meet the balance criteria. He appeals to the myth of mathematics as objective and neutral by emphasizing how those who know mathematics would be able to see the crisis of overcrowding and the urgent need for a solution. This discourse silences those who feel less comfortable with mathematics by making clear that anyone who sees the data should understand it and come to the same conclusion. Moreover, he sets up the need for a mathematical counterargument because one can only argue numbers with other numbers (see Ewing, 2018; Mudry, 2009).

Included within these discourses was the relationship between whiteness and owning property (see Harris, 1993). Two public speakers explicitly discuss how the usage of the school's capacity was being questioned and the school board members, as elected officials determining the use of the taxes collected, have a responsibility to them as taxpayers and property owners (i.e., their power in having whiteness). This is further evidenced by how historically white people have used paying taxes to assume entitlements to better education than those who presumably do not (see Walsh, 2017). The speakers leaned on their property ownership to push their mathematical model of equality regarding the utilization of the schools in terms of their capacity.

> *I'm a homeowner in this neighborhood and I pay property taxes to fund these schools* and there's a duty here to *utilize these school*s. (Kayla, Feb. 13 Boundary workshop, emphasis added)
>
> Also there's a *fiscal responsibility to the taxpayer to balance* the school so I'm convinced this is the logical solution and I commend the administration on their recommendation. (Mason, Jan. 27 Boundary Meeting, emphasis added)

Kayla and Mason flaunt their whiteness to demonstrate why the mathematical model for equality is legitimate and through the model a mathematical solution can be reached. As taxpayers, they make a "claim of privileged public position that obscure[s] class divisions while simultaneously elevating those with 'more' taxable income to a position of 'more' rights, particularly education rights" (Walsh, 2022, p. 239). The model, therefore, deems the solution appropriate.

The inclusion criteria discussed provided the advocates of the shifting the color-line to promote their idealized version of (white) reality. By flattening the overcrowding issue to one of fairness and capacity or balance between the schools, the parents maintain the whiteness of Wilhelm Elementary. They are able to provide a mathematical solution to a social problem fitting the criteria included in the mathematical model and argue for why it is sufficient. As part of their argument of included variables, there also needs to be claims about excluded variables.

Excluded variables: The lived experiences of policy

As the meetings continued, more and more of the families of Figure Eight gave public comments to argue against the mathematical model of equality constructed by those for shifting the color-line of Wilhelm Elementary. As a result, white parents shifted their argument to be more explicit about the excluded criteria and how the exclusion was beneficial to their mathematical model. The justification of the included and excluded variables aligned with the myth of mathematics as objective and neutral and provided power to their arguments for race (and other social identities) and emotion to be excluded. In other words, the lived experiences of the policy were to be excluded. Two discourses surrounding this excluded variable emerged from the analysis: 1) That it is unnecessary to consider race and 2) that emotions as harmful to decision-making. The myth of mathematics as objective and neutral provided the necessary legitimization for the exclusion.

Unnecessary to consider race. From a CRT perspective, "race is biologically insignificant, but it doesn't follow that it is *socially* insignificant. Race is politically and socially real because, as with currency, people have imbued the concept with a value" (Ray, 2022, pp. 6–7, emphasis in original).

Race, therefore, is central to the conversations about the demarcated attendance zone and who gets to stay at Wilhelm Elementary. While the white parents' arguments would result in shifting the color-line, their public comments explicitly worked to devalue the necessity to consider race in the school board's decision-making. When emphasizing their mathematical model of equality, race and other social factors were irrelevant to the quantifiable measures leading to a solution. The problem was framed as a numerical one, and therefore, race—as not quantifiable—was an excluded variable. Jack and Joy provided direct disregard for race in their public comment:

> These are awesome kids. Nobody here is saying that these students are bad students, bad for the school, *that this is a class decision, or a race decision. This is simply a matter of numbers.* I love all these kids. I know a lot of these kids….it hurts me that somebody has to go. But the fact of the matter is *we can't continue at this rate*, this neighborhood is also expanding. This problem is only going to get worse. (Jack, Jan. 27 Boundary Hearing, emphasis added)

> The idea that kids will be ripped away from their friends or that this has anything to do *with race, ethnicity, or socio-economic status just simply isn't true. This is a logistical numbers problem* and you, as the trustees, have a duty to help facilitate *what is in the best interest for our kids*…. (Joy, Feb. 20 Regular Board Meeting, emphasis added).

Jack, like other parents, first praised the learners as good kids, but then proceeded to erase their social identities to claim those aspects should not be considered within their model of mathematical equality. Joy emphasized the school board's responsibility to take action, and therefore, the necessity to exclude unnecessary variables like race, ethnicity, and socio-economic status. No matter what the school board's decision is, some good students will be removed and it just happens to also be mostly South Asian and Latinx learners. The mathematical problem of space at Wilhelm Elementary took precedence to the identities and needs of the learners and parents of Figure Eight Apartments.

Emotionless Mathematics. DeBellis and Goldin (2007) wrote, "mathematics, unlike the humanities, music, or the arts, is commonly understood as 'purely rational', with emotion playing no role" (p. 131). Although DeBellis and Goldin along with others (Gomez, 2016; Hannula, 2012; Martinez-Sierra & Garcia-González, 2016) stress the importance of emotions in mathematics, the white parents stressed it was an excluded variable in their model of mathematical equality. Emotions were seen as outside of mathematics and it was only through emotionless mathematics that rational unbiased decisions could be made. Emotions distort one's ability to make the appropriate decisions according to the constructed mathematical model of equality. Therefore, the mathematical model is appropriate because it allows the school board to make a more objective decision on the color-line. The exclusion of the emotion variable was only possible due to the myth of mathematics as objective and neutral. Adrian and Kayla explicitly discuss the exclusion of this variable:

> We're asking you to make *a simple decision here that removes emotion and all the other class, culture considerations. We asked you to do the math. It's first grade math.* There are almost 300 students too many at Wilhelm. There is capacity for 300 students at Nuno Pereira Elementary. My first grader could solve that problem. (Adrian, Feb. 13 Boundary Workshop, emphasis added)

> I implore you to continue to *look at the facts and data regarding this issue and to not let emotions cloud the decision* that is in the best interest *of all students.* (Kayla, Feb. 13 Boundary Workshop, emphasis added)

The exclusion of emotions was purposeful in dehumanizing mathematical activity and erasing socio-political considerations in the school board's decision-making processes. It stressed the simple quantifiable measures over socio-cultural qualitative considerations. Adrain invokes the audiences' level of mathematics needed—equivalent to a 1st grader—to demonstrate the straightforward nature of the solution and require counterarguments to address the numbers. The mathematical model of equality constructed intentionally perpetuates the myth of mathematics as neutral and objective to maintain white institutional spaces.

Discussion

At the February 20th school board meeting, the trustees voted to shift the color-line of Wilhelm Elementary even though Creator ISD administration recommended the board reject the proposal.

> But in the end, in looking at everything and the project, our recommendation—because by policy we have to make a recommendation to you on anything you ask us to look at—was to not adjust Wilhelm at this time. Let the 2018 bond project move forward with the planning and see how we can adjust maybe in the future. (Senior Chief of Schools and Innovation, Feb. 20 Regular Board Meeting)

The administration wanted more time for a 2018 bond—approved by voters to expand the number of classrooms at Wilhelm Elementary—to be completed before determining the need to shift the attendance zone. When asked for a specific timeline, the Creator ISD administrators could not provide one because they claim timelines for attaining approved permits for construction are unpredictable, but that they hoped to have construction completed by Fall 2022. The board, however, felt an urgency to resolve the issue of overcrowding as captured by a trustee's questioning of the administration:

> So I'm confused here. So your recommendation is that we do—is we leave all the kids at Wilhelm. And we have almost 300 spots available over at Nuno Pereira. Like I don't understand that. Can you explain to me a little more how that makes sense? I don't understand. (Trustee, Feb. 20 Regular Board Meeting)

The school board ultimately rejected the administration's recommendation.

Wilhelm Elementary's shifted color-line positioned the white student population as the dominant population of the school compared to previous years where Asian learners were the majority. According to student data from Creator ISD records, the Asian learner population decreased by 68% from 390 learners in the 2019-2020 school year to 125 learners in the 2020-2021 school year. The number of white learners, however, became a majority of the student body (from 34.1% in 2019-2020 to 47.2% in 2020-2021); thereby, benefiting the most from the resources at Wilhelm. While the percentage of the student population was greater for Latinx learners, economically disadvantaged, and those receiving services through special education, it is deceptive as the number of students in each of those categories decreased by 22%, 16%, and 32% respectively. This follows a history of school board decisions involving attendance zones preserving the interests of white parents (see Castro, 2022; Mendez & Quark, 2022; Walsh, 2017).

This work contributes to the field of mathematics education and mathematics-related disciplines broadly as an example of how the myth of mathematics as objective and neutral served to reinscribe and reify racialization, segregation, and educational injustice in one central Texas district. Therefore, the results of this study provide space to discuss with teachers, policy makers, mathematics educators, and other researchers the power of supposed neutral quantification and how neutrality relates directly to the maintenance and perpetuation of white supremacy (see Espeland & Sauder, 2016; Zuberi, 2001). The results of the study provide advocates one way to understand and deconstruct the mathematical model of equality widely used in political discourse, drawing attention to the flattening done by quantitative models. This can illuminate tactics used by those with whiteness and prepare community actors with tools to challenge problematic mathematical models of equality. For example, Gómez Marchant et al. (under review) describe three community members' contestation of their school district's dehumanizing mathematical model of equality during school closure debates. Their strategies are multiple including both the outright refusal of the mathematical model presented as insufficiently attending to human affect and experience and the presentation of an alternative model with the inclusion of community selected quantitative measures. We call for more research on mathematics within the tapestry of civic engagement and what it could mean for families, community members, administrators and the professional development of teachers.

A critical race spatial perspective provided insight into the mathematical model of equality constructed by the white parents and their discourses about racialized space, in this case the attendance zone of Wilhelm Elementary. Tate et al. (1993) guided us in bringing to the forefront the racial components of the white parents' construction of a mathematical model of equality that emphasized their idealized (white) reality. Our focus on the included and excluded variables shows how the myth of mathematics as objective and neutral empowered their arguments to maintain Wilhelm as a white institutional space and legitimize their privilege as property owners and entitlements as taxpayers. Quantifiable measures were powerful in flattening the phenomenon being modeled; consequently, erasing and dehumanizing the families from the Figure Eight Apartments, but at the same time providing a cloak to white supremacy ideals in a guise of neutrality. The white parents in favor of shifting the color-line were ultimately successful in arguing their mathematical model of equality as being sufficient in modeling the phenomena and providing a solution to the issue.

Conclusion

The strategies used by those maintaining/increasing the whiteness of Wilhelm would not have been possible if mathematics was not promoted as an acultural, emotionless, objective subject. In endeavoring to solve the rezoning problem, these public comments dealt with the messiness and multidimensionality of their issue by presenting a new, single-dimension problem to the board. By reducing the students and their addresses, race, culture, class and more to a single variable—a number—a once complex concern now has a simple solution. The problem these parents attempted to solve is not the one originally presented. The public commenters highlighted in this piece used numbers of their own creation, and a solution for their abstraction may not be the best solution for the students, their families, or the community. A numerical version of events might be something a 'first grader could solve,' but a numerical version of events is entirely different from the complex and multifaceted issue of rezoning real students. In other words, mathematics was used to describe a phenomenon while purporting to have no connection to humanity (see also Bos, 1991; Rubel & McCloskey, 2021). These discursive moves dehumanized the learners and community of Figure Eight Apartments. Working together, each public comment made by white parents used mathematics as a way to grant permission to distance oneself from issues of race, class, and ethnicity (see also Ewing, 2018). Mathematics permits one to have no emotional connection to the erasure of the identities of the learners and community of Figure Eight Apartments. The myths about mathematics were weaponized by white parents to construct a new racialized space maintaining their dominance.

We conclude by turning to the question of resistance. Resistance was not absent. The South Asian and Latinx parents from the Figure Eight Apartments—along with some white allies—did construct their own counter-model, but it was not a mathematical model, and thereby, silenced by quantifiable measures (e.g., capacity, balance). Future work should explore the weaving of the construction of these models during civic engagement in a variety of political spaces. Mathematics has shown to be very powerful in political spaces. As a field, we must continue to develop a richer understanding of the discourses revolving around mathematics to counter white supremacist narratives and prepare teachers, administrators, and other researchers to resist and advocate for humanizing mathematical ideals.

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More Complexity, Less Uncertainty: Changing How We Talk (and Think) about Science

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ABSTRACT

This article focuses on the phenomenon of complexity in scientific communication. The article argues that shifting frames in science communication and the rhetoric of science from *uncertainty* to *complexity* can benefit audience understanding of scientific issues, and can also prevent bad-faith uptake of these issues that can be used to stoke political divisions. Such detrimental uptake happened with several science communication issues related to the COVID-19 pandemic. The rhetorical strategy detailed here balances the need to support mainstream science, while also incorporating some critique of it. Such a balance can be beneficial for the *ethos* of scientists and science communicators, and can result in more robust public engagement.

Keywords: rhetoric of science, science communication, rhetorical theory, uncertainty, complexity, stasis theory, COVID-19

Introduction

In early December, 2022, the Governor of the State of Florida, Ron DeSantis, filed a petition to the state Supreme Court to start an investigation into the safety and efficacy of the mRNA vaccines for COVID-19, or, as his office's press release put it, to "investigate crimes and wrongdoing committed against Floridians related to the COVID-19 vaccine" ("Governor," 2022). Despite credible estimates that COVID-19 vaccines had saved an estimated three million lives in the US alone (Trang, 2022), the Governor argued that "truthful communication" about the vaccines from federal government officials had been "obscured" ("Governor," 2022).

As some observers of the episode pointed out, public perception of the success of the vaccines had been complicated by "misleading messaging from public health experts and from the White House," which has "created confusion that's left fertile ground" for political acts like DeSantis's (Flam, 2022). This example, though, is indicative of a broader issue: that problems with messaging, and with science communication, in general, has been a prominent feature of public discourse around the COVID-19 pandemic. Especially in the United States, debates over vaccines, over masks, over pandemic mitigation measures, lockdowns, travel bans, school closures, and a variety of other, related, issues have splintered public opinion on these subjects, have led to divergent and contradictory state-level rules and responses to the pandemic, have caused overt mass protests, and overall have fragmented the country's political, social, and, especially, rhetorical landscape.

While the pandemic may seem a unique event, it is arguably just a significantly high-profile example of a recurring problem in science communication. That is, the pandemic has sharply illustrated the difficulty in effectively communicating and discussing complex subjects, especially scientific subjects. Such a problem is hardly new in the field of science communication itself, which has long stressed this difficulty of effective communication of complex scientific subjects, and which continues to grapple with this issue in contemporary science communication training (Bennett et al., 2020), and in science education more generally (Cook & Oliveira, 2015). Honing both scientists' and the public's ability to communicate and to understand difficult subjects in a robust way is widely seen as vital to public knowledge, and to a productive and healthy political sphere (Spoel et al., 2009).

Over the past two decades, for scholars in fields of science communication and the rhetoric of science, the particular problem of conveying nuance and complexity in scientific topics has often been framed as a question of communicating "risk," and more recently, as a question of communicating "uncertainty" (Walsh & Walker, 2016). Walsh and Walker, in their (2016) discussion of scholarship in this area, note that while there remains some "inconsistent treatment" and a lack of "principled, rhetorical frameworks" in the scholarship on uncertainty (and risk) (p. 71), the field has nevertheless gravitated toward the concept of uncertainty as a "boundary object" (p. 79), thus making it a key label for scholars studying the complex indeterminacies featured in scientific discourses.

In this article, I argue that the framing of "uncertainty" in scientific communication discourse, while still an important and useful lens for the field, can also have deleterious effects on science communication efforts and on public discourse itself. Both scientist communicators and science communication scholars, in this sense, can benefit from thinking of this thorny issue in scientific discourse as a problem of *complexity* rather than uncertainty. As I will explain in later sections, this terminological shift has significant implications, as changing the terms we use for a concept can fundamentally change our understanding of that concept, and change how we both talk and think about it. However, the main argument I make in this article is that scientific *uncertainties* have entailed *reductive* rhetorical treatments by scientists and scientific communicators, and that this aversion to complexity has harmed public discourse about these topics. Time and time again with scientific issues, communicators take a simplified approach to a scientific concept, either to facilitate public understanding, to promote agreement and spur action, or, from a rhetorical point of view, to address particular stasis points (more on this later) in order not to complicate the message. By doing this, though, communicators attenuate the message and cause the science to be understood in an oversimplified way. In turn, that science is unable to be engaged fully in public discourse, or worse, can be picked up and wrongfully appropriated to sow political dissent—such as with Governor Ron DeSantis's usage in the example that opened this article. The simplification of topics thus results in anemic science communication that can promote or exacerbate both political division and rhetorical disengagement.

In what follows, I first elaborate on the urge to be reductive, which is discussed to some extent in scientific communication scholarship, but is treated more specifically and extensively in scholarship on complexity and complex systems. Subsequently, I explore the implications of shifting terms as a rhetorical strategy, and specifically, of shifting from uncertainty to complexity. In the later sections of the article, I examine a few specific examples of science communication related to the COVID-19 pandemic, and argue that avoiding complexity in these cases harmed public understanding and created political division that might have been avoided with a different communicative strategy. As well, I discuss the implications of this argument for science communication and the rhetoric of science, and make the case that communicators cannot ignore certain stases of argument in their rhetorical approaches (cf. Ceccarelli, 2011). Ultimately, I argue that my approach strikes a middle ground between critique of scientific discourse and a full uncritical embrace of its conclusions.

Keep It Simple, Short-Sightedly

It should be noted that when discussing complex concepts, it is natural, and often even desirable, to be reductive. When there is a lot to be explained, or when there is a great deal of information about a topic—either because of its complexity or because of its breadth—it makes sense, rhetorically, to simplify it for an audience. Audiences can't always sit through book-length dissertations on a subject; they typically need (and respond more positively to) more concise, relatable, and often simpler explanations. As prominent science/technical communication journal *Technical Communication Quarterly's* recent call for papers put it, there is an urgent need for "durable" and "portable" approaches to science communication that can "resonate" with its audience (St. Amant & Graham, 2019). Reduction can, in this sense, be a viable strategy for this kind of effective communication that resonates.

Many specific tactics of reduction are often useful in a science communication context. As Groves (2021), writing on the difficulty of communicating with skeptical audiences about controversial scientific topics, argues, "esoteric language" can be "ostracizing for non-specialists" (p. 78). There are also "surprising [. . .] gaps in technical literacy" among laypersons that make even slightly-complex explanatory elements illegible to the general public—for example, Groves mentions the poor comprehension of logarithmic graphs that were widely used to communicate disease information during the COVID-19 pandemic (p. 78). As well, as Walsh and Walker document, "scientists who speak in public experience enormous pressure to eschew uncertain expressions" (p. 78), in part because it is perceived that highlighting nuance, or any kind of doubt, may diminish scientists' credibility, weakening their rhetorical *ethos* by implying that they may not be as sure about a topic due to its complexity. Even worse, many scientists feel that any admission of uncertainty can be weaponized by opponents or those who disagree with the speakers, and can thus backfire and create more doubt and dissent among audiences (pp. 78-79). Considering these communicative exigencies, simplifying communication is often a logical rhetorical path.

On the flip side, however, there is much literature on science communication that does push against this drive to reduce. The National Academy of Sciences' (NAS) report on "Communicating Science Effectively" (2017), for example, explains that:

> scientific "facts" not only are complex but also can often be interpreted in more than one way. Effective science communication conveys both complexity and nuance, and does so in a way that is understood by and useful to the audience to which it is directed. (p. 21)

Here, the NAS emphasizes what rhetoricians of science know "in their DNA" (St. Amant & Graham, 2019, p. 101): that facts are not pre-given unassailable bits of knowledge, but rather, are the product of debate and science that is, as the NAS puts it themselves, "seldom settled" (p. 21), and that can be interpreted differently by different audiences with differing viewpoints and in divergent contexts.

These competing ideas—that foregrounding complexity and uncertainty are seen as rhetorically problematic, and that avoiding complexity and uncertainty does not do justice to the malleable and complex nuance of actual scientific facts—is where the problem lies in science communication. The conflict between these ideas, in fact, continues to pervade contemporary scientific discourse, including discourse related to the COVID-19 pandemic. In general, despite the repeated urges of science communicators and some scientists, actual science communication—especially about fraught or contested topics—still tends to reduce, to its great detriment. The story about the Governor of Florida that opened this article is an excellent example of the problems with reduction. In that case, by emphasizing the benefits of the vaccine far more than the potential harms, the science communication *did* avoid complexity. As will be discussed later in this article, a more complex discussion of this issue would have bolstered the credibility of the scientists by creating an ethos of honest communicators, and could have better supported the point those scientists were trying to make in the first place—that the potential benefits of the vaccine outweighed the potential harms. Note that this conclusion wouldn't have been proven beyond a shadow of a doubt; as with all science, there is never complete certainty. But, as this article argues, foregrounding exactly this complexity is a more fruitful rhetorical strategy than is suppressing it.

The remainder of this article will address a specific terminological shift that would focus beneficial rhetorical attention on complexity, instead of shying away from it. This shift involves moving from a rhetorical framework that revolves around "uncertainty," to one that revolves around "complexity." Such a shift recognizes existing scholarship on complexity and complex systems, which has problematized the tendency to reduce in scientific accounts of complex phenomena. This shift also recognizes a systems view of knowledge as emergent from a context rather than as a fixed quantity. This systems perspective strikes a balance between the view of science communication as a critique of scientific epistemologies that hold knowledge as stable and fixed, and an embrace of the certainty of scientific conclusions (as discussed in Ceccarelli, 2011). As I suggest below, foregrounding complexity amounts to a kind of inoculation against future attempts to misuse or discredit scientific communications.

The Importance of Terminology

"Systems theory" is a broad term that represents a variety of theoretical perspectives on complex systems. One thing that is in common to the approaches to the topic, though, is an appreciation of the incredibly high level of complexity, interconnection, and uncertainty present in systems that encompass a high degree of variables and interrelations. Everything from traffic patterns to cities to ecosystems can, from a given systems theory perspective, be considered as a complex system. The scope of this article prevents further differentiation of the overlaps and divergences of these perspectives, but suffice to say, this body of theory deals with phenomena that are, much like many scientific topics, incredibly complex.

Kauffman's (1995) notion of an "ideal of reductionism in science" is a useful point to consider in this article, as it describes much of the tendency toward reduction that occurs when discussing complex and nuanced subjects. Much like scientific concepts, conceptions and descriptions of complex systems are plagued, he argues, by a tendency to reduce to the simplest terms—in the case of much of science, descriptions of complex multi-factor systems get reduced to descriptions of basic physical elements. As Kauffman puts it, the ideal is a product of the desire in the sciences to take complex phenomena such as "economic and social phenomena" and explain them "in terms of human behavior." In turn, that behavior is to be explained in terms of biological processes, which are in turn to be explained by chemical processes, and they in turn by physical ones. (p. 16)

Kauffman does allow that this ideal is to be "respect[ed]" for its many benefits, such as creating explanations that are understandable to a wider audience, and, facilitating scientific development predicated on mechanistic, physical processes (several scientific discoveries were born out of simplified versions of complex systems). However, Kauffman also argues that this kind of reduction can be quite problematic, as it can lead to an elision of the "multitudes" of nuances in complexity (pp. 16– 23). In Kauffman's account of complex systems, such descriptive and conceptual elisions deprive us from full appreciation and understanding of the intricacies of complexity. For science communication, then, the same impulse to reduce can have a similar detrimental consequence, but also, can create unintentional negative attitudes among audiences (as will be detailed later in this article).

It is important to distinguish here that I do not argue in this article for a reduction of *uncertainty*. To do so would be to commit an error that Walsh and Walker identify: to presume that uncertainty is "an epistemological gap that can and should be reduced to zero" (p. 72). Uncertainty, per se, is not a problem in science communication. What I argue needs to be revised are the terms we use to talk about uncertainty. As I explain in the next section, uncertainty can never be reduced, ultimately. In this complex systems point of view, knowledge and uncertainty are both properties emergent from a particular system configuration, and while uncertainty can be rearranged or superficially hidden, it does not completely disappear.

Instead, then, of advocating against the reduction of uncertainty in scientific communication, I advocate in this article against the aversion to *complexity* in scientific communication. Such a terminological shift, I argue, can have significant implications on our conception of science, on our audience's conception of science, and on the efficacy of science communication.

Such a terminological shift from uncertainty to complexity may seem trivial; however, as both rhetoric scholarship and scholarship on science communication frequently emphasize, even a small change in terms can have major effects. Kenneth Burke (1966) is perhaps the most well-known rhetorician to advocate for this position. His conception of "terministic screens" summarizes it best: as Burke puts it, every choice of terminology is both a "selection of reality" and a "deflection of [other aspects of] reality" (p. 45). Terminology shapes our perception in ways that bring some aspects of the world instead of others to the forefront of our thinking. For the success of science communication, this can make all the difference. As Groves (2021) argues, making sure to "choose [one's] words carefully" is among the most important strategies for effective communication (p. 78).

As the next section explores, choosing to frame—and to emphasize rather than avoid—the notion of complexity as central to science communication, instead of uncertainty, can have beneficial consequences for an audience's understanding of not only the topic at hand but also of complexity and uncertainty itself.

From Uncertainty to Complexity

The notion of uncertainty as a thing that can be reduced recalls the scholarly discussion of the concept of "ignorance," which is itself a contested concept. For example, many treatments of ignorance regard it as a quantifiable lack of knowledge about a stable reality (Peels, 2017, p. 2; Rescher, 2009) that can be overcome with the collection of more knowledge (McGoey, 2014). Such a conception of the world, and of knowledge itself, as stable and quantifiable is contrary to many rhetoricians' view (a view that can be traced back to the sophists) of knowledge as shifting and contingent. Other treatments of the concept of ignorance, in fact, explicitly contest its ability to be quantified (e.g., Treanor, 2013), and describe it as a perpetual condition that shifts when contexts shift (e.g., Mays, 2021).

The idea that ignorance can be remedied by acquiring more knowledge is flawed in the same way as is the (repeatedly debunked, but still pervasive) idea that giving the public more knowledge and "facts" about science will increase public support for that science—a theory known as the "deficit model" (see, for example: Bauer et al., 2007; Bennett et al., 2020; Besley & Nisbet, 2013; Dozier & Ehling, 1992; Fischhoff, 1995). In this sense, thinking of ignorance as a mathematical quantity, able to be reduced by adding more knowledge, is flawed in a similar way as is the consideration of uncertainty as a quantity that can be reduced by providing more knowledge, which is similar in turn to the idea that antipathy toward science is a deficit that can be reduced with more facts.

If conditions such as ignorance, uncertainty, or antipathy cannot be remedied by countering them with their opposite, it may seem counterintuitive that I am here calling for more complexity in scientific communications. After all, my point might seem to resemble the deficit model, which, as mentioned, calls for more information as a strategy to gain support among audiences. However, the argument here is not that more=better, it is that complexity is not quantifiable in the first place, but rather is a quality that should be emphasized and elaborated in scientific communication.

In fact, I argue that none of the qualities discussed here are mathematical at all. For example, as Hogg and Blaylock (2012) point out, feelings of uncertainty often result in the amplification of certainty elsewhere (p. xxiii), and not necessarily in a linear or mathematical way. Uncertainty, in this sense, is emergent from contexts; it is greater than the sum of its parts, and therefore it can evolve in ways that defy linear accounting. To be sure, there are things that we know we don't know (known-unknowns) that can in a sense be itemized. But, in the end, a mathematical view of uncertainty suggests that there is a stable quantity of knowledge in the world that can in theory all be acquired. And, according to this view, this knowledge is universally accessible and is the same in all contexts. This is not, I argue, how knowledge works. The view of knowledge as finite and sortable into discrete quantities of information directly opposes a view of knowledge as a shifting flux that changes as contexts change. This latter view is a rhetorical view, and it is what I advocate here. Uncertainty manifests in a communicative situation unpredictably, and it is not a stable or quantifiable thing that can be countered or eliminated.

Instead of treating uncertainty as something isolatable that can be reduced linearly, the proposal here is to use a different term altogether to talk about the concept. The use of "uncertainty" as a frame is a problem precisely because it suggests that it *can* be reduced or countered with certainty. This is borne out by the science communication literature on uncertainty, which, as Walsh and Walker point out (and as was discussed previously), is pervaded by the belief that it can be brought to zero. Using the term complexity, however, avoids this tendency. To be sure, one can still discuss complexity as being reduced, and this can slip into mathematical thinking—but I argue that complexity is less prone to be characterized as a quantifiable quantity since complexity is ever-present. Thinking of complexity as a quality to be embraced, rather than a thing that can be quantified, of course, is a key part of this terminological shift. But the argument here is that an issue that is complex cannot be magically made less complex, it can only be discussed in ways that work to reduce the impression of its complexity.

Overall, though, splitting hairs about the possibility of quantifying uncertainty versus complexity is not the point. While I argue that neither is mathematical, the important part of my argument here is aimed at science communicators themselves. That is: audiences don't need to grasp whether uncertainty is a quantity or not; I argue that the term complexity better captures the situation, and it is *less likely* to be seen as something that can be balanced with its opposite. Thinking of complexity and simplicity suggests a rhetorical emphasis rather than a mathematical one.

In addition to the argument that complexity as a frame is better suited than is uncertainty in discussions of scientific communication, this article has a second, potentially more important argument: that avoiding complexity in scientific communication is detrimental. The next section of this article explores this second premise.

Simple Messages Gone Wrong

The development of the COVID-19 vaccines was, by many accounts, an unprecedented scientific achievement. Overall, the trials for the Pfizer and Moderna mRNA vaccines showed remarkable promise: upwards of 90% efficacy of protection against symptomatic disease, with a relatively low rate of side effects (Flam, 2022). Given that success, it was scientifically reasonable that the vaccines should be made available to the general public as soon as possible, and that the public would be well served by taking the vaccine. Sure enough, in December 2020, within a year of the pandemic becoming widespread, scientists had developed, trialed, and produced a vaccine with significant efficacy against severe disease, and to some extent, against symptomatic illness as well (Trang, 2022).

Here, though, is where the communication problems came in. The speed of the vaccines' rollout, while a huge benefit in many ways, was also a potential drawback. The clinical trials for the vaccines were conducted over a period of several months (the Pfizer trial, for instance, followed 50,000 people from July to November in 2020), and largely didn't measure asymptomatic infections (Flam, 2022). So, that it was scientifically reasonable (i.e., that it was supported by credible scientific evidence) to support the vaccines is clear. That individuals would be well-served from taking the vaccine—given the current knowledge about the success rate of the vaccines, the potential lethality of the virus, its capacity to make people quite sick even if the sickness wasn't fatal (not to mention the at-the-time new reports of long-lasting COVID-19 symptoms)—is also clear. However, this does not mean that there were no complications, or, complexities, in that calculus. Because the vaccines were trialed over a period of months, researchers weren't able to determine the duration of protection, nor the nature of that lasting protection. Certainly, there were reliable predictions about duration based on settled immunological and epidemiological principles—specifically, that vaccines can provide lasting protection against severe disease because of our immune system's ability to remember the shape of prior infections, which the vaccines are able to mimic. But, this conclusion was not something that was proven by the trials themselves, and therefore, the nature of that lasting immune protection for this novel virus wasn't completely assured. Neither was there a way to predict all of the side effects—nor the specific extent of them—that would show up from vaccines, given that their administration would effectively be scaled up from a sample size of tens of thousands (in the trials) to the hundreds of millions (in the general population).

As it turns out, there were some complications and imperfections in the vaccine rollout. Side effects did occur (though relatively rarely), protection did wane (though mostly against symptomatic, but not severe, disease), variants did evolve that were more able to avoid the vaccine, and vaccine protection and disease severity was uneven across age groups as well as across other individual risk factors. Even that assessment over-simplifies the state of the science, which was continually evolving and subject to some debate—there was some disagreement even among scientists as to whether, for example, vaccine boosters were needed for all age groups (just to name one prominent disagreement) (Flam, 2022).

The main issue here is that the science of the vaccines, while suggesting one major important takeaway—that people should take the vaccine—was complicated; it was *complex*. The public messaging about the vaccine, however, largely eschewed this complexity in favor of that one takeaway. The Johns Hopkins Medicine information page (most recently updated in January of 2022) bears the title "Is the COVID-19 Vaccine Safe?," and has a communication style and strategy representative of mainstream science communication about the vaccines. In the first paragraph, the site answers that question:

> Yes. The two mRNA vaccines, Pfizer and Moderna, authorized by the U.S. Food and Drug Administration (FDA) and recommended by the Centers for Disease Control and Prevention (CDC), are very safe and very good at preventing serious or fatal cases of COVID-19. The risk of serious side effects associated with these vaccines is very small. (para. 1)

Further down the page, the site goes on to give more information about vaccine safety (it says that the vaccines are safe), reported side effects (it says that these are rare and only occur in certain populations), risk of allergic reactions (it says that if you are allergic to injectables, you should talk to you doctor, and all other individuals are safe), why the vaccine was developed so quickly (the mRNA technology made this possible), whether one has to wear a mask (this is somewhat ambiguous, but the gist is that they recommend it), and their ultimate recommendation (yes, one should get the vaccine). Nothing on the page is wrong, nothing is false, and nothing is overtly misleading about this information. It is clear science communication that shows no uncertainty, and projects the utmost confidence in the message.

I argue, though, that this kind of communication is precisely what led to the divergent interpretations of different audiences, and to the aggressive resistance to the dominant official messaging about the vaccines. It is also what allowed political figures like Florida Governor Ron DeSantis to exploit this divergence for political gain, as he did in filing his petition to investigate the safety of the mRNA vaccines.

This kind of messaging—one that reduces uncertainty and projects confidence—was the norm for several related issues during the pandemic. On masks, the questions of whether and to what extent they work, and under what conditions, were largely answered in simple, straightforward, and reductive terms, with inconsistencies minimized. On social distancing, the questions of how much space is enough, and whether and to what extent the environmental contexts matter, were largely eschewed in favor of simple admonishments to keep one's distance (disapproving pictures of crowded beaches, for example—often taken using telephoto lenses that exaggerated the closeness of the crowd—became a staple on social media sites).

This kind of communicative strategy, of course, is decried by many science communication scholars and rhetoricians of science, who argue that reducing uncertainty is not a viable way to gain adherence to one's arguments. Instead of thinking in terms of uncertainty, though, these communicative situations could have been better addressed by *explicitly embracing their complexity*. Not necessarily by expressing uncertainty—as in, "we don't know whether [for instance], vaccines will work as we expect." But rather, to actually broadcast the *specifics* of why the vaccines were being promoted, and why there might be complications of the vaccines, especially down the line. Giving the public a view "under the hood" of the science, showing them how and why the controversies among scientists are happening (if they are happening in a substantial way), and overall, not taking shortcuts in discussions of the complexity of the situation—and even emphasizing that complexity, *as* complexity, not as uncertainty—could serve several practical benefits for the public's understanding of, and reaction to, that science.

Shifting Stases or Skipping Steps? Implications and Benefits of Embracing Complexity

Rhetoric scholar Leah Ceccarelli (2011) has famously advocated that scientists and science communicators not shy away from acknowledging scientific controversies. Instead of dwelling in those controversies and ceding ground to their detractors, though, Ceccarelli argues that these communicators should emphasize that these debates are "fairly settled" (p. 217). In this way, she argues, science communicators can shift the debate from those already (fairly) settled stasis points to ones that are more important to their objectives. To briefly explain this idea: points of stasis are rhetorical concepts that refer to the *kinds* of arguments one can have about a subject. The stasis of "fact" (i.e., arguing about what are the established facts of a situation) is considered a lower, and more primary stasis, whereas the stasis of "policy" (i.e., arguing about what should be done about those facts) is considered higher, and more secondary. As Walsh (2009) explains, public debates about science tend to have an "upward pull" on the stasis points (p. 42), wherein matters of fact (e.g., do vaccines help prevent disease) are inexorably drawn toward—and conflated with—matters of policy (e.g., should we mandate the vaccines for all United States citizens).

Ceccarelli argues that science communicators should explicitly shift to those higher stasis points, effectively bracketing off the questions of fact as already settled, so as to avoid getting into thorny debates about facts that obscure these communicators' purpose—which is typically to suggest a course of action, and to determine policy (pp. 212-13). As she writes (Ceccarelli's point was about global warming debates, but the same applies to pandemic debates):

> For example, arguers who disagree about whether global warming is happening might find a point of contact in support of a policy to promote the development of alternative energies, regardless of where they stand on the technical issues surrounding climate science. (p. 213)

This strategy, though, directly contributes to the divisive nature of these debates, precisely because it suppresses complexity. These matters of fact are, for the opponents, *not* "settled," and so they aren't able to get past those stasis points to argue about policy. If I don't agree with you that global warming is happening, I'm unlikely to agree with you about any policy related to that issue—in other words, contrary to Ciccarelli's assertion, interlocutors are often not able to get past what is often a complex

”facts” debate and stasis point. Similarly, in the case of the pandemic debates, if I don’t agree with you that vaccines are safe, I’m not going to agree with you about the scope of vaccine mandates. In fact, I argue that skipping steps (and stasis points) here exacerbates these disagreements over latter stasis points, because the skipping over of the matters of fact debates is perceived as patronizing and duplicitous. If communicators want to establish trust, they need to acknowledge the complexity of the lower stasis debates. Even if, for them, those questions are “fairly settled,” being transparent as to how they became settled (i.e., walking the audience through the issue), and even acknowledging that there is some dissent (and being clear as to why that dissent is overruled by the consensus view) would go a long way toward establishing that trust in the conclusions of these communicators. Treating audiences as capable of handling complexity avoids them feeling as if they are being talked down to.

It is no coincidence that global warming debates took quite a long time to get to substantive policy actions (and there is still, to be clear, a long way to go). The dynamics of the debate around the facts had to play out first, before policy could even begin to be agreed upon (again, to be clear, total agrement hasn’t happened yet, but there is more movement on policy related to global climate change than there was when Ceccarelli was writing). For the pandemic, there wasn’t nearly the same timescale to get past matters of fact to get to matters of policy: public pandemic policies needed to be decided very quickly, almost on the spot. In communicating the rationales for these policies, an emphasis on the complexity of the science, then, could have at least gotten audiences to understand that there wasn’t a perfect solution, but that the vaccines, for instance, were, on balance, beneficial. This kind of discussion of complexity could have foregrounded the point that yes, there *was* the potential for harm from the vaccine, but that the CDC and the federal government were recommending its use because the potential for harm from *not* taking the vaccine was greater than the potential for harm from taking the vaccine.

Of course, one could argue that this strategy brings us back to a focus on relative risk, which is often paired with uncertainty in science communication scholarship (e.g., Beck, 1999; Grabill & Simmons, 1998; Sauer, 2003). But here, the choice of terminology is important. The rhetorical strategy should be to emphasize the complexity of the debate, not to emphasize the risks, nor the uncertainty of the scientists. In part, this strategy is about ethos-creation for scientists and science communicators. By emphasizing complexity, and by being transparent about the immense complexity of the situation, there is less of a chance that they will be perceived as dishonest, or as acting in bad faith.

Such a negative reaction is precisely what the DeSantis petition depends on: the public feeling like they are being patronized, at best, or lied to, at worst. The Johns Hopkins site about vaccines conveys a message that getting the vaccine is simply a good thing to do. There is no mention, though, that the trials were on a time scale that meant that long-term side effects could go undetected (even if the science suggests this is unlikely). Nor is there specific and candid acknowledgment that infections still happen, nor that there is debate over the efficacy of additional vaccine booster doses for some cohorts. This information is certainly available to readers of the Johns Hopkins site, but not from the site specifically. A significant potential rhetorical effect of the rhetorical strategy employed by this page is to make audiences feel as if they can’t handle complexity, and that they need to be spoon-fed curated information that leaves out any negativity.

Supporting Science, But Not Uncritically

Embracing complexity, then, avoids the kind of subtle evasion that arouses suspicion, distrust, and animosity. Moreover, using complexity as the primary frame for this rhetorical strategy applies in situations where the issue is not quite uncertainty, per se, nor is it “risk.” Complexity is broadly applicable, and for the most part, has much less negative connotation than does uncertainty (or risk). Complexity is—as it should be—a good thing, and both scientists and science communicators should operate from that premise.

Ceccarelli's discussion of science controversies and global warming also argues for a "supportive orientation toward mainstream science" (p. 200), which can counter the danger that overly critical rhetoricians of science have opened up an avenue for bad actors to exploit that skepticism and to discredit important—and potentially life-saving—scientific progress. Embracing complexity, though, is not the same as being critical of mainstream science. Rather, this embrace foregrounds the nuance of complicated subjects, and entails that we not wholeheartedly accept mainstream science, but neither should we mindlessly discount it either.

The embrace of complexity detailed here also has the potential benefit of exposing publics to the very same dissenting arguments and critique that might be used by the abovementioned bad actors, but also, exposing these publics to the precise counter-arguments that were used by scientists to debunk those dissenting views. In this sense, embracing complexity, and walking audiences through the scientific debates that got scientists to their current consensus view, constitutes a kind of inoculation against any bad faith attempts to use these dissenting views as a rhetorical wedge in public opinion. If the public already knows why the dissents were overcome, there is no potential to frame those dissents as having been suppressed by a nefarious group of mainstream scientists—which is precisely what DeSantis's complaint alleges.

This approach, of course, wouldn't have guaranteed that the vaccines would have been universally accepted. However, providing transparency by embracing complexity in the communication of the scientific issues surrounding the pandemic could have elevated the complexity of public discourse about vaccines, and avoided at least some of the stark disagreements and extreme politicization of practically every issue related to the pandemic. While such an embrace is not a cure-all, it could be a beneficial strategy for science communicators, especially when they are dealing with concepts or issues that are, in fact, quite complex. It is commonly believed that in the face of uncertainty, people try to find certainty. In the face of complexity, though, they may be more willing to accept it.

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Elementary Students' Representations of Scientists and Engineers: Disciplinary Conflations and Confusions Before and After a Semester with an Engineer

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ABSTRACT

Current U.S. science education reform efforts call for engineering to be included as part of science instruction at all grade levels. As students experience engineering instruction alongside science, an important question is how students conceptualize the nature of those two fields, and especially the extent to which they differentiate science and engineering. In this study, grades 3-5 students in thirteen classrooms participated in engineering activities as part of their science instruction for a 16-week semester. During that semester, students also interacted with an engineering graduate student who regularly visited the classroom to plan and implement science and engineering activities. Before and after that semester, we analyzed students' responses on the Draw-A-Scientist Test and the Draw-An-Engineer Test. Unlike prior analyses of those instruments, our approach focused on the alignment of students' drawings with the activities and processes of professionals in those two fields. At the time of the pretest, students' representations of scientists and engineers were often misaligned with the targeted fields, and overall alignment improved modestly from pretest to posttest. An important question was whether students would conflate the fields of science and engineering, especially after experiencing engineering as part of their science instruction. Although some evidence of conflation exists, we did not find an increased prevalence of conflated drawings from pretest to posttest. The results indicate that the inclusion of engineering activities in the science classroom does not necessarily lead students to confuse science with engineering, but also that significant work needs to be done to help students accurately conceptualize the nature of work in those fields.

Keywords: nature of science; nature of engineering; elementary education; engineering education; draw a scientist; draw an engineer

Introduction

An enduring goal for K–12 science education efforts has been to help students better understand the nature of science, which includes an understanding of what science is, how scientific knowledge is developed, and how scientists do their work (Clough, 2006; Lederman & Lederman, 2014; McComas & Clough, 2020). Unfortunately, many students have misconceptions about science and scientists, likely developed from exposure to inaccurate portrayals found in science instruction, curriculum materials, and the media (Clough, 2006; Finson, 2002; Schibeci, 1986). For decades, a common method of probing young students' conceptions of scientists has been the "Draw-A-Scientist Test" (DAST), which is an open-ended instrument in which students draw a scientist engaged in scientific work. The task was originally developed by Chambers (1983) and has been modified multiple times, often by changing the prompt or adding additional written items to the task (e.g., Christidou et al., 2016; Farland-Smith, 2012; Losh et al., 2008). A long line of research has indicated a consistent stereotypical view of scientists. With minor variations, the "standard" image of a scientist produced by young students is a white male, robed in a lab coat, engaged in mysterious laboratory work with bubbling chemicals (Barman, 1999; Chambers, 1983; Christidou et al., 2016; Flick, 1990; Finson, 2002; Kelly, 2018; Mead & Metraux, 1957; Schibeci, 1986).

A complexity that has been introduced by current science education reforms, such as the Next Generation Science Standards in the United States (NGSS; NGSS Lead States, 2013), is that engineering is expected to be taught alongside science (Moore et al., 2015; National Research Council [NRC], 2012). As engineering has become more common in K–12 classrooms, interest has grown in K–12 students' conceptions of engineering (e.g., Capobianco et al., 2011; Lachapelle & Cunningham, 2014; National Academy of Engineering [NAE] & National Research Council [NRC], 2008). To investigate students' conceptions of engineering, Knight and Cunningham (2004) modified the DAST to develop the "Draw-An-Engineer-Test" (DAET), which prompts students to "Draw an engineer doing engineering work" and also provides space for respondents to explain what the engineer is doing. Studies using the DAET have found that students tend to erroneously represent engineers engaging in manual labor tasks rather than the more "mental" process of technological design and development (Capobianco et al., 2011; Chou & Chen, 2017; Fralick et al., 2009; Knight & Cunningham, 2004; Rynearson, 2016; Weber et al., 2011). Although misunderstandings have been documented using the DAET, researchers have not yet found a "standard" stereotypical image for engineers like that found for scientists (Capobianco et al., 2011).

The erroneous conceptions revealed by students' representations of scientists and engineers are concerning for multiple reasons. Cultivating student interest in science and engineering is a common educational objective, but such efforts are undermined when students misunderstand the nature of those disciplines (American Society for Engineering Education [ASEE], 2020; Finson, 2002; Montfort et al., 2013; NAE & NRC, 2008, 2009; Reinisch et al., 2017). Students who, for instance, think of science mostly as a solitary activity of mixing chemicals in a laboratory might wrongly conclude that science is of little interest to them (Luo et al., 2021). Similar issues would likely arise for students who associate engineering mostly with car repair (ASEE, 2020; Capobianco et al., 2011; Fralick et al. 2009). A broader concern is that because engineering is now often being taught in the science classroom, students might not develop accurate distinctions between the two fields, despite their substantial differences. McComas and Nouri (2016) argue that the way in which engineering is treated within the NGSS, as well as recent efforts to promote STEM integration (e.g., Kelley & Knowles, 2016; Roehrig et al., 2021), is part of the problem:

> Science and engineering are quite distinct disciplines both philosophically and practically. Therefore, we should be much more focus directed to help all those involved with science teaching understand the important engineering/science distinction. With this in mind, it is problematic that the Nature of Science distinction between science and engineering appears only twice in the NGSS. Also, many have suggested that blending science and engineering, and even adding the other two parts of STEM in the elementary grades, is a good idea. However, we see no note of concern in the NGSS that learners – particularly those in the early grades – understand the separate roles of science and engineering (McComas & Nouri, 2016, p. 571).

Why exactly *should* students differentiate science and engineering, given their clear relationship? Of course, pointing out the differences between science and engineering is not a denial of the connections and similarities between the fields. The reason to highlight distinctions, as well as connections, is that both are vital to the longstanding efforts to promote scientific and technological literacy. An essential part of those literacies is an understanding of the roles that scientists and engineers play in society (ASEE, 2020; International Technology and Engineering Education Association [ITEEA], 2020; NGSS Lead States, 2013). To imply that there is no difference between the roles of scientists and engineers is contrary to a foundational educational goal. Put another way, ignoring or eliding the differences between science and engineering misrepresents both the nature of science and the nature of engineering – both of which are valued parts of STEM education (McComas & Burgin, 2020; NAE & NRC, 2009; NRC, 2012; Pleasants & Olson, 2019a).

Prior studies indicate that targeted interventions can reduce students' stereotypical images of scientists, particularly those that provide students with more accurate exemplars of scientists and scientific inquiry (Christidou et al., 2016; Finson et al., 1995; Flick, 1990; Huber & Burton, 1995; Sharkawy, 2012). Similarly, interventions that provide students with authentic engineering experiences and more accurate examples of engineers can reduce certain erroneous images on the DAET (Lachapelle & Cunningham, 2007; Rynearson, 2016). However, few studies have compared students' representations of both scientists and engineers, and none have explored how science instruction that incorporates engineering affects students' representations of both fields. In general, investigations of how students differentiate science and engineering are lacking, which is problematic given the concerns raised about disciplinary confusions (McComas & Burgin, 2020; McComas & Nouri, 2016; Zeidler et al., 2016).

In one of the few studies to examine students' views about science and engineering, Karatas et al. (2011) used interviews as well as a drawing task to investigate sixth-grade students' understanding of the nature of engineering. The interviews they conducted with students included a question probing students' thinking about the difference between engineering and science. They found that many of the students confused science and engineering. Although some articulated differences between the fields, many did so by indicating that scientists and engineers are both researchers, but that they research different things: scientists study natural things, whereas engineers study machines. A study by Fralick et al. (2009) also provides some insight into how students think differently about scientists and engineers. They gave the DAET and DAST to a large group of middle school students and compared students' representations across the two instruments. The main difference they found between students' representations was that scientists were more often shown indoors working in a laboratory, whereas engineers were shown outdoors doing manual labor. A limitation of their study was that the DAET and DAST were completed by different groups of students. A further limitation of both studies is that neither examined how students' views *changed* as a result of instruction.

Missing from the literature are comparisons of how the same group of students represent scientific and engineering work, the extent to which those representations accurately reflect the differences between those fields, and the ways in which those representations change after instruction. The goal of the present study is to address those gaps in the literature. In this study, grades 3–5 students' drawings of scientists and engineers were examined before and after a 16-week semester during which they received science instruction that included multiple engineering experiences, as advocated by the NGSS. The study was conducted within the context of a professional development project in which participating teachers were teamed with engineering graduate students who supported the integration of engineering content and activities into science instruction. The students in the study therefore not only experienced science and engineering instruction, but also interacted with an engineering expert.

In this study, we examine changes that occurred in students' conceptions of scientists and engineers over the course of the project, focusing on the extent to which students' representations of scientists and engineers accurately reflect each field. In the following section, we describe the framework that we used to conceptualize the distinctions between science, engineering, and technology, which we then used as a basis to analyze students' representations. Our study seeks to address the following research questions:

1) How do grade 3-5 students' representations of scientists and engineers align with scientific, engineering, and technological activity both before and after receiving science and engineering instruction?
2) To what extent do grade 3-5 students' representations demonstrate conflation between the work of scientists and engineers before and after receiving instruction?

Conceptual Framework: Differentiating Science, Engineering and Technology

Because our study seeks to examine the extent to which students' representations of scientists and engineers accurately represent work in those fields, clarity is needed regarding the distinguishing characteristics of those fields. This task also requires that attention be paid to the field of technology, given the many intersections and interactions that exist between technology, engineering, and science. Much can be said about the interactions and similarities between science, engineering, and technology, but our primary goal in this section is to focus on *distinctions,* given the research questions we seek to address. The approach we take to drawing distinctions is to focus on the divergent goals and purposes of science, engineering, and technology (McComas & Burgin, 2020; Pleasants, 2020; Vincenti, 1990).

Technology occupies an unusual status in that it is less a field or discipline than it is a set of products, systems, and processes (Dusek, 2006; Mitcham, 1994). Taking an approach common in the philosophy of technology (e.g., Kroes, 2012; Mitcham & Schatzberg, 2009), we adopt a broad definition of technology that includes all human creations that are oriented toward practical purposes. In terms of goals and purposes, therefore, technological activity includes all human activity oriented toward the creation and maintenance of human-made products and systems. The field of engineering is similarly concerned with the design and development of technological systems (Kroes, 2012; Mitcham, 1994). The field of engineering is therefore a subset of the broader array of technological activities. Many technological activities, of course, are not engineering; engineers focus on design and development of technological systems rather than the actual work of production, repair, or maintenance (Bucciarelli, 1994; Dym & Brown, 2012; Kroes, 2012; Mitcham, 1994). A car mechanic, therefore, is engaged in technological activity, but not engineering. To enable further design and development, engineers also engage in "engineering science," research work that produces knowledge about technological systems (Bucciarelli, 2009; Houkes, 2009; Mitcham & Schatzberg, 2009). Although many technologies are designed and developed by engineers, engineering is not the sole source of novel technologies; invention is not synonymous with engineering (Newberry, 2013). A craftsperson who creates a novel piece of furniture or a tinkerer who creates a new tool might be said to have engaged in invention, but not engineering.

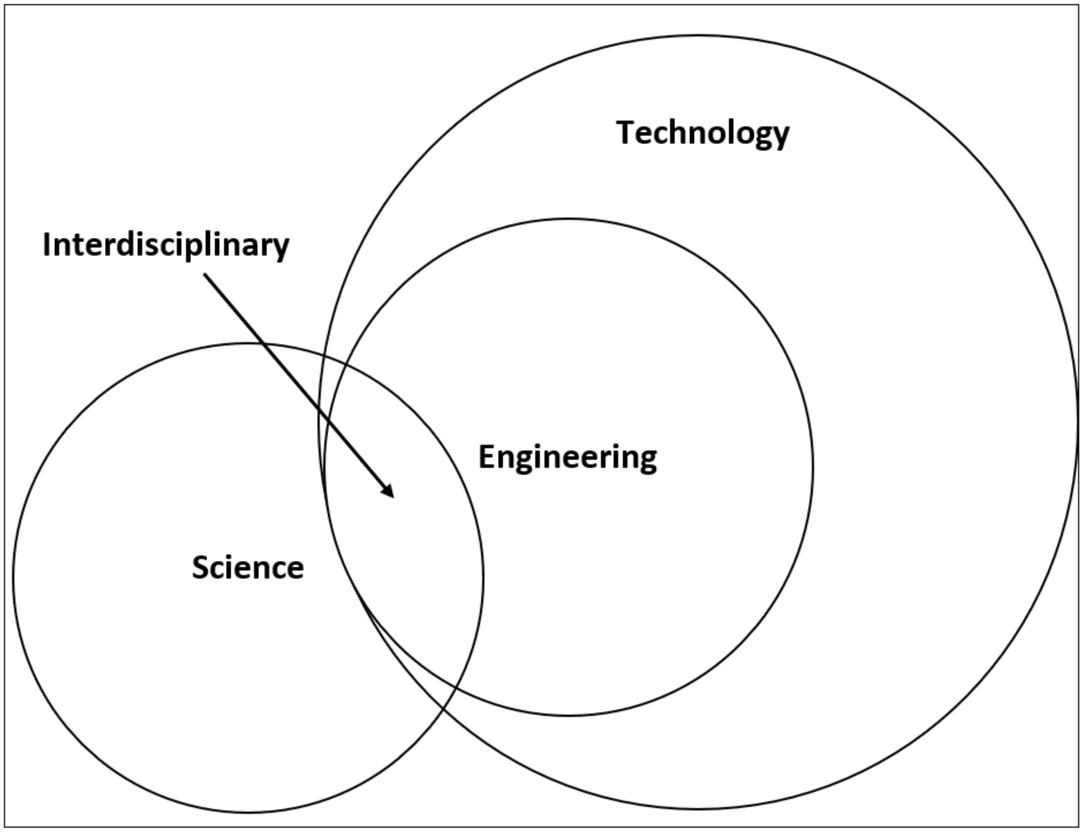
In contrast to engineering and technology, the goal of science is primarily to produce knowledge of the natural world. In conducting their work, scientists of course make extensive use of technological equipment and thus frequently repair, refine, and develop their instruments in order to advance their research into the natural world (Ihde, 2009; Pitt, 1995). Scientists might also create things that did not exist before, as when a chemist synthesizes a novel substance in a laboratory. Yet even when scientists engage in technological activities, their work differs from that of an engineer or technologist in that for the scientist, the goal is to produce scientific knowledge rather than a novel technology (Vincenti, 1990). Galileo and Newton both made considerable contributions to the development of the telescope, but for both scientists the telescope was foremost an instrument of scientific knowledge production (Pitt, 1995). Similarly, when a chemist produces a chemical in the laboratory, they do so in order to advance chemical knowledge. When a chemical engineer produces a new chemical, they do so for practical reasons; perhaps the chemical has properties that are useful for the industry in which they work, and is therefore highly marketable, provided it can be produced cheaply enough.

The distinctions drawn above are necessarily simplifications of what is, in reality, a more complex state of affairs. Scientists are not blind to practical considerations, and much scientific research is done with potential applications in mind. Scientific and technological goals can also be simultaneously pursued within the same project (Vincenti, 1990; Volti, 2005), as illustrated by "big" projects such as the development of the atomic bomb (Hoddeson et al., 1993). Even if disciplinary borders are blurry, rather than sharply defined, important differences nevertheless exist between science, engineering, and technology (Pigliucci, 2013; Pleasants & Olson, 2019b; Vincenti, 1990). While not a philosophically perfect method of demarcation, a focus on the divergent goals of the fields is conceptually useful, particularly from an educational perspective (McComas & Burgin, 2020; Pleasants, 2020). Understanding the goals of science and engineering allows learners to better comprehend the different, though at times overlapping, roles played by each within society – an important component of scientific and technological literacy (ASEE, 2020; ITEEA, 2020; NRC, 2012). Differentiating the fields based on their goals is also likely to be more comprehensible to students than focusing on more subtle epistemological or methodological distinctions.

Our conceptual framework is summarized in Figure 1. Engineering is shown as a subset of technological activity. Science is shown as distinct from engineering/technology while also allowing for an overlapping interdisciplinary space.

Figure 1

Conceptual Relationships Between Science, Engineering and Technology



Applying the Framework to Student Representations

We developed the conceptual framework described above to ground our approach to analyzing students' drawings and support the construct validity of our methods (Schreier, 2012). As described in the following section, we used it as a starting point to categorize students' representations of scientists and engineers and determine the extent to which their representations were accurately aligned with those fields. Because our research questions also seek to identify instances of conflation, within our framework, we operationally define "conflation" as an instance where a representation portrays scientific work as engineering or vice-versa. For example, a portrayal of a scientist engaged in technological design or development in pursuit of practical goals, rather than for the goal of knowledge production, would qualify as an instance of conflation.

In developing our conceptual framework, we necessarily attended to many nuances and complexities that inevitably arise when trying to draw distinctions between fields that are as interconnected as science, engineering, and technology. Attention to complexity is crucial when establishing conceptual categories from a research perspective. However, we do not imply that *students* ought to understand the disciplinary landscape at that level of detail, particularly at the elementary level. Indeed, there is not yet consensus regarding the depth to which students at varying age levels ought to understand multiple aspects of the nature of science, engineering, and technology (Abd-El-Khalick, 2014; ITEEA, 2020; Pleasants & Olson, 2019a). Our conceptual framework, therefore, is not necessarily a prescription for what students *ought to learn*. Rather, it is a tool to make sense of what students' views *currently are*.

Methods

This study took a sequential mixed methods approach in which qualitative data (students' drawings of scientists and engineers) were transformed into numerical data for quantitative analysis

(Teddlie & Tashakkori, 2009; Fetters et al., 2013). The transformation process took place via a qualitative content analysis (Schreier, 2012). This methodology allowed us analyze a large data set of student drawings and identify patterns and differences between pretests and posttests as well as between the DAST and DAET.

Context of the Study

The study took place as part of a broader teacher education and professional development research project aimed at supporting the incorporation of engineering into elementary science instruction. Student teachers and their cooperating teachers, all in grades 3–5 classrooms, participated in the project and each pair was teamed with an engineering graduate student (called "engineer" hereafter) to form a triad for a 16-week semester. The engineers spent one full day per week in the classroom with their triads to help plan and implement science and engineering lessons. The triads completed a 2-day professional development workshop before the semester, a 1-day workshop midway through the semester, and were also provided ongoing support from project staff. Additionally, the engineers also participated in an on-campus course that met weekly to support their work in schools, provide ongoing instruction in pedagogy, and help them navigate their roles as content experts within their triads.

The primary objective given to participants was to modify their science curricula to better support students' learning of science concepts while also incorporating engineering experiences into instruction. The project workshops modeled inquiry-based science and engineering design activities, with emphasis on the ways that engineering can be meaningfully connected to science in the classroom. The project gave participants access to *Engineering is Elementary* curriculum materials developed by the Boston Museum of Science (2007), and the school district provided teachers with *Full Option Science System* (FOSS) curriculum (Lawrence Hall of Science, 2015). Teachers were not required to use the provided curricula; rather, they were given discretion to modify and develop materials suited to their individual contexts.

The project provided participating teachers with several experiences to help them better understand the relationship and difference between science and engineering as well as accurately communicate the disciplines to their students. The pre-semester workshop included a 30-minute presentation by an engineering faculty member on the nature of the engineering discipline and the ways that scientific knowledge is used in engineering. It also included a 60-minute session during which a science educator addressed the nature of science and ways that science and engineering are connected, yet different. After participants experienced the modeled science and engineering lessons, the facilitators also raised the issue with participants that students might conflate the two disciplines and encouraged participants to address that issue with their students. Given their expertise, the engineers were called upon to communicate the characteristics of engineering to help students see the ways in which it is similar to, yet different from, science.

Triads were not required to incorporate specific engineering activities into their classrooms or follow a prescribed approach to instruction. Because triads were situated in different grades and different schools, the specific science content they addressed and the learning activities that they implemented varied. Nevertheless, there was a relatively consistent pattern of implementation that occurred across the triads in terms of how they implemented engineering activities and how the engineers engaged with the students.

Triads' science instruction typically followed the FOSS curriculum aligned to their grade-level science standards. During science instruction, the engineers assisted the teachers in modifying the FOSS activities to be more engaging for students, and also often co-taught those activities. During science, the engineers were therefore positioned primarily as "science co-teachers" or "science experts" in the classroom rather than "engineering experts." The triads typically incorporated engineering into their science instruction by placing an engineering design challenge at the end of one of their FOSS units. The engineers took substantial roles in terms of conceptualizing and planning those activities, linking them to the science units being taught, and implementing the activities with students. All triads implemented at least one engineering design challenge with their students, and most implemented between two and four over the course of the semester. With few exceptions, the design challenges followed a common structure: students were tasked with designing a technology to solve a problem, given a set of criteria and constraints, then worked in teams to generate ideas, test those ideas, and improve them.

Observations of triads' instruction indicated that explicit conversations about the nature of engineering (or the nature of science) were rare during engineering design challenges (Pleasants, 2018; Pleasants & Olson, 2021). The only substantial amount of time that triads typically devoted to explicitly addressing the nature of engineering occurred when the engineers initially introduced themselves to their students. During their first visits to their classrooms, the engineers typically delivered a presentation on their field of engineering, what engineers in their field do, and what projects they were working on. Those presentations often included some discussion of how the engineers used scientific knowledge to do their own work, but rarely overtly addressed how their work differed from science.

Instruments

The instruments used in the present study were versions of the DAST (Chambers, 1983) and the DAET (Knight & Cunningham, 2004). Because we sought to compare responses on the two instruments, we used forms of the DAST and DAET that were as similar as possible. In our version of the DAST, respondents were tasked with drawing "a scientist doing science," and below the drawing area, space was provided to explain in words what the scientist was doing. The DAET was identical to the DAST, except that it asked the respondent to "Draw an engineer doing engineering work." Like the DAST, the DAET provided the respondent with space below the drawing to explain what the engineer was doing. The exact forms of the DAST and DAET used in this study are provided in the appendix.

Participants and Data Collection

Triad members administered the DAST and DAET to their students at the beginning and end of their semester of participation in the project. Electronic copies of the DAST and DAET were made available to the triad members, along with instructions for their administration, and those who chose to use them did so as part of their normal classroom instruction. Although the authors were not present during administration, triad members indicated that they gave the DAST and DAET in a manner consistent with the guidelines of the instrument developers (Knight & Cunningham, 2004), taking about 10 to 15 minutes to do so (no time limits were imposed by the triads). In several cases, we received data from triads who modified the instruments or did not follow the guidelines for their administration; data from those triads were not included in the study. Both the DAST and DAET were administered as pretest and posttest; pretests occurred before or on the day of the engineer's first visit to the classroom, and posttests on or after the day of the engineer's final visit. Triads sent copies of completed responses to the researchers after de-identifying them by replacing student names with ID numbers so that students' pretest and posttest responses could be matched.

We obtained complete data sets from 13 different triads located in 10 different schools, all within the same large, diverse urban school district located in the Midwestern United States. Demographic information about the students, triad members, and schools are provided in Table 1. In total, 280 students completed one or more drawings, but some student responses were missing, either because they did not complete a drawing or produced a drawing that was too unclear to interpret. After accounting for missing data, 244 DAST pretests, 236 DAST posttests, 252 DAET pretests, and 232 DAET posttests comprise the data set for the study.

Table 1

Overview of Participant Demographics

			School		Engineer		Cooperating Teacher	
Triad	**Grade**	**Class Size**	%Nonwhite	%Free/Reduced Lunch	Field	Gender	#Years Teaching	Master's Degree
15	3	25	48.6	55.0	Mechanical	M	21	Yes
22	3	21	56.0	85.5	Materials	F	14	Yes
24	4	23	48.6	55.0	Mechanical	M	22	Yes
25	4	25	43.1	39.7	Mechanical	M	8	No
32	3	24	21.6	42.1	Mechanical	M	11	Yes
34	5	24	86.8	38.6	Mechanical	M	10	Yes
41	3	25	46.7	76.4	Materials	F	6	Yes
42	3	24	25.6	24.6	Chemical	M	15	Yes
51	4	17	43.1	39.7	Materials	F	6	No
52	5	23	56.0	85.5	Chemical	M	5	No
53	4	24	38.4	72.9	Agricultural	F	5	No
56	4	21	69.0	80.4	Mechanical	M	14	No
58	4	24	69.0	80.4	Biochemical	F	10	No

Analysis

A qualitative content analysis approach (Schreier, 2012) was used to analyze the data, transforming the qualitative DAST and DAET responses into quantitative data via a coding process. The overarching objective was to create a coding system that could be applied to either a DAST or DAET response to indicate its alignment with science, engineering, or technology (see Figure 1).

Coding systems have been previously developed for the DAST (e.g., Farland-Smith, 2012; Finson et al., 1995) and for the DAET (e.g., Capobianco et al., 2011; Thomas et al., 2020; Weber et al., 2011). Approaches to assessing DAST responses have typically focused on quantifying the number of elements in a drawing that reflect stereotypes about scientists (Chambers, 1983; Finson et al., 1995; Flick, 1990; Huber & Burton, 1995). More recent coding systems characterize the activities represented in the drawings rather than just the number of stereotypical elements (e.g., Farland-Smith, 2012). Approaches to analyzing the DAET also aim to describe the activities represented in the responses, along with descriptive features such as the gender of the engineer, the tools used by the engineer, and the skin tone of the engineer (e.g., Capobianco et al., 2011; Fralick et al., 2009; Thomas et al., 2020; Weber et al., 2011). The coding system used by Fralick et al. (2009), for instance, describes the portrayed activities with codes such as observing, experimenting, explaining, and designing.

Although existing coding systems have utility, none were well-suited to addressing our research questions because our study takes a different focus than previous examinations of students' drawings. Characterizing the activity represented in a DAST/DAET response, as done by Farland-Smith (2012) and Fralick et al. (2009), is to some extent relevant to our study, but not sufficient for making the kinds of determinations needed to address our research questions. We therefore developed a new coding system for the purposes of the present study, built from the foundation of our conceptual framework. Importantly, the circumscribed focus of our study means that we did *not* attend to certain aspects of students' drawings that previous studies have examined. The coding system that we developed, for instance, did not seek to describe the race or gender of the scientist/engineer shown in the drawing, nor did it attempt to provide a global rating of "accuracy."

Coding Frame Development

Coding frame development began with a set of *a priori* codes that were grounded in the conceptual framework (summarized in Figure 1). For a given response (whether DAST or DAET), the initial coding frame categorized that response as aligned with either science, engineering, or technology. An "Interdisciplinary" code was also added to account for the possibility that a representation could show work that could be regarded as both science and engineering. A code of "None" was used for representations that showed an activity belonging to neither science, engineering, nor technology or portraying someone engaging in an activity unrelated to the goals of engineering or science. For instance, a portrayal of an everyday activity such as raking leaves would receive a code of "None." Although scientists and engineers might rake leaves, they do not do so as part of their scientific or engineering work. The initial coding frame is shown in Table 2. Note that the categories are mutually exclusive, and that the coding frame was designed to apply to either a DAST or DAET response.

The first and second authors iteratively tested and refined the initial coding frame by applying it to small subsets of the full data set. During each round of testing and refinement, the first two authors independently applied the coding frame and then met to compare codes and discuss issues that arose during coding. They tested and discussed each new coding frame until they encountered no drawings that could not be coded and reached a suitably high level of intercoder agreement during independent coding. Intercoder agreement was determined using Krippendorff's α (Krippendorff, 1970; 2004) because it accounts for chance agreements between the coders and can be used on categorical data such as those in the present study. For the final version of the coding frame, the value of α was 0.77 (Krippendorff, 2012; Schreier, 2012), indicating a high degree of interrater reliability. After establishing the final version of the coding frame, the first two authors applied it to the full data set.

Table 2

Initial Coding Frame

Category	Category Description
Science	A clear and unambiguous representation of scientific work. The person in the drawing is engaged in the study of natural phenomena or is developing explanations of the natural world.
Engineering	A clear and unambiguous representation of engineering work. The person in the drawing is engaged in technological design and development. The work is clearly distinguished from crafts work, tinkering with materials, or other skilled labor.
Technology	The person in the drawing is engaged in technological activity that is clearly **not** engineering work. Examples include routine repair and maintenance, construction work, or crafts work.
Interdisciplinary	The person in the drawing is engaged in work that simultaneously represents science and engineering.
None	The person in the drawing is doing something that falls outside the scope of any of the other categories. For instance, the scientist/engineer is teaching a group of children or reading a novel.

The final coding frame that emerged from the iterative process is summarized in Table 3, and examples of each of the assigned codes are shown in Figure 2.

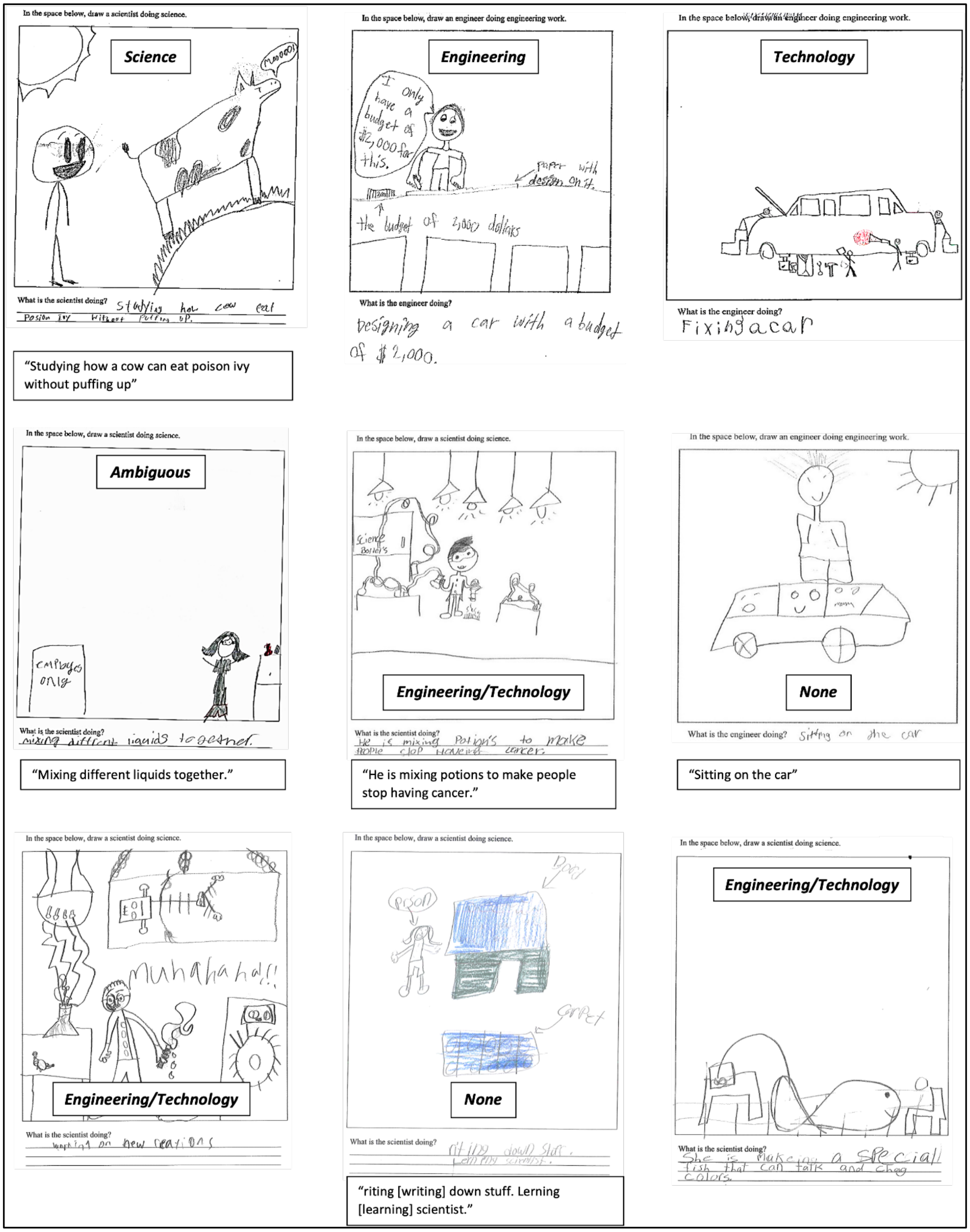
Table 3

Final Coding Frame

Category	Category Description
Science	A clear and unambiguous representation of scientific work. The person in the drawing is engaged in the study of natural phenomena or is developing explanations of the natural world.
Engineering	A clear and unambiguous representation of engineering work. The person in the drawing is engaged in technological design and development. The work is clearly distinguished from crafts work, tinkering with materials, or other skilled labor.
Technology	The person in the drawing is engaged in technological activity that is clearly **not** engineering work. Examples include routine repair and maintenance, construction work, or crafts work.
Ambiguous	The person in the drawing is engaged in activity that is sufficiently vague, that could potentially represent scientific, engineering, or technology work. Examples include "mixing chemicals" without any additional context, or "testing" something that is not described.
Engineering/ Technology	The person is engaged in work that is clearly **not science** and is consistent with either engineering **or** technology. More detail would be needed in order to definitively categorize the response as specifically "Technology" or "Engineering." Examples include making new inventions or "making potions."
None	The person in the drawing is doing something that falls outside the scope of any of the other categories. For instance, the scientist/engineer is teaching a group of children or reading a novel.

Figure 2

Representative Examples for Coding Categories



Several important modifications to the coding frame occurred during the refinement process. First, we found that many drawings showed a scientist or engineer engaged in a vaguely described activity that could potentially be consistent with science *or* engineering. For instance, many drawings showed a scientist/engineer "mixing chemicals" or "mixing liquids" in a laboratory. Without a clear and distinguishable intention for those activities, we could not categorize them as science or engineering. However, they also did not reflect "Interdisciplinary" work. A truly "Interdisciplinary" response would need to convey that *both* scientific and engineering goals were being pursued, but in the case of a vague and generic activity there was *no* clearly identifiable goal. We therefore created an "Ambiguous" code to account for those drawings. Although we found many instances of ambiguous drawings, we did not find any that were clear instances of interdisciplinary work. Although a part of our conceptual framework, the non-use of the "Interdisciplinary" code led us to eliminate it from our coding frame, consistent with our qualitative content analysis approach (Schreier, 2012).

An additional modification was made to account for the substantial number of drawings that showed individuals engaged in technological work that could not be unambiguously identified as engineering. As discussed in our conceptual framework, technological work or invention is not synonymous with engineering. New technologies can be utilized, modified, and developed by engineers, but also by tinkerers and craftspeople. In order for the creation of a novel technology to be a clear case of engineering, more contextual information would be needed. For example, several drawings depicted an individual "working on" or "making" a technology like a "robot" or some new "creation," but lacked any additional clarifying details. Without a clear goal or intention, we could not differentiate between an individual engaged in design (i.e., engineering) versus one engaged in building or fixing (i.e., non-engineering work). To account for those responses, we created the "Engineering/Technology" code. Importantly, this code differs from the "Ambiguous" code in that the response clearly does *not* represent science.

Capturing Emergent Patterns

Consistent with our qualitative content analysis approach (Schreier, 2012), during the process of developing our coding frame we were open and sensitive to emergent patterns in the data set. The emergent patterns described below are not directly related to our research questions but provide additional information about students' drawings that we reasoned might help us more fully interpret the results from our study. One pattern that we identified was a very high frequency of DAST responses showing a scientist working with chemicals. Those responses were assigned a range of codes, depending on what objectives the scientist was pursuing with the materials. For instance, as noted above, many responses were "Ambiguous" because they showed a scientist simply "mixing chemicals;" others showed a scientist using chemicals to create a medicine or "potion" of some kind (those responses were coded as "Engineering/Technology"). Even though representations of scientists working with chemicals could be aligned with different fields, we found the overall ubiquity of chemicals to be interesting in itself; we therefore decided to track its frequency on both the DAST and DAET.

We found a similarly ubiquitous representation on the DAET: a large number of students showed engineers working on vehicles – primarily cars, but also sometimes trains or airplanes. Most often, those representations showed the engineer as a mechanic and thus were coded as "Technology," but other codes were also possible if, for example, the engineer was shown engaged in the design of a vehicle rather than maintenance. Like representations of chemicals on the DAST, the ubiquity of vehicles on the DAET was interesting enough in itself that we also tracked its frequency on both the DAET and the DAST. Worth noting is that both of the emergent patterns identified here are consistent with prior research that used the DAST and DAET with young students (e.g., Capobianco et al., 2011; Finson, 2002; Fralick et al., 2009; Kelly, 2018; Thomas et al., 2020; Weber et al., 2011).

Quantitative Analysis of Coded Data

After all student DAST and DAET responses were coded, the proportion of pretest and posttest responses receiving each of the codes shown in Table 3 was determined for both instruments. For each instrument, statistically significant pretest-to-posttest changes in the proportion of responses assigned each code were determined using two-proportion *Z* Tests. For both instruments (pretest and posttest), we also calculated the proportion of responses that showed the common representations that emerged during our analysis: working with chemicals and working with cars.

Our research questions aim not to simply describe students' responses, but to assess the extent to which they align with the field in question. Table 4 summarizes the extent to which each of the assigned codes indicates alignment or misalignment for DAST and DAET responses.

Table 4

Extent of Alignment Indicated by Codes Assigned to DAST and DAET Responses

Assigned Code	DAST	DAET
Science	Fully Aligned	Misaligned
Engineering	Misaligned	Fully Aligned
Technology	Misaligned	Misaligned
Ambiguous	Partially Aligned	Partially Aligned
Engineering/Technology	Misaligned	Partially Aligned
None	Misaligned	Misaligned

The Ambiguous code is regarded as partially aligned for both instruments because it signifies a response that requires additional detail to determine whether it is a representation of science, engineering, or something else. The Engineering/Technology code is considered partially aligned for the DAET in that it signifies a response that has some elements of engineering while not being a clear case. Note that the Engineering/Technology code is **not** aligned for the DAST. To analyze the alignment of students' DAST and DAET responses, we determined the proportion of pretest and posttest responses that were Fully Aligned, Partially Aligned, and Misaligned. To compare the distribution of alignment levels from pretest to posttest, a Chi-Squared Test was conducted for both instruments. We also compared the distribution of alignment levels between the two instruments using a Chi-Squared Test.

The aggregate-level comparisons described above show which codes became more or less frequent from pretest to posttest, but they do not show what changes were actually occurring in individual students' responses. To provide a more in-depth examination of changes at the student level, a follow-up analysis was conducted using data only for students from whom we obtained linked pretest and posttest responses. Excluded from this analysis were three sets of classroom data (out of 13 total) in which the teachers removed student names from the drawings, but did not assign each student a unique identifier, thus making it impossible to link student pretest and posttest responses. After excluding those cases, 168 students had both pretest and posttest responses for the DAET, and 166 students had both for the DAST.

For each instrument, we tabulated how many students showed each possible "code shift," here defined as pretest-to-posttest change in assigned code (e.g., a change from the "Technology" code on the DAET pretest to an "Engineering" code on the DAET posttest). Because there were 6 codes that could be assigned to each response, a total of 36 different code shifts were possible (6 of which represent no change). To identify meaningful changes from pretest to posttest, we looked for code shifts with relatively high frequencies when compared to code shifts that occurred in the reverse direction (e.g., students who changed from "Technology" to "Engineering" versus students who changed from "Engineering" to "Technology").

Acknowledging the Limitations of Drawing Tasks

As we followed the analytical approach described above, we kept in mind several limitations that have been previously identified with drawing tasks. Researchers have emphasized that a single drawing does not necessarily reflect the full complexity of students' thinking (Bielenberg, 1997; Christidou et al., 2016; Finson, 2002; Samaras et al., 2012). Further, Reinisch et al. (2017) caution that assessing the accuracy of a given DAST response is challenging because scientists engage in a broad array of activities (Irzik & Nola, 2011) and therefore no one "ideal" DAST response exists. Engineers similarly engage in a broad range of activities (Pleasants & Olson, 2019a; 2019b). We designed our analysis to avoid the most significant of these limitations. Our analysis does not aim to fully characterize students' thinking about scientists and engineers, but rather to determine the extent to which their representations are aligned with different fields. Our coding frame allows for many different activities to potentially align with each field, thus addressing the concern raised by Reinisch et al. (2017). Further, we do not make overly expansive claims about students' knowledge of the nature of science or engineering based on their drawings; an accurately aligned representation is not necessarily indicative of a fully informed student. At the same time, a misaligned drawing is likely indicative of misconceptions.

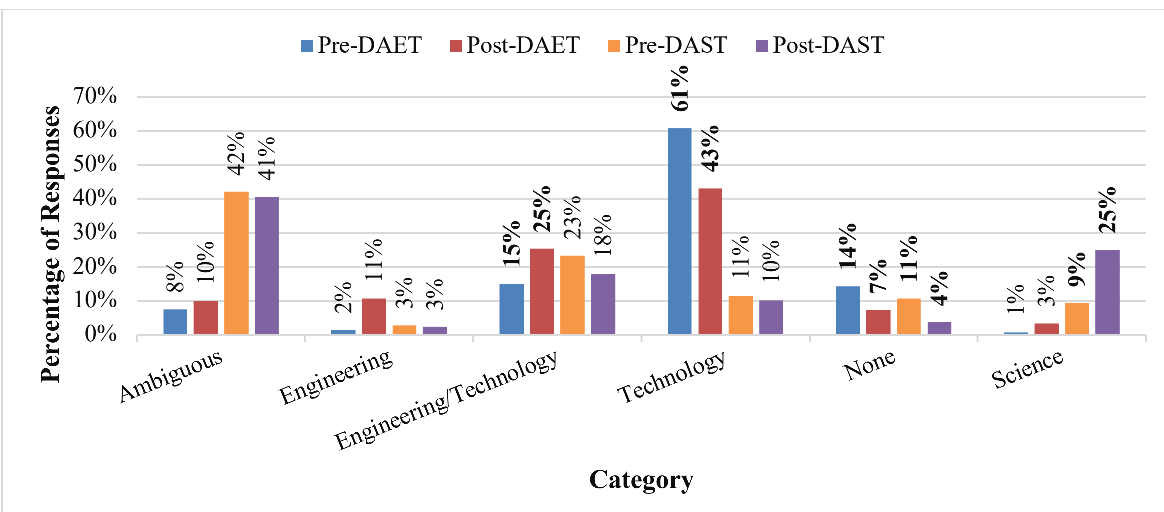
Results

Research Question 1: Alignment of Students' Representations with Science/Engineering

For each of the six possible codes that could be assigned to a student response, Figure 3 summarizes the percentage of responses that were assigned that code on each instrument for pretest and posttest.

Figure 3

Percentage of Student Responses Assigned Each Code



Note. Bolded percentages represent statistically significant changes from pre- to posttest based on a two-proportion Z test ($p < .05$).

Not surprisingly, a different overall pattern of codes can be seen for the DAST and the DAET. The Ambiguous code was common for the DAST on both pretest (42%) and posttest (41%) but rare for the DAET (8% of pretests and 10% of posttest). In contrast, the Technology code was common on the DAET for both pretest (61%) and posttest (43%), but relatively rare on the DAST (12% of pretests and 10% of posttests).

Table 5 summarizes the extent of alignment for students' responses on the DAET and DAST for pretest and posttest.

Table 5

Percentage of Student Responses at Each Alignment Level

	DAET (pre) n=252	**DAET (post) n=232**	**DAST (pre) n=244**	**DAST (post) n=236**
Fully Aligned	2%	11%	9%	25%
Partially Aligned	23%	35%	42%	41%
Misaligned	76%	54%	48%	34%

A Chi-Squared Test indicated that the pretest-to-posttest change in the distribution of alignments was statistically significant both for the DAET (X^2(df=2) = 32.7, $p<.001$) and the DAST (X^2(df=2) = 22.8, $p<.001$). On the pretest, fully aligned responses were rare for both instruments, but rose modestly from pretest to posttest. Similarly, on both instruments there was a modest decline in misaligned responses from pretest to posttest. We also compared the distribution of alignment levels between the two instruments and found a statistically significant difference in the distribution both at the time of pretest (X^2(df=2) = 43.7, $p<.001$) and posttest (X^2(df=2) = 22.8, $p<.001$). At both time points, students' responses showed an overall greater level of alignment on the DAST than they did on the DAET.

Pretest/Posttest Changes at the Student Level

Tables 6 and 7 summarize the code shifts that occurred for the DAET and the DAST, respectively.

Table 6

DAET – Number of Students with Each Pretest-to-Posttest Code Shift

	Posttest Code					
Pretest Code	Ambiguous	Engineering	Eng./Tech.	None	Science	Technology
Ambiguous	0	1	5	2	1	2*
Engineering	0	0	3	0	0	1
Eng./Tech.	5	3	8	1	1	9*
None	2	1	6	1	1	10
Science	0	0	1	0	0	0
Technology	12*	5	20*	8	5	54

Note. Shaded cells represent no code shift.
*These cells represent substantial net movements from pretest to posttest

Table 7

DAST – Number of Students with Each Pretest-to-Posttest Code Shift

Pretest Code	Posttest Code					
	Ambiguous	Engineering	Eng./Tech.	None	Science	Technology
Ambiguous	35	1	9*	1*	11	6
Engineering	1	0	3	0	1	0
Eng./Tech.	17*	1	14	2	4	5
None	8*	1	4	3	1	3
Science	10	0	1	1	4	2
Technology	5	0	5	0	2	5

Note. Shaded cells represent no code shift.
*These cells represent substantial net movements from pretest to posttest

Each cell gives the number of students showing each of the 36 possible code shifts that could have occurred from pretest to posttest. The shaded cells along the diagonal are instances where no shift occurred. For the DAET, 38% of responses received the same code on pretest and posttest, mostly in cases where the Technology code was assigned both times. For the DAST, 37% of student responses received the same code, typically in cases where both were coded as Ambiguous or Engineering/Technology. For students who *did* change codes, most notable are shifts that not only occurred with high frequency, but also occurred with a *greater* frequency than the reverse shift. Two such shifts occurred on the DAET: 12 students shifted from a Technology code to an Ambiguous code, whereas only two students shifted from Ambiguous to Technology; and 20 students shifted from Technology to Engineering/Technology, while only nine students showed the reverse. Both of those shifts represent a net increase in alignment in that both are from a misaligned code to a partially aligned one. These results indicate that the overall increase in alignment for the DAET (see Table 5) was driven in large part by the changes in these particular codes. For the DAST, two notable shifts were identified: 17 students shifted from Engineering/Technology (a misaligned code) to Ambiguous (a partially aligned code) while only nine showed the reverse shift. In addition, eight students shifted from a code of None (a misaligned code) to Ambiguous (a partially aligned one) while only one showed the reverse. Like the DAET, then, these results point to specific code shifts that played a large role in the increased alignment shown in Table 5.

Although the student-level analysis here gives insight into some of the code shifts that drove the overall increase in the alignment of students' responses, it does not fully explain the changes in alignment that were found. The two shifts identified for each instrument represent net movements from misaligned codes to partially aligned ones. However, there was no singular code shift that accounted for the increase in fully aligned codes. That is, for students who shifted to Engineering on the DAET posttest or Science on the DAST posttest, there was no clear pattern in terms of which codes they came from on the pretest. One point to emphasize when interpreting this part of the analysis is that the alignments in Table 5 utilize all student responses, whereas Tables 6 and 7 show only a subset of responses from students who had complete sets of linked data.

Research Question 2: Evidence of Conflation in Students' Representations

As noted above, the overall pattern of codes assigned to students' responses differed between the DAET and the DAST (see Figure 3). In aggregate, therefore, students do seem to perceive differences between the two fields. That result is further supported by examining the two common representations that emerged during our coding process: working with cars and working with chemicals. Table 8 shows the percentage of responses that showed those common representations for each instrument, which indicate that students associate scientists and engineers with different sorts of materials.

Table 8

Percentage of Student Responses Showing Common Representations

	DAET (pre) n=252	DAET (post) n=232	DAST (pre) n=244	DAST (post) n=236
Working with Chemicals	1%	0%	52%	57%
Working with Vehicles	42%	24%	2%	2%

However, even though students do not necessarily view engineering and science as identical, cause for concern still exists regarding the issue of conflation, particularly in students' representations of scientists. In the case of their representations of engineers, very few instances occurred where students' DAET responses showed scientific work (1% of pretests and 3% of posttests). In contrast, although students' DAST responses were rarely coded as Engineering (3% on both pretest and posttest), they were often coded as Engineering/Technology: 23% of pretests and 18% of posttests received that code, a change that was not statistically significant (p = .131; see Figure 3). The Engineering/Technology code was typically applied to instances of portrayals of scientists as inventors, rather than as investigators of the natural world. While invention is not necessarily engineering, it is an activity that is better aligned with engineering than with science. Consistent with the patterns in Table 8, students often depicted scientists inventing *with chemicals*, whereas chemicals were largely absent from students' representations of engineers. Yet even if students associate scientists with different *materials* than engineers (a dubious distinction, given that chemical engineering exists), the fact that many students conceptualize both as being involved in technological development is evidence of conflation.

Discussion

Prior studies of students' drawings of scientists and engineers have established the existence of multiple misunderstandings and stereotypes (Capobianco et al., 2011; Christidou et al., 2016; Finson, 2002; Fralick et al., 2009; Huber & Burton, 1995; Kelly, 2018; Sharkawy, 2012). Those common misrepresentations emerged in the present study as well, but the goal of the present work was not to replicate or confirm those well-established patterns. Rather, our investigation extends research on the DAST and DAET in multiple ways. First, we focus on the extent to which elementary students' representations of scientists and engineers are accurately aligned with the kinds of work done in those fields—an important issue given that science and engineering are now being taught in the same classroom space, possibly even in the context of integrated "STEM" (Kelley & Knowles, 2016; Moore et al., 2015; NRC, 2012; 2014). Second, unlike prior studies, we conducted a side-by-side comparison of students' DAST and DAET responses. Third, we explored how those responses changed after the students experienced a semester of science instruction that included engineering design activities as well as interactions with an engineer through a triad teaching model.

We found that students' drawings showed more accurate representations of scientists than of engineers, both at the time of pretest and posttest. That result is perhaps not surprising, as young students are not likely to have had much contact with engineers or the field of engineering either in or out of school (Capobianco et al., 2011; Fralick et al., 2009; NAE & NRC, 2008; 2009). For students in the present study, the engineering instruction they received during the project was generally their first such educational exposure to engineering. The better alignment found in students' DAST responses might therefore be attributed primarily to the larger number of experiences students have had learning about science and scientists both in and out of school. Of course, such experiences do not necessarily result in *fully accurate* views about science (Clough, 2006; Kelly, 2018; Finson, 2002; Sharkawy, 2012). We emphasize that even though students' representations of scientists were *better* aligned than their representations of engineers, their *overall* alignment was relatively low. Students in our study showed many of the same misunderstandings about science (and engineering) that appear in previous research.

We found an overall increase in alignment from pretest to posttest on both the DAST and the DAET. Those increases, however, were relatively modest. Even on the posttest, very few students produced representations that were completely aligned with the targeted field, especially on the DAET (only 11% of posttest responses). Much more common than fully aligned responses were partially aligned ones. The student-level analysis of code shifts from pretest to posttest also showed that the largest net movements on both instruments were from misaligned codes to partially aligned codes. While partially aligned responses do not show overtly erroneous ideas, their ambiguities mean that they are not necessarily indicative of accurate conceptions of science or engineering. Thus, while students might have let go of certain misconceptions about science and engineering, they might not have replaced them with views that are fully accurate or that enable distinctions to be made.

Although the increases in alignment were modest, the fact that they occurred across both instruments is interesting given that the project involved interactions with a graduate student in *engineering*, not science. Most likely, the gains in alignment are attributable to certain components of our professional development project. Like many projects that form partnerships between teachers and STEM professionals (e.g., Houseal et al., 2014; Mitchell et al., 2003; Thompson & Lyons, 2008), an overt goal of our project was for the engineers to serve as ambassadors of their field and communicate to both the teachers and students the kind of work that engineers do. Throughout the professional development activities, we also emphasized to both the teachers and engineers the importance of helping students recognize the differences between science and engineering. The triads did work toward those objectives to some extent. The time when triads most deliberately addressed those ideas was near the beginning of the semester, when the engineers introduced students to the details of their own work. Those presentations typically included the most substantive discussions about the nature of engineering and how science is important for engineering work.

However, the majority of students' contact with engineering over the course of the semester came in the form of the engineering activities in which they participated. In prior examinations of those activities, we found that the teachers and engineers rarely engaged students in conversations about the nature of engineering or science; in the few instances that conversations did occur, they were typically very brief (Pleasants, 2018; Pleasants & Olson, 2021). Thus, students were mostly left to draw their own conclusions about the nature of those fields based on their classroom experiences. Research makes clear that in such situations, the conclusions that students draw will not necessarily be accurate ones (Clough, 2006; Lederman & Lederman, 2014; McComas et al., 2020). Thus, the somewhat limited ways that the teachers and engineers addressed the nature of science and engineering with their students are likely to account for both the overall gains in alignment, as well as the modest size of those gains.

The present work was motivated by the concern that students could potentially conflate science and engineering, especially after experiencing engineering lessons alongside science instruction (McComas & Nouri, 2016). That concern was only somewhat borne out by our results. The pattern of codes assigned to students' DAST responses differed substantially from those assigned to their DAET responses, and we found very few instances where students represented scientists engaged in engineering or vice-versa. We also found that students tended to associate scientists and engineers with different materials (chemicals for scientists and vehicles for engineers). Those results indicate that while students may not hold accurate views of scientists and engineers, they do not view the two as being one and the same.

We did, however, find one avenue of potential conflation in that a significant proportion of students showed scientists engaged in the invention or creation of new technologies (approximately one fifth of responses on pretest and posttest), an activity more accurately associated with engineers. The view of scientists as inventors is consistent with images of the "mad scientist" that often appear on the DAST and that are pervasive in the media (Finson, 2002, Kelly, 2018). It is a problematic perspective because it wrongly conveys that both scientists and engineers are largely concerned with the same thing: creating novel technologies. Even if students associate scientists and engineers with different materials (i.e., scientists invent things using chemicals, whereas engineers invent machines), that difference is neither an accurate one nor does it avoid the conflation problem. Importantly, though, we did not find that the percentage of students showing this problematic conception *changed* from pretest to posttest. It is therefore unlikely that the notion was *caused* by the incorporation of engineering into their science instruction. Rather, it is a pre-existing and persistent conception, given that it was not *improved* by the instruction that students received.

Limitations and Future Directions

Drawing tasks such as the DAST and DAET have inherent limitations that restrict our ability to make claims about how students conceptualize science and engineering. Drawing tasks do not necessarily elicit the full complexity of students' ideas about scientists or engineers (Bielenberg, 1997; Christidou et al., 2016; Reinisch et al., 2017). The reasoning underlying what students choose to draw is also inevitably hidden from the view of the researcher (Finson, 2002).

Despite the limitations, we used the DAST and DAET in this study largely because alternative research tools are lacking. Many instruments exist to probe students' views of the nature of science (Abd-El-Khalick, 2014), but there are few extant methods for examining students understanding of the nature of engineering and, more importantly, their understanding of the differences between science and engineering. Interviewing students (e.g., Karatas et al., 2011) can provide more in-depth views of their thinking but is impractical for studies with large numbers of students, such as the present one. This issue could be mitigated by conducting follow-up interviews with a subsample of students after administering the DAST or DAET. This approach has been suggested by other researchers as a way to validate the interpretations of student drawings (e.g., Hammack et al., 2020; Reinisch et al., 2017). Even more valuable would be an instrument that more directly elicits students' thinking about the differences between science and engineering. An instrument that tasks students with categorizing (rather than generating) different STEM-related activities, for instance, is a potentially fruitful option that has already been shown to be insightful within nature of science research (Walls, 2012).

Another limitation of our study is that it occurred within the context of a resource-intensive professional development project. The educational experiences provided to the students in the study are therefore not representative of typical elementary classrooms. Elementary teachers who incorporate engineering into their science instruction are unlikely to receive extensive professional development support (Banilower, 2019), and are especially unlikely to have access to an expert in engineering. Thus, future research ought to examine how students in more typical classroom conditions, develop in their understanding of scientists and engineers as engineering is incorporated into science instruction.

Conclusions & Implications

Including engineering as part of science instruction will not necessarily result in issues of conflation, nor will it, by itself, assist students in making sense of the nature of engineering or science. Helping students to develop more accurate views about the nature of science and engineering, and particularly the ways in which the two fields differ, requires explicit instruction (Lederman & Lederman, 2014; McComas et al., 2020). Engaging students in engineering activities does create opportunities for teachers to have explicit conversations about what engineering is and how it differs from science. Teachers ought to draw students' attention to how the goals of an engineering design activity are different from a science inquiry activity. The materials might be similar, and students might also use similar techniques in both, but the purposes are not the same. Such explicit conversations would do much to address the misunderstandings that were found in the present study.

Bringing STEM experts into the classroom creates further opportunities to help students develop more accurate views of science and engineering. STEM experts, such as the engineering graduate students in the present study, have knowledge and experiences that can help students connect classroom activities to the real world of disciplinary practice. They can share examples of their own work and use those examples to highlight key features of how science and engineering work, how they are related, but also how they are different ways of knowing. However, all of those promising possibilities can easily become *missed opportunities*; this was largely what occurred in the present study. Unless explicit conversations are had about the nature of disciplinary work, mere proximity to and interactions with STEM experts will not be sufficient (Sadler et al., 2010). Projects that bring STEM experts into science classrooms should prepare them to have productive conversations about the nature of science and engineering and find ways to ensure that those conversations take place.

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Appendix: DAST and DAET Instruments

Name: ______________________________

In the space below, draw a scientist doing science.

What is the scientist doing?

Name:____________________

In the space below, draw an engineer doing engineering work.

What is the engineer doing?

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Knowledge Analysis of Chemistry Students' Reasoning about the Double-slit Experiment

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ABSTRACT

Previous work has highlighted the difficulties students have when explaining wave behavior. We present an investigation of chemistry students' understanding of the double-slit experiment, where students were asked to explain a series of PhET simulations illustrating a single continuous light source, single-slit diffraction, and double-slit interference. We observed a variation in student reasoning and students were categorized into groups based on their ability to explain and generate a mechanism for the double-slit experiment. Some students struggled to explain the features of waves which impacted their reasoning about interference and caused them to rely on intuition to generate explanations. Other students were able to productively incorporate their previous knowledge about wave behavior, with their observations from the simulations, to build a robust mechanism for wave interference. However, students generally exhibited a limited understanding of interference, and specifically attending to the key features of waves during instruction can promote more sophisticated reasoning about this phenomenon.

Keywords: light-matter interactions, double-slit experiment, knowledge analysis, knowledge-in-pieces, postsecondary chemistry, scientific reasoning

Introduction

In first-year chemistry courses, the double-slit experiment is introduced to illustrate the wave nature of light and is further built upon to introduce the wave nature of quantum particles. Students are asked to extend the dual nature of light to the dual nature of matter, where matter can exhibit both wave and particle behavior. Understanding how light behaves in the double-slit experiment is a necessary first step in understanding the wave nature of matter. Specifically, students need to have a basic understanding of wave phenomena like diffraction or interference (Vokos et al., 2000). Henriksen et al. (2018) showed that students' productive explanations of the dual nature of light rely on interference patterns to explain wave behavior.

Research in physics education has investigated how physics students understand and interpret the double-slit experiment. In one qualitative investigation, three broad difficulties were identified: misapplication of geometrical optics, reliance on algebraic formulas without a conceptual understanding, and difficulties with understanding light as photons and electrons as waves (Ambrose et al., 1999). High school and university level students also displayed difficulties in meaningfully interpreting interference and diffraction patterns for a single and a double-slit during an eye-tracking study (Susac et al., 2021). Difficulties with these concepts highlight the intricate nature of understanding the double-slit experiment, and ultimately how the experiment has implications for understanding duality. In a study investigating students' understanding of the wave nature of matter, students had difficulties with basic wave behaviors such as interference, which further impacted their ability to extend wave behavior to matter (Vokos et al., 2000). In a mixed-methods study describing student performance on a light interference assessment, students performed better on assessment items targeting phase differences and interference patterns than items targeting changes in wavelength and changes in the direction of propagation (Dai et al., 2019). In this same study, the qualitative investigation revealed that novice students memorized equations without demonstrating a conceptual understanding. Dai and colleagues (2019) observed students struggle to apply interference to novel or atypical problems, such as changing wavelength or direction of propagation, and possessed a limited conceptual understanding of interference. Further, advanced physics students showed evidence of employing phenomenological primitives (p-prims) while explaining the double-slit experiment of single particles (Sayer et al., 2020). This was evident when students explained that if two particles had the same wavelength, they would have the same amount of kinetic energy regardless of particle size. This study showed that the extension of wave behavior to particles is difficult and resulted in a reliance on intuitive reasoning. Extending wave properties to matter can be further exacerbated when a student's understanding of wave behavior is limited.

While this topic has been investigated in a physics context and provides important insights into how students understand light, it remains important to investigate in a chemistry context because of how differently chemists use and approach light in chemistry instruction. Additionally, many studies in physics have focused more on investigating students' alternative conceptions regarding the double-slit experiment (Ambrose et al., 1999; Yalcin. et al., 2009). This study focuses on how students understand and reason about this experiment and the variation in their knowledge structures. Here we describe our investigation of chemistry students' understanding of the double-slit experiment. This is part of a larger project looking at how chemistry students understand the nature of light and light-matter interactions using a developmental perspective (Balabanoff et al., 2020). Specifically, this investigation was framed by the following research question:

> **RQ:** How do postsecondary chemistry students reason about light behavior in the double-slit experiment?

Theoretical Framework

This study of students' understanding of light and the double-slit experiment is framed by Knowledge in Pieces (KiP). This framework describes learner's knowledge as fragmented, where fragments of knowledge are considered the finest grain of cognitive units and can be activated in a range of contexts (diSessa, 1993, 2018; Hammer et al., 2005). KiP provides a framework for investigating how students reason in the context of the double-slit experiment and a way to evaluate the variation in students' knowledge structures.

Knowledge in Pieces

The KiP framework is grounded in the constructivist paradigm, where students' knowledge is considered *rich* and *productive*. This is because as students learn, new pieces of knowledge or information are integrated with previous knowledge. New fragments can be added to generate more complex and organized systems of knowledge. KiP considers knowledge to be multi-scaled in nature where smaller knowledge pieces are added, displaced, connected, or isolated within a larger knowledge system (diSessa, 2018). As such, the goal of our analysis focused on how students combined fragments, and whether the incorporation or displacement of specific pieces of information supported or hindered their explanations of the double-slit experiment.

Students' knowledge is described as small fragments that are often context dependent within the KiP framework (diSessa, 1993, 2018; Hammer et al., 2005). *Contextuality* is the idea that students' fragments are neither fixed nor stable, with some explanations appearing in specific contexts and not others (diSessa, 2018). Phenomenological primitives (p-prims) are abstract and intuitive ideas about how things work. Structurally, p-prims are small in nature and often isolated from other pieces of information. They are irreducible knowledge elements in that they typically cannot be further explained. One example of a p-prim is "more is more", or Ohm's Law, where more of a cause is connected to more of some effect. These intuitive elements often guide a student in the sense-making process without the student recognizing the p-prims are doing so because they are deeply ingrained (diSessa, 1993; Hammer, 1996). Students may rely heavily on *intuitive knowledge,* or p-prims, when relevant prior knowledge is inaccessible.

Coordination Classes

Within the KiP framework, a second model of cognition describes and defines the properties associated with expert-level thinking. In contrast to p-prims and fragmented knowledge, coordination classes are structurally distinct in that they consist of a complex system of knowledge elements. Coordination classes are reliable across contexts, unlike the fragmented and context-dependent nature of p-prims. The function of coordination classes is to extract some "class" or network of information from the world that is characteristic of a particular concept (diSessa & Wagner, 2005; Thaden-Koch et al., 2006).

Coordination class theory has distinct structural and architectural features to categorize students' knowledge. The two main features are *extractions* and *inferences* (diSessa & Wagner, 2005; Levrini & diSessa, 2008). Extractions correspond to observations of the world where one coordination class may use multiple observations within one single situation. Inferences are the part of the knowledge system that draws conclusions about the extractions, also referred to as the *causal* or *inferential net*. One key aspect of students making inferences is that they must first determine which extractions or observations are relevant for that particular situation (Thaden-Koch et al., 2006).

Within coordination class theory, there are two processes for how learners determine how prior knowledge is used. The first is *incorporation,* where prior knowledge and a new conceptualization is merged. The second is *displacement*, where prior knowledge is dismissed from a new conceptualization. Both of these processes involve the learner determining if the prior knowledge is relevant to the new conceptualization (Barth-Cohen & Wittmann, 2017). Other architectural features of coordination classes describe how learners consider knowledge across contexts. For instance, *span* refers to the ability to recognize and access relevant knowledge across a range of contexts. In addition, *alignment* refers to learners determining which information from different situations is actually the same information and relevant. Those who have an advanced coordination class surrounding a concept demonstrate both span and alignment when generating inferences (Barth-Cohen & Wittmann, 2017; diSessa & Wagner, 2005; Thaden-Koch et al., 2006).

A coordination class has distinct architectural specifications. In some cases, a student may possess a coordination class for only certain concepts and not others. There may also be situations where a student has a knowledge system that does not meet the strict requirements of an advanced coordination class. For example, a student could have accurately coordinated extractions with relevant prior knowledge, which indicates alignment. However, if that student inconsistently applies the coordinated extraction and prior knowledge, they would not demonstrate span. Examples such as these can be described as *developmental coordination classes.* Learners with developmental coordination classes are often in the beginning stages of generating a more complex class with limited success (diSessa, 2002; Thaden-Koch et al., 2006).

Method

Knowledge Analysis

This study is guided by the Knowledge Analysis methodological framework focusing on modeling students' knowledge. The guiding principles of this methodological framework are: (1) that the aim is to model students' thinking and learning, (2) developed models are content specific, (3) intuitive knowledge is important, (4) analysis requires capturing the thinking and learning processes, and (5) intellectual performance is context dependent (diSessa et al., 2016).

Participants

Participants in this study were recruited from multiple chemistry classes ranging from introductory chemistry through quantum chemistry during the Fall 2019 semester. Students were recruited from multiple chemistry courses because it was important to capture a variation of understanding. Students were recruited from general chemistry (GC, N=10), general chemistry for chemistry majors (GCM, N=10), organic chemistry (OC, N=11), and physical chemistry (PC, N=1). General chemistry surveys chemistry very broadly, including atomic structure, molecules, structure-property relationships, and chemical reactions (including thermodynamics, equilibrium, and kinetics). Where general chemistry serves all STEM majors, general chemistry for majors serves primarily chemistry, biochemistry, chemical engineering, and physics majors. Organic chemistry surveys molecular structure, reactivity, chemical reactions, and reaction mechanisms. In the laboratory component of organic chemistry, students encounter common techniques for characterizing and identifying molecules, many of which rely on interacting electromagnetic radiation with matter. Finally, physical chemistry introduces students to quantum mechanics, including the dual nature of light and matter, the hydrogen atom, multiple quantum mechanical models, and an introduction to spectroscopy. Physical chemistry represents the most advanced treatment of the nature of light and its interaction with matter in the undergraduate chemistry curriculum.

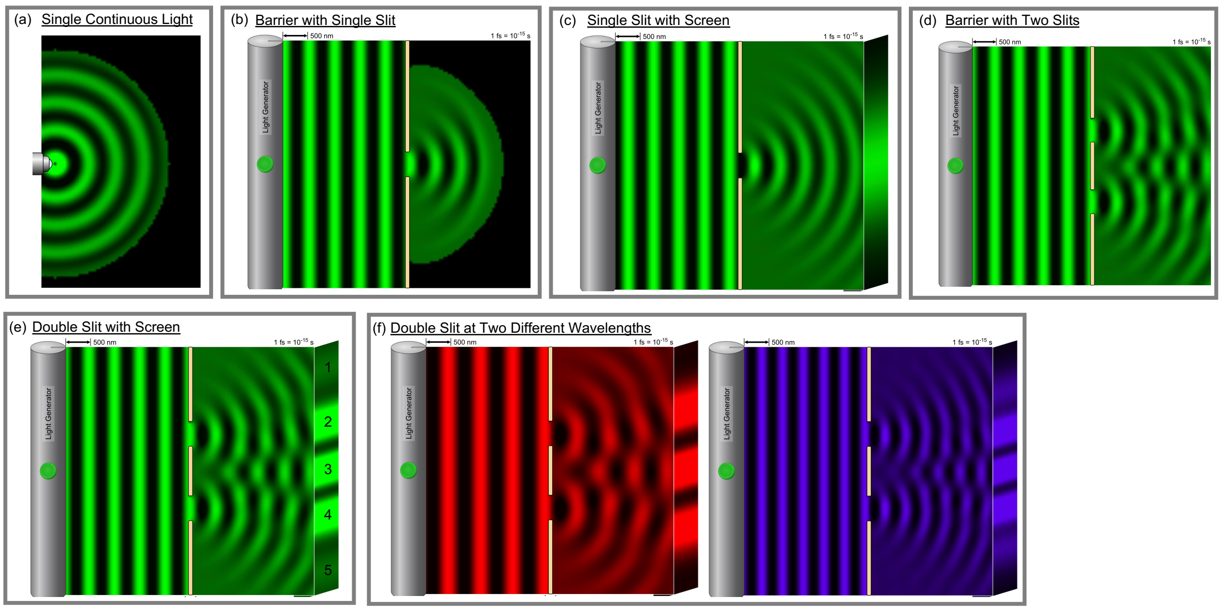
Data Collection

Data was collected via semi-structured interviews which allowed for an in-depth investigation of students' understanding of the double-slit experiment and their reasoning. Audio and video recordings were collected for each interview, which ranged from 25 to 73 minutes. Student drawings and notes were collected after each interview and scanned. The study was approved by the Institutional Review Board.

During the interview, students were shown a series of PhET simulations (Figure 1) that depicted light waves traveling without any barriers (Figure. 1a), and light waves traveling through barriers with single (Figure 1b and c) and double-slits (Figure 1d - f). The interview consisted of four parts: (1) describe the light behavior from a single continuous light source (Figure 1a), (2) make predictions and describe a single light source that is shining on a barrier with one slit (Figure 1b and c), (3) make predictions and describe a single light source that is shining on a barrier with two slits (Figure 1d - f), and (4) draw conclusions about the nature of light. Students were asked to make predictions prior to observing simulations and asked to provide explanations after observing simulations. Parts 1 and 2 of the interviews were designed to elicit students' understanding of how a single light wave travels and interacts with a barrier. Part 3 was designed to elicit students' understanding of interference caused by two sources of light. Part 4 was designed to elicit students' ideas about how light behaves and their conclusions about the way light behaves based on observations of the simulations.

Figure 1

Sequence of Simulations Shown to Students During the Interview



Note: Students first observed a single continuous light source (a), barrier with a single slit (b), the single slit with a screen (c), barrier with two slits (d), the double-slit with a screen (e), and finally, observed the double-slit experiment with red and violet wavelengths (f).

Data analysis

Interviews were transcribed verbatim using a transcription service. Gestures captured from the videos were added to the transcript and images of student work were embedded at the appropriate time points. The transcripts were open-coded using a constant comparison approach (diSessa et al., 2016; Strauss & Corbin, 1990) where codes were refined by multiple analysts (first and second author). Trustworthiness was established through an iterative series of applying and refining codes. All authors were involved throughout the entire process to ensure that the first author's codes were applied consistently and meaningfully interpreted (Golafshani, 2003).

The coding took place in two stages where transcripts were initially open coded to develop content-based codes and the second stage of coding focused on students' reasoning. The codes focusing on students' content knowledge were organized into the following categories: general light properties, light behavior, and deductions from simulations. General light properties included codes describing properties that were scientifically accepted such as light having no mass, or the amplitude corresponding to the intensity. This category also included non-normative scientific ideas such as the number of photons corresponding to the intensity of light and brightness relating to the energy of light. The light behavior category codes described students' overall ideas of how light behaves, including both normative and non-normative ideas. Some examples are light travels linearly, the wavelength of light changes as it radiates outward, and a change in frequency changes how quickly light travels. The last category of the content-based codes, deductions from simulations, described the explanations students generated based on extractions from the simulations and inferences. The deductions were further categorized to align with the four parts of the interview, as outlined in the Data Collection section. Again, the deductions from simulations coding during analysis included both normative and non-normative ideas. Some examples include interacting with the barrier results in light scattering, interference causes light to change direction, and illuminated regions on the screen represent instances of constructive interference.

The second stage of coding used the KiP framework to code students' explanations and reasoning. Because of the architecturally strict nature of this framework and explicit definitions, the codes used in the second stage were directly developed from this framework. These codes included p-prim, intuitive reasoning, reasoning grounded in experiences, contextuality, extraction, inference, incorporation, displacement, span, and alignment. Definitions for these codes can be found in Table 1.

Table 1

Codes and Associated Definitions for the Second Stage of Coding Using the Knowledge in Pieces Framework

Code	Definition
Phenomenological primitive (p-prim)	Abstract or intuitive idea about how things work, often structurally small and isolated from other pieces of information
Intuitive reasoning	Explanation where a student does not know exactly where it comes from or why
Reasoning grounded in experience	Explanation built on an experience with the physical world
Contextuality	Fragment that is not fixed nor stable, an explanation that appears in some contexts and not others
Extraction	Observation of the world
Inference	Conclusion about an extraction or observation
Incorporation	Prior knowledge and a new conceptualization that is merged
Displacement	Prior knowledge is dismissed from a new conceptualization
Span	Recognizing and accessing relevant knowledge across a range of contexts
Alignment	Determining which information from different situations is the same information and relevant

Students were then grouped based on how they drew conclusions, whether they relied more on intuitive reasoning and p-prims or the degree to which they coordinated their extractions and inferences. The grouping of students was based on the qualitatively different ways in which students reasoned about the double-slit experiment. Specifically, the analysis centered around the consistency or lack thereof across the explanations provided for the range of simulations. For instance, we looked at how students' explanations changed with the introduction of each simulation or how they built upon explanations generated from previous simulations. This resulted in three categories (Table 2): primarily fragmented (*N*=13), developmental (*N*=13), and coordinated (*N*=6).

Table 2

Distribution of Students in Assigned Levels

	Fragmented	Developmental	Coordinated
GC	7	2	1
GCM	1	6	3
OC	5	4	2
PC	0	1	0

Using some examples from Table 1, if a student's interview transcript consistently highlighted their use of p-prims, intuitive reasoning, or reasoning based on experiences, this student was categorized as primarily fragmented. In contrast, a student's transcript that was frequently coded with span, alignment, and inferences built upon prior knowledge and earlier observations if the simulation elicited a coordinated classification. In the developmental category, these students exhibited a mixture of both fragmented-type codes (e.g., p-prims) and coordinated codes (e.g., span and alignment). Each student was grouped based on their overall reasoning structure surrounding the double slit experiment.

Results

Each category (fragmented, developmental, and coordinated) will be described and explained through vignettes from an exemplary participant in each category. The generation of vignettes for exemplary students was informed by our methodological framework with the aim of modeling students' knowledge and to capture students' understanding over the course of the interview. We have selected three students because they are representative of their category and enable a detailed discussion of the variation in student reasoning across categories. Below in Table 3, the general trends and features associated with each category are outlined.

Table 3

General Trends Described for Each Eategory: Fragmented, Developmental, and Coordinated

Fragmented	Developmental	Coordinated
• Limited or absent prior knowledge • Inconsistent use of prior knowledge • Focused on visible light or shadows • Inconsistent explanations throughout the interview • Absence of mechanism	• Relied on correct relationships • More comfortable with constructive interference than destructive • Correct predictions with limited mechanistic understanding	• Easily accessed relevant prior knowledge • Comfortable with constructive and destructive interference • Detailed and accurate mechanism of double-slit experiment

Fragmented – Ramona

Students categorized as fragmented typically had limited prior knowledge relating to the double-slit experiment and inconsistently applied that prior knowledge. Students in this category heavily relied on their experiences when reasoning about the phenomenon and tended to use intuition when their understanding was limited. This limited their ability to generate explanations for their predictions or observations. In addition, the limited prior knowledge resulted in inconsistent explanations which indicated a lack of alignment and span. Ultimately students were categorized as fragmented due to the inconsistent application of prior knowledge, overreliance on intuitive reasoning in the absence of prior conceptual knowledge, and not incorporating mechanistic reasoning of the experiment. Some students were able to recall the term interference upon observing the double-slit experiment, but were not able to further explain the details of interference or which observations from the simulations corresponded to evidence of interference.

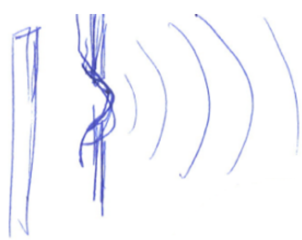
One student who exemplified the fragmented category was Ramona, a student in the general chemistry course for chemistry majors. Throughout the course of the interview, she relied on her physical experiences to explain her observations of the simulation. Because she relied on experiences, Ramona's reasoning was fragmented and grounded in intuition. Additionally, her explanations of the simulations lacked any mechanism of interference or light interactions.

After showing Ramona the simulation depicting a Single Continuous Light (Figure 1a), she was asked to predict what would happen if the single light source were shined on a barrier with a single slit. She used what she previously observed from the simulation shown in Figure 1a to inform her prediction and drew Figure 2:

> It'll probably still keep moving out, but it'll be bent because of the obstacle… Sort of how I pictured in my head, which might be wrong, it's like the lines (of light) are flat, so it's going… and [at] the barrier, it kind of gets pushed a little bit.

Figure 2

Ramona's Drawing Illustrating How Light Bends When It Meets the Barrier



Ramona explained that she expected the light to continue to move out from the source and upon hitting an obstacle, the light will bend. Ramona generated a prediction that aligned with her previous observation of light traveling with no barriers. When Ramona was shown the Barrier with Single Slight simulation (Figure 1b), she remarked:

> Yeah, looks [the same]. The fuzzy regions, I think it kind of loses its intensity as it goes out because the barrier kind of blocks some of it.

Ramona indicated that the simulation looked as she predicted and explained that because the light is going through a single slit, the barrier blocks some of the light, which caused a decrease in intensity.

Ramona constructed a prediction based on her previous observations of light continuously moving outward and subsequently generated an explanation of the simulation that highlighted a loss of intensity.

Immediately following the Barrier with Single Slit simulation in Figure 1b, Ramona was asked to predict what she expected to see once a screen was added to the simulation. She offered a prediction for two types of light behavior: light acting as particles and light particles traveling in waves. Ramona first brought up the idea that light is made up of particles. She explained that if light particles did not move like waves and traveled in a straight trajectory, she would expect to see individual dots on the screen. However, she further explained she did not expect to see dots because the light particles are in fact moving like waves.

> I know we used to think that light was just a bunch of particles, but it's particles that move like waves. So, if that were true [light being made up of particles] …the screen would just show a bunch of dots because it would just be particles and they would be individual and in different spots, **but it's not**.

This prediction indicated that Ramona considered both wave and particle behavior and ultimately decided that wave behavior was more appropriate in this context. She affirmed that she did not expect the light to behave like particles, rather the light particles are moving like a wave.

After her prediction, Ramona observed the Single Slit with Screen simulation in Figure 1c and agreed with her previous prediction that light particles move like waves because she did not observe individual dots on the screen. She then described her observations of the intensity of light on the screen (Figure 1c):

> Yeah. It shows a little bit more of the intensity and how it blurs out on the sides I guess… because there's going to be a little bit of shadow, kind of blurring out from the obstacles. There's still a little bit of light showing, but it's not going to be as strong.

When describing the intensity of the light on the screen, Ramona grounded her explanations in physical experiences by the invocation of shadows. Ramona described the barrier as creating a shadow on the screen, with some light passing through the single slit that is not as strong. Ramona connected her experiences with shadows to her observations of the simulation and explained that the blurred regions on the screen are a shadow of the barrier.

Later in the interview, Ramona was asked to make a prediction about what she expected to see on the screen now that the barrier had two slits (Figure 1e). She based this prediction on what she observed on the screen when the barrier had a single slit. She explained that she expected to see three separate regions on the screen:

> Something similar as before but it's going to be stronger right here and stronger right here and here (pointing at Regions 2, 3, and 4 of Figure. 1e). But a little blurry here and here, and here and here (pointing between Regions 2 and 3 and between Regions 3 and 4 of Figure 1e). Because the same as before, because of the obstacles in the middle. They're still going to have some effect on putting a shadow in the middle of the light. But because it's the two waves of light crashing into each other, they're going to have that spot in the middle.

Ramona used her previous observations of the Single Slit with Screen simulation and predicted to see three regions illuminated on the screen. Two of the illuminated regions were a direct result of light shining through the slits, just as she had observed with the Single Slit with Screen simulation. Unique to the Double-slit simulation, she explained that there will be a third "middle" region of light because the light waves will crash into each other. Ramona made a productive prediction by drawing on the idea of light radiating outwards. However, in this prediction she did not further explain why the waves crashing into each other results in an illuminated region. Ramona's invocation of the shadow to explain darkness and blurriness on the single slit simulation also informed her prediction for the double-slit simulation. She explained that there would be a "shadow in the middle of the light" due to the barriers.

After making her prediction, she was shown the Double-slit with Screen simulation (Figure 1e). She explained:

> These two parts (Regions 2 and 4) are still strong, bright, because that's exactly where they're showing right through in the middle of the slits... But right there (Region 3) they're going to be just as strong because that's where they meet. That's like kind of the peak of where they meet. [The gaps] are because since the waves still move out but meet in the middle, that's kind of like the remainder of the shadow from that middle part. If the waves didn't move the way they [do], that whole part (Region 3) would be shadow. But since they still like move out and hit each other, it's just going to kind of show the edges of what the shadow would be if it was a particle.

Ramona explained the illuminated regions across from the slit openings (Regions 2 and 4) are a result of light traveling unobstructed through the slit to the screen, thereby confirming her prediction. She also explained that the middle region is just as strong as the other regions because that is where the two sources of light meet, also confirming her prediction. She used prior knowledge of light radiating outward to help her explain how the light waves meet in the middle. Her explanation of the gaps between the illuminated regions was rooted in shadows, similar to her explanation for the single slit. She expected Region 3 of the screen to be a shadow if the light behaved only like a particle because she associated light radiating outward with wave behavior. But because the particles move like waves, Ramona explained that we see the region in the middle illuminated and the "edges" of the shadow.

Despite making a correct prediction, Ramona displayed a limited understanding of why her prediction was correct. Her explanation of the interference pattern relied on the intuitive expectation that if light waves continued to expand outward, they would eventually meet. Ramona's explanation also lacked any kind of mechanism or ideas about *how* the waves were combining. Many other students within the fragmented category used water waves to explain waves joining together, where students built off their experiences and connected them to light waves. This could be related to the p-prim "more is more" where two waves adding together can create a brighter region of light than a single wave. Further, we observed where her limited prior knowledge of interference impacted her explanation of the dark regions on the screen (Figure 1e) and caused her to rely on her experiences with shadows, even though shadows did not serve her very well. That is, she struggled to use shadows beyond attributing them to the barriers. Additionally, she did not connect wave behavior to the gaps on the screen. Rather, she only incorporated wave behavior to explain the illuminated regions on the screen.

Throughout the interview with Ramona, she focused on observations that aligned with wave-like behavior. For instance, she first activated wave behavior when she spoke about light bending around obstacles and light radiating outward. She continued to think about waves when she explained light radiating outward and meeting in the middle of the double-slit experiment. However, in the absence of an explanation of interference, it was evident that Ramona used intuition to predict the peak on the screen in between the slits. The reliance on intuition was further evidenced by her reasoning about dark regions on the screen; that is, dark regions were caused by shadows from the barrier. In this case, intuition was specifically grounded in physical experiences.

While the *predictions* Ramona generated of the later simulations (e.g. Double-slit with Screen) were relatively productive and built on her previous observations and extractions of earlier simulations, her *explanations* of the simulations were fragmented and intuitive and lacked any incorporation of wave behavior when explaining the dark regions on the screen (Figure 1e). Ramona's consistent predictions, but inconsistent explanations of the simulations, indicated a limited understanding of interference. Ramona, like many other students in this category, focused on her previous experiences with visible light and shadows. As the interview progressed, Ramona became more dependent on her intuition, and specifically shadows, even though shadows failed to explain the patterns observed on the screen. This reliance on intuition and her limited prior knowledge resulted in Ramona generating explanations that lacked any kind of mechanism of interference. Generally, students in this category could rely on intuition to make predictions regarding constructive interference, perhaps relying on a "more is more" p-prim to do so, but intuition failed to predict or explain destructive interference. The distinction between students in the fragmented category and the more sophisticated categories is that students did not engage in generating a mechanism. Specifically, students in the fragmented category either provided an explanation based on intuition or experiences without providing details with some students remarking *"this is just how it is."*

Developmental – Arthur

The developmental category describes students who provided accurate predictions of the simulations and who were able to generate more mechanistic explanations of light interactions, even if some knowledge elements were incorrect. Students in this group were also generally more comfortable with describing how constructive interference occurs compared to destructive interference. These students often considered fundamental wave behaviors and relied on relationships between frequency and wavelength, however, often lacked some relevant prior knowledge that would result in a detailed and accurate description of the double-slit experiment.

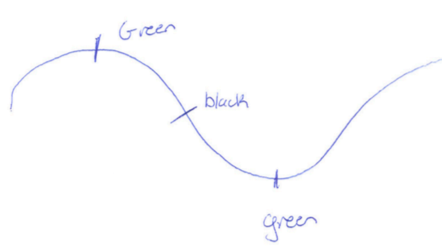
One student in the developmental category, organic chemistry student Arthur, relied on particle behavior to explain the single and double-slit experiment despite demonstrating a sophisticated understanding of wave features in the context of a single light source. Throughout his interview, Arthur tried to fit his ideas about light particles into his ideas about light waves and his observations of the simulations. He used light particles to explain interference and frequently made attempts to organize his ideas about particle behavior into his ideas about wave behavior.

At the beginning of the interview, Arthur explained the Single Continuous Light (Figure 1a) and attributed the pattern he observed to the nodes and antinodes of a wave. Drawing Figure 3, Arthur says:

> Green is the top of the peak. And then the black, I'm assuming would be the region between the two peaks.

Figure 3

Arthur's Drawing Depicting the Features of a Wave



Note. Arthur labeled the green and black regions of the Single Continuous Light simulation.

For the first simulation (Figure 1a), Arthur demonstrated a sophisticated understanding of wave features by assigning the green regions to the antinode and the black regions to the node (Figure 3).

After discussing Figure 1a, Arthur was asked to make a prediction about light passing through a single slit (Figure 1b). Arthur explained that as the light exits the slit, it will expand and emanate outward because light particles are energized and do not want to be next to another particle:

> It'll fan out in half circles again, sort of emanating from that slit and then expand out because light is energy. If you have all of these particles that are energized in one small space, they obviously don't want to be next to each other.

Arthur was correct in thinking about light as energy, however, he further explained that light radiates outward due to the light particles being energized. Here, he seemed to be conflating energy and charge, where energized particles do not "want" to occupy the same space. Despite productively describing the wave behavior of light earlier, Arthur generated an explanation rooted in the particle behavior of light to explain the outward radiation. While his prediction was correct, it did not build upon his previous knowledge about wave features. Rather, Arthur's explanation of energy repelling served as a p-prim. The use of the word "obviously" is a cue for identifying p-prims (diSessa, 2018) because to students, p-prims are generated from seemingly obvious observations of the world.

After his prediction, Arthur watched the Barrier with Single Slit simulation (Figure 1b). He remarked that the simulation somewhat showed what he expected but he was not expecting to see the "blurred region" near the top and bottom. He interpreted the blurred regions as "chaos" with particles spreading out. He then goes on to explain that it seems like lines are being reformed which he inferred as particles reforming as waves.

> That's kind of what I expected but there's this blurred region on the top and bottom which I was not expecting… You have these clean bars of light that are moving through [the slit] and then when they come out on the other side, it's kind of just mayhem and chaos as they're spreading out…It looks like it's starting to form lines again with the green-black alternation. So I'm guessing somehow it goes from being a mess of particles into somehow forming waves again. I suppose it has something to do with the attraction between the energized particles.

Arthur's interpretation of the simulation showed that he was thinking about particles of light and how that relates to light waves. He postulated that the light particles are spreading out and go on to reform into waves because there is some sort of attraction between light particles. In Arthur's explanation, he continued to build on the idea of energized particles. While he now considered energized particles both attracting and repelling, he still aligned this explanation to previous explanations by conflating energy and charge. We also can see that Arthur used particles of light to think about some big picture ideas regarding light behavior, in this case, diffraction.

Next in the interview, Arthur watched the Single Slit with Screen (Figure 1c). He explained that the brighter region on the screen is due to "clear waves" that have a certain amount of energy to make it to the screen, and the edges of the screen are not illuminated because the light particles have spread out and lost energy.

> I think that hazy portion is like a mayhem of particles and because it's not really organized into those individual unique distinguishable waves, it's not producing a lot of light [on the screen] because those energized particles are just spreading out, dissipating, and losing the energy. Whereas in the middle, because they're very clear waves, they have the energy and the endurance to make it all the way to the [screen].

Arthur connected his observations of the simulation to both particle and wave behavior. For instance, he attributed the hazy regions of Figure 1c to light behaving like a particle and described them as being disorganized. He later explained that the middle area of Figure 1c depicted light waves. Arthur's explanation of the Single Slit with Screen simulation showed that he tried to organize the ideas he has about particle and wave behavior by assigning particle behavior to certain aspects of the simulation and wave behavior to others.

After observing the single slit simulations, Arthur watched the Barrier with Two Slits (Figure 1d). He explained that the light exiting the slits collides, and when the light collides, they begin to move forward.

> So as the light exits both of those openings, it collides. And I suppose when it collides it, they kind of push off each other and start going in that forward direction instead of going straight outward.

Arthur explained that rather than light radiating outward, it has now collided with another light source which caused the light to travel forward.

After observing the Double-slit with Screen simulation (Figure 1e), Arthur explained that it reminded him of something he had seen in high school. Upon observing Figure 1e, he remembered previous observations of the double-slit experiment.

> I'm starting to think back to like high school when we did some experiment like this where we had those two slits in a sheet of paper. It makes sense that of course we get two light sources on that screen from each of the openings. But then those waves are colliding in the middle and kind of combining to organize themselves into a third [region].

Arthur described an experiment he observed in high school where he had a piece of paper with two slits. He went on to explain that Regions 2 and 4 are illuminated from the two slits, building off his earlier prediction. He also explained that Region 3 is a result of waves colliding and organizing into an illuminated region. Arthur recognized that some regions are a result of light sources interacting with each other. However, he did not extend that interaction to all illuminated regions.

When asked to further explain how light was combining to organize into a third illuminated region, Arthur brought up two analogies, sound waves and pool balls to help him describe his thinking. At this point in the interview, Arthur introduced a particulate explanation of light behavior that he relied on for the duration of the interview. He explained:

> If you have two sound sources, they kind of collide and then merge into one. Um, and I guess it's kind of the same thought process as if you had two pool balls and you were to push them toward each other. They're going to collide and then start moving forward because when they collide, they cancel out the side-to-side motion, if you will. And the only thing that's left is that forward motion.

Arthur's explanation of how light collides highlights his attempt to connect and fit particle behavior into wave behavior in the context of the double-slit experiment. This explanation also shows how Arthur used particle behavior, in this case, the analogy of pool balls, to think about light waves colliding. Arthur built upon his previous explanations grounded in particle behavior to explain how the collision of light results in the illuminated middle region (Region 3 on Figure 1e).

Arthur was then asked to explain why some regions on the screen were dark on the Double-slit with Screen simulation (Figure 1e). He built upon his ideas about light organizing into waves and how it collides and ultimately moves forward. He goes on to further explain that the remaining light that is not organized into waves is scattered light. The scattered light resulted in dark regions on the screen and Arthur related that to intensity:

> Some of [the light] is organized and moving forward and then some of it is thrown off from the collision and scattered. And so it's much lower in intensity than the [light] that was organized in a single wave.

This explanation provided by Arthur shows that he continued to fit particle-like behavior, specifically collisions and scattering, into his explanation of light waves. Further, he connected the ideas of scattering and colliding to intensity. This explanation highlights that Arthur was considering the observations from the simulation and aligned to his prior knowledge of wave-like properties such as intensity. It also shows that Arthur relied on particle behavior to generate a mechanism of light interactions. However, Arthur's knowledge of wave interference was limited, which resulted in a reliance on particle behavior to generate explanations for his observations.

Towards the end of the interview, Arthur was asked to draw conclusions about light behavior based on his observations of the simulations. He concluded that these simulations are evidence of light acting as particles and explained that light particles organize themselves into waves.

> This supports the theory that light acts as particles. Somehow those particles come together and organize themselves into light waves. And that's why we see [the diffraction pattern]. Because even though they're acting in that weird manner when they first exit the two openings, they still are able to come together and form very distinct [lines].

Arthur conclusively stated that the observations from the double-slit experiment show that light acts as particles. Throughout the interview, Arthur continued to build upon the idea that light acts as a particle and frequently tried to fit his ideas about light particles into big picture ideas about light waves. Here, Arthur discussed how the particles exit the slits and eventually organize into distinct lines.

When asked how the light particles organize themselves into lines or light waves, Arthur postulated that the organization was related to the frequency of the light particles. He explained that because the light particles are all traveling with the same frequency, they can organize themselves resulting in an increase in intensity:

> I suppose it has something to do with the frequency of the light particles. If they're traveling at that frequency before they hit the opening, and then if they're still maintaining that frequency as they're moving through the opening, then they should align with the other particles because they're moving the same frequency. Even though they're spreading out and dissipating, you still have all these particles traveling at the same frequency. So some of them that are going in the same direction are going to line up and their intensity will be increased as a group.

Arthur explained that upon exiting the slit, the particles of light are spreading out, but all of the particles of light are traveling with the same frequency. Because of this, the particles that either move forward directly out of the slit, *or* collide and move forward, will line up and increase the overall intensity. In this explanation, Arthur combined ideas productively to explain his observations of the double-slit experiment. He considered the role of intensity and how that was related to the amount of light on the screen. He further connected that to how particles collide as they exit the slits to move in a forward direction. He also correctly incorporated the relationship between the number of light particles and intensity.

During Arthur's interview, he initially described the simulations using wave-like properties and then transitioned into explaining light as particles, indicating context dependent explanations and fragmentation. Arthur transitioned from wave to particle behavior once the slits were introduced in the simulation and light was now interacting with another object. He began to generate explanations heavily focused on collisions and energized particles. While Arthur's ideas of energized light particles are scientifically non-normative, he consistently returned to particle behavior to explain his observations of light behavior. When explaining the double-slit experiment, Arthur recalled conducting a similar experiment in high school. This was productive for Arthur, indicating he connected his observations during the interview to prior observations of the experiment. Following this connection, Arthur continued to organize his ideas about light particles into his observations of wave behavior. For instance, he considered wave-like properties such as frequency and intensity to explain how light particles organized themselves into waves resulting in the pattern on the screen.

While Arthur's explanation of wave interference is not correct, he made productive attempts to generate a mechanism for the simulations he observed. He aligned his explanations of the simulations with his prior knowledge of particle behavior and wave features such as frequency and intensity. He used particle behavior to help explain the wave behavior he observed in the double-slit experiment and to generate a mechanism of the resulting diffraction pattern. Arthur is evidence of the developmental category because his ideas are still partially fragmented with his use of p-prims. However, Arthur coordinated knowledge elements together by continuing to use energized particles as an explanatory tool and coordinated this with extractions from the simulations. Despite his inferences about particle behavior being incorrect, they are temporally stable and served as a tool for generating a mechanism of his observations of the double-slit experiment. Like Arthur, other students in the developmental category relied on particles to explain the merging or canceling of waves and often connected that to the dual nature of light. Other students in the developmental category also displayed a less sophisticated understanding of interference, which surfaced when they incorporated ideas of interference when considering both the single and double-slit experiment.

A key distinction between developmental students and fragmented students was their ability to engage in the generation of a mechanism of interference. With their mechanisms, we observed students making connections between the simulations they observed and including elements of prior knowledge of light behavior. However, the developmental students' explanations are less sophisticated and often included incorrect knowledge elements, which set them apart from the most sophisticated group of students.

Coordinated – Destiny

The third category, coordinated, describes students who easily accessed and applied useful prior knowledge and generated a mechanistic explanation of the double-slit experiment. Students in this category explained both constructive and destructive interference and the effect of changing the wavelength of light. Students were categorized as coordinated when they provided explanations that included relevant prior knowledge and built on that prior knowledge throughout the course of the interview. Destiny, a student in the general chemistry course for chemistry majors, fell into the coordinated category. Throughout the interview, Destiny incorporated relevant pieces of knowledge and displaced pieces that were not relevant. Additionally, she exhibited span across simulations by recognizing the role of interference in each simulation.

After observing the Single Continuous Light (Figure 1a) and the Barrier with Single Slit (Figure 1b), Destiny was asked to predict what would appear on the screen after the Barrier with a Single slit (Figure 1c). In her prediction, she described light passing through the center of the slit where the light hitting the edge of the barrier would deflect or diffract. Here, Destiny was unsure of which vocabulary word best describes what she was predicting. However, she was able to describe what she was thinking:

> I guess just that if it passes through the center of the slit, then it has minimal impact or deflection or diffraction or whatever the proper word is to use. And so essentially, you know, the very center will keep on a straight path, but beyond that it will kind of be turned outwards.

Despite the vocabulary barrier Destiny experienced, she explained that she would expect to see the light interacting with the edges of the barrier and turning outwards which indicated her understanding of light diffraction.

Destiny then watched the Single Slit with Screen simulation (Figure 1c) where she explained why she observed light radiate outwards:

> [With] only one slit, there isn't actually, there's no other wave to interfere with, just kind of photons amidst their own wave, there isn't really another wave for them to interfere.

Destiny explained that with only one slit, there was only one light source, or one wave, exiting the barrier. Because of this, she recognized there was no reason for interference with another wave to occur. Destiny's explanation highlighted her displacement of interference in this context. Specifically, she recognized that interference was not a relevant resource to employ.

Later in the interview, Destiny was asked to predict and explain the Double-slit Experiment with the Screen (Figure 1e). She described that she expected to see an interference pattern with alternating bars of light and dark regions. She further explained that the pattern is a result of constructive and destructive interference because there are now two waves exiting the barrier:

> An interference pattern of bright and dark little bars not continuing on since there's only the two slits, but like kind of getting dimmer and like less distinct as you move out (motions with both pointer fingers spreading out to represent the spreading of light). [There will be] bright spots and dark spots intermittently as the lights interact and to have constructive or destructive interference with the two different waves that come from the individual slits.

Here Destiny appropriately incorporated her prior knowledge of constructive and destructive interference. She recognized and explained that because the barrier with two slits resulted in two different waves exiting the barrier, interference occurred. This aligned with her explanation and understanding of interference earlier in the interview, where she determined interference was not occurring in the simulation depicting Barrier with Single Slit (Figure 1b) and Single Slit with Screen (Figure 1c).

After Destiny predicted and explained the double-slit experiment, she was then asked to predict what she would expect to happen if the frequency of light changed. Up until this point in the interview, each simulation depicted a green light. When asked to consider the effect of frequency, students compared a red and violet light (Figure 1f). Destiny explained that with a change in frequency, she would not expect to see any changes in intensity. Instead, she predicted that changing the frequency would impact the angle of interference by considering Equation 1.

$$d \sin \theta = m\lambda \qquad (1)$$

> So, I'm trying to think here. Frequency. If you raise the frequency, obviously you would lower the wavelength. I guess you would see a change in the angle at which the bright spots appear depending on if we raised or lowered the frequency … But you wouldn't see a change in like brightness or anything like that…Constructive and destructive interference is dependent on the wavelength. If you think of it like a wave, then you want the crest and the trough to cancel

> out. And when you change the frequency, then you change the distance between those crests and troughs. And so when you change the frequency, you in turn change the wavelength, which impacts that distance. And that distance is related to the angle (of interference) at which the bright spots and dark spots appear.

In Destiny's prediction, she explained that the instances of interference were related to the wavelength or the distance between the peaks and troughs. She exhibited an understanding of the relationship between wavelength, frequency, and intensity by explaining that the brightness will remain the same. Destiny incorporated relevant prior knowledge of how interference occurs and aligned that with her understanding of changes in wavelength. This resulted in a prediction accompanied by a thorough explanation of constructive and destructive interference.

Following her prediction, Destiny observed the Double-slit at Two Different Wavelengths simulation (Figure 1f). She saw that the simulation matched her prediction, and she further explained why she observed more violet bars than red bars:

> I see more bars [with violet] because the angle at which those bright spots and dark spots appear is smaller. You know, it's the distance between, um, it's like there's a relationship, it's not random where they appear at what angle because they maybe are closer together. [For the red] there's more out here [pointing off screen], you just can't see them because the screen is too small in a sense that the angle (of interference) is larger and so it kind of goes off of your visible area.

Destiny explained that if the screen had been larger, she would have observed more red bars. Because Destiny was able to recognize that the wavelength impacts the spacing between the instances of interference, she has shown that she has a mechanistic understanding of interference. Further, all of Destiny's explanations were grounded in the wave behavior of light, which indicated her understanding that the simulations are evidence of wave behavior rather than particle behavior. Additionally, Destiny exhibited span throughout her explanations of each simulation by applying the resources of interference across the simulations through both displacement—recognizing the inappropriateness of the resource—and alignment—recognizing the relevance of the resource.

Over the course of Destiny's interview, she displayed a robust understanding of how light waves travel and how they interfere with one another. Destiny exhibited a coordinated framework of ideas by incorporating and displacing relevant knowledge elements and aligning her prior knowledge with extractions from the simulation. She accurately determined the relevancy of interference across contexts by displacing interference with one source of light and incorporating interference with two sources of light. Her ability to recognize the relevancy across contexts also corresponded to a detailed mechanism of how interference occurs. Her understanding of the mechanism of interference allowed her to explain that interference requires two light sources. Additionally, Destiny's framework is considered coordinated because she exhibited span across the multiple simulations because her understanding of interference is consistent across contexts (i.e., the multiple simulations over the course of the interview). This indicates that she was able to access relevant prior knowledge and coordinate with extractions across simulations, which ultimately led her to generate scientifically normative explanations of the double-slit experiment.

Conclusions

Based on our qualitative investigation of students' understanding of the wave nature of light, we found a variation in student reasoning ranging from fragmented to coordinated. Informed by our theoretical framework, we categorized students' knowledge structure as fragmented, developmental, or coordinated.

We observed students using their daily observations of light to explain wave behavior with some observations being more relevant and productive than others. One productive experience that was frequently used by students to explain the single slit was light coming into a dark room with the door ajar. The observations from this experience were transferred to the single slit often resulting in an accurate prediction. Other experiences were less productive. For example, some students relied on shadows to explain the dark regions on the screen due to destructive interference. Rather than developing an explanation grounded in light waves interfering, students relied on their experiences and intuition. Similar to other studies with physics students, the absence of a basic wave model or a limited understanding of wave phenomena prompted intuitive reasoning (Henriksen et al., 2018; Singh, 2020).

In the developmental knowledge structure category, many students showed productive reasoning and attempts at aligning relevant prior knowledge. However, their limited understanding of wave behavior often resulted in partially correct explanations. Many students within this category were comfortable with the concept of constructive interference, but only applied it to the middle-illuminated region, rather than all illuminated regions. In addition, destructive interference proved to be particularly challenging with some students introducing particle-like behavior to explain the dark regions on the screen which could be influenced by a chemistry context requiring students to consider the behavior of small particles.

Students who exhibited more expert-like knowledge structures were able to accurately displace and incorporate relevant prior knowledge to explain the mechanism of interference. Students in this category provided detailed explanations of how interference was occurring, and which regions on the screen corresponded to constructive or destructive interference. Students verified a mechanistic understanding by generating accurate predictions when considering two different frequencies of light and further explaining the relationship between wavelength and instances of interference. Students who demonstrated a more robust understanding of wave phenomena are likely to be more comfortable extending these ideas when considering the dual nature of matter.

After grouping the students into three categories, we examined the distribution of students by course (see Table 2). The majority of general chemistry students were categorized as fragmented, which could be because this is the first time many of these students have been introduced to light behavior. In the general chemistry for majors course, some of the students we interviewed were either double majoring in physics (e.g., Destiny) or enrolled in other physics courses, which accounts for a large number of GCM students falling in the developmental or coordinated groups. We saw that most organic chemistry students were categorized in fragmented or developments. This may be because OC students have likely not received additional instruction on the conceptual basis of light behaviors, but rather, received more instruction regarding the light-based instrumentation techniques such as spectroscopy. Finally, our sample of physical chemistry students was limited to one student and therefore limits the drawing of any conclusions about which group physical chemistry students at large may belong to.

Based on our observations of student reasoning and the structures of their knowledge frameworks, certain areas can be specifically addressed in instruction. We observed many students struggle with the features of waves, particularly nodes and antinodes, which limited their understanding of interference. This resulted in students generating explanations for constructive interference rooted in intuition because it is relatively intuitive that the region on the screen directly across from the barrier is a result of waves joining together. This was further evidenced by the fact that many students did not assign constructive interference to all regions on the screen and provided non-normative explanations for destructive interference regions. It is important to note that across all levels, students incorporated instances of intuition. However, there were differences in the use of intuition. Students who exhibited more novice-like knowledge structures relied on intuition to explain observations where they had limited prior knowledge. In contrast, students exhibiting more expert-like thinking incorporated intuitive reasoning with their prior knowledge and those students could provide more detailed explanations following their intuitive explanations.

Understanding light waves has implications for learning the dual nature of matter. In first-year chemistry, the double-slit experiment is one of the first steps to introducing the dual nature of light and matter. We expect students to transfer their ideas of light waves to a new context, specifically electrons. When introducing the complex idea of duality, it might be assumed that students already understand basic wave phenomena. However, it is important to first make sure students possess a detailed understanding of the wave model in the context of light before expecting them to apply that model to matter. Students in this qualitative investigation, regardless of their knowledge structure, used the PhET simulations to generate informed predictions and explanations. Presenting wave behavior in this sequential manner could be beneficial in promoting a coherent understanding of wave behavior. Finally, it would be beneficial for the chemistry and physics community to discuss light-matter interactions, as this is a central idea across science disciplines. Together both communities can better support students by having conversations about how the double-slit experiment is introduced across introductory courses and the features each community attends to when considering the wave behavior of light and matter.

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The Genesis of Routines: Mathematical Discourses on the Equal Sign and Variables

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ABSTRACT

This study explores high school students' and their mathematics teachers' mathematical discourses on the equal sign and variables in the classroom context. The participants in this case study were ninth graders and their mathematics teachers. The data were analyzed in terms of participants' routines, specifically ritual and explorative routines in the classroom context, through a commognitive perspective. Results indicate that teacher discourse governed process-based routines that have roots in elementary grades. Students focused more on operational approaches instead of algebraic reasoning while solving equations, and this finding conveys challenges in constructing variables.

Keywords: equal sign, variables, routines, high school

Introduction

Equal signs and variables are essential in international and national K-12 curricula and mathematics education research (Ministry of National Education (MoNE), 2018; National Council of Teachers of Mathematics (NCTM), 2005). Students are exposed to the equal sign and variables beginning in the early grades (primary school) and in most mathematical topics (e.g., operations, ratios, first-order equations, and functions) throughout the later grades. The equal sign's pervasiveness at all levels of mathematics emphasizes its significance (Stephens et al., 2013). The equal sign and variables are fundamental components of algebraic thinking and reasoning, which enable students to consider mathematical concepts in an objectified manner instead of as an arithmetic problem that needs to be solved (Kieran, 2004). Kieran (2004) recommends focusing on algebraic thinking and not just correct calculations when working with numbers and variables. To develop students' algebraic thinking, Kieran also suggests tasks that involve comparing algebraic expressions to determine equivalence and investigating the meaning of the equal sign.

Studies have focused on elementary mathematics teachers' knowledge of student thinking on core algebraic concepts, such as the equal sign and variables (Asquit et al., 2007). Among students' struggles when dealing with equations that include unknowns is the concept of variables, which are an integral part of equations, especially starting in the elementary grades. Further, variables play a substantial role in mathematics education research (Küchemann, 1978; MacGregor & Stacey, 1997; Philipp, 1992; Usiskin, 1988). These studies' findings indicate that students encounter challenges using literal symbols in algebra. Küchemann (1978) explored students (13-, 14-, and 15-year-olds) who recognize literal symbols as objects. In contrast, few interpret such symbols as unknown numbers with a fixed value as specific unknowns, a generalized number representing multiple values, or variables representing a range of numbers. Moreover, interpreting letters as variables involves systematic value changes (Küchemann, 1978). In sum, understanding equivalence and variables is essential to algebra, along with the ability to use these concepts with algebraic thinking and reasoning (Knuth et al., 2005). It is significantly more typical for young pupils to perceive the equal sign operationally, as a command to perform computations, rather than relationally, as a signal of equivalence or sameness (e.g., Kieran, 1981; McNeil & Alibali, 2005a; Renwick, 1932). Researchers have shown that using the equal sign operationally can reveal difficulties with algebraic reasoning and errors when solving equations with missing values (McNeil & Alibali, 2005a; Powell, 2012).

Given these challenges for students regarding variables and the equal sign, students may hesitate to join classroom discourse. It is daunting for students facing difficulties to join conversations with their teachers and peers. Without participating in classroom discourse and discussing mathematical concepts, students may struggle to move from ritual routines to explorative routines. Rituals are process-oriented routines, representing the first step in joining a classroom discourse and entail rigidly following a previously performed procedure. On the other hand, explorations are outcome-oriented and self-oriented flexible routines (Lavie et al., 2019). To fully comprehend and help students overcome their challenges, teachers have a vital role. So, relating students' thinking to teachers' teaching from a socio-cultural perspective can provide us with important insights. By extensively considering the literature, researchers have separately explored students’ and teachers’ understanding of equations and variables through cognitive perspectives, mainly in elementary and middle grades. Kieran (2004) has suggested that a more comprehensive analysis, than that reported in the existing literature, is needed to generate rigorous and thorough findings. Little is known about classroom discourse as a means to understand students’ thinking by relating it to teachers’ instruction about equations and variables. Although several studies on equations and variables are based on elementary and middle grades (Asquit et al., 2007; Carpenter & Levi, 2000; Kieran, 2004; Schifter, 1999), few studies have focused on the high school level. This study investigates classroom discourse on the equal sign and variables at the high school level. Thus, our research question is: What are the characteristics of high school students and their teacher’s mathematical discourses on the equal sign and variables in the classroom context?

Theoretical Framework

Operational View to Relational View

The operational view has a process-oriented structure such as “doing the operation,” “adding the numbers,” and “calculating the answer.” On the other hand, the relational view expresses equivalence in an objectified manner (Stephens et al., 2013). Skemp (1976) summarized operational conception as a rule without any reasoning.

The equal sign is fundamentally a relational symbol that denotes the similarity or interchangeability of the quantities that either side of an equation represents (Carpenter et al., 2003; Fyfe et al., 2020). Research has indicated that it is significantly more typical for young pupils to perceive the equal sign operationally, as a command to perform operations, rather than relationally, as a signal of equivalence or sameness (Kieran, 1981; McNeil & Alibali, 2005a). Students face challenges transitioning from the operational view to algebraic thinking (Asquit et al., 2007; Bednarz et al., 1996; Kieran, 1992; Wagner & Kieran, 1989). Kilpatrick et al. (2001) observed that students consider the equal sign to be a left-to-right and/or right-to-left directional signal. Similarly, researchers have found that algebra in elementary grades is mainly based on equation manipulation instead of algebraic reasoning (Asquit et al., 2007; Carpenter & Levi, 2000; Kieran, 2004; Schifter, 1999). Steinberg et al. (1990) concluded that most students do not interpret the meaning of two given equations that are equivalent; instead, they focus on solving equations as an operation. Researchers have argued that treating the equal sign operationally can convey challenges in algebraic thinking and mistakes in solving equations with missing numbers (McNeil & Alibali, 2005b; Powell, 2012). Researchers have also mentioned that students should interpret the equal sign from a relational view, that considers the symbol to mean the same, rather than from an operational perspective, that considers the symbol as an arithmetic operation (Falkner et al., 1999; Jacobs et al., 2007; Kieran, 2004; Rittle-Johnson & Alibali, 1999; Knuth et al., 2005; Sherman & Bisanz, 2009).

The overall findings show how deeply ingrained the operational view is and how crucial it is to keep investing effort into supporting students' development of a relational understanding (Stephens et al., 2013). It is important to encourage teachers to see challenges regarding equation tasks as opportunities to use and develop an understanding of the equal sign, rather than just as a chance to practice typical equation-solving methods (Asquit et al., 2007). Studies have suggested that it is vital to discuss both the operational view and the relational view of the equal sign with students while performing equation tasks. Visually supported tasks can encourage students with limited early algebraic knowledge, who often use the equal sign as computing, to interpret the equal sign and equations relationally (Stephens et al., 2013).

To achieve a profound depth of conceptual knowledge, students need a practice that goes beyond memorization of rules and processes. There is a need for professional development initiatives that concentrate on building relationships between middle school algebra instruction and concepts once thought of as belonging to the area of arithmetic (such as understanding the equal sign and developing number sense; Asquit et al., 2007). Understanding how students think about variables and equal signs is crucial for teachers, and expanding their understanding will make it possible for them to focus more closely on students' struggles and communication in relation to the equal sign and variables (Asquit et al., 2007).

Signs, Symbols, and Objects

By nature, mathematics comprises mathematical signs, symbols, and objects. Brousseau (1997) argued that the primary pedagogic goal of mathematics instructors' symbolic practices is to communicate mathematics. Mathematical signs and symbols are the main sources for characterizing mathematical knowledge, communicating mathematical arguments, and performing and generalizing it (Steinbring, 2006). While teaching and learning mathematics, we need to use mathematical signs as an instrument for communicating with other people, ourselves, textbooks, and curricula (Sfard, 2008; Vygotsky, 1987). It is therefore fundamental to understand the importance of signs and their relationship with mathematical objects. A sign can be a mathematical symbol, statement, expression, or object in the context of mathematics education (Berger, 2004). According to Tachieli and Tabach (2012), a mathematical object exists between symbols rather than within any of them; hence, no mathematical object can be defined through a concrete object. Students can engage with the mathematical object and communicate with other participants in the discursive community to develop mathematical ideas using mathematical signs (Berger, 2004).

Commognitive Perspectives

I utilized the commognitive perspective in this study. “Commognition” is a hybrid word that is a combination of “communication” and “cognition” (Sfard, 2007). The commognitive view formulates thinking as self-communication, and this formulation eliminates the dichotomy between thinking and communication (Sfard, 2008). Thinking is the activity of communicating with oneself (Sfard, 2012) and mathematics can be seen as a discourse, a particular type of communication (Sfard, 2008). Participants do not have to communicate to be part of the same discourse community; however, participation in communication activities enriches participants’ sense of belonging to the broader discourse community (Sfard, 2007). According to commognition, mathematical discourse can be identified through the use of mathematical keywords; visual mediators that refer to graphs, diagrams, algebraic notations, and figures; routines that are repetitive patterns; and endorsed narratives that are substantiated by other elements of discourses (Sfard, 2008). An endorsed narrative is "regarded as reflecting the state of affairs in the world and labeled as true" (Sfard, 2008, p. 298). For instance, endorsed narratives are theorems, definitions, and lemmas in mathematics (Sfard, 2008).

Routines are defined as a set of meta-rules that explain a repetitive action (Sfard, 2008), and a routine is a known pattern of action in a task situation (Lavie et al., 2019). In this study, I focus on examining routines concerning rituals and explorations. Rituals are "sequences of discursive actions whose primary goal (closing conditions) is neither the production of an endorsed narrative nor a change in objects, but creating and sustaining a bond with other people" (Sfard, 2008, p. 241). In a ritual, participants align with other participants in routines in the community, which can be considered social approval (Berger, 2013). Regarding exploration routines the "goal (closing condition) is the production of [an] endorsed narrative" (Sfard, 2008, p. 298). Explorations are routines encompassing solving equations, proving a mathematical result, or generating and investigating a mathematical conjecture (Berger, 2013). For example, during ritual participation, the learner will ask themselves, "How do I proceed?" In explorative participation, the learner will inquire, "What is it I want to get?" (Lavie et al., 2019).

Routines involve not only procedures (the course of action) but also tasks, so analyzing the patterns in a task situation is essential (Lavie et al., 2019). Completion of routines indicates circumstances constituting successful completion of the performance, including how the routines ended in a task situation (Sfard, 2008). Explorations and rituals differ mainly by the types of tasks and their closure (Sfard, 2008). Students' previous experiences establish precedence and heavily influence the routines they use in a new task situation; further, learning occurs through the routinization of students' actions (Lavie et al., 2019). Sfard (2008) asserted that ritual is an inevitable stage in routine development, and new mathematical routines, which are rituals, may evolve into explorations. The participant who does not have a clear idea of when a routine can be implemented may eventually be capable of implementing it independently (Sfard, 2008).

A commognitive perspective provides a lens to understand communicational bindings on mathematical concepts between people and/or materials. Catching communicational failures provides us a perspective on students' understanding and teachers' teaching. If mathematically inconsistent concepts are found in communication, which is labeled as communication failures, the reasons for communicational failures should be explored to comprehend the discourse of the community in general and student discourse in particular. Communicational failures provide a means to catch students' struggles with specific mathematical concepts. Communicational failures are generally observed in rituals because rituals are process-oriented routines to obtain social approval.

Nachlieli and Tabach (2019) presented a methodological lens about ritual-enabling and exploration-requiring learning occurrences. Initiations and closures show when, and the mechanism for how, a routine occurs (Sfard, 2008). Ritual-enabling learning opportunities are described as using a previously recognized procedure. In contrast, the term "exploration-requiring opportunities to learn" is defined as pupils being unable to complete a task just by following a ritual; rather, they need to participate exploratively in developing mathematics narratives concentrating on predicted results (Nachlieli & Tabach, 2019). The teacher will prompt usage of words such as "what," "why," "find," and "explain" during explorative engagement.

In this study, I use Nachlieli and Tabach's (2019) methodological lens to analyze classroom discourse on the equal sign and variables. Analyzing students' routines in classroom discourse enables us to examine the characterization of the classroom discourse. Having investigated students' discourses within their context, I understand its features by relating them to teaching and interaction in the classroom. Due to the nature of this investigation, we can provide suggestions for teachers' discourses as facilitators of students' discourses on the equal sign and variables.

Methodology

Context of the Study

Constructivist perspectives have governed the mathematics curriculum in Turkey (Zembat, 2010); however, the primary teaching approach in Turkey is still direct instruction (Emre-Akdoğan et al., 2018). Learning outcomes for equations and variables start in middle school according to Turkey's education curriculum, specifically Grade 6. At this grade level, students are expected to comprehend algebraic expressions. The letters in algebraic expressions represent numbers and are defined as variables (MoNE, 2018). Algebraic expressions, equity, and equations are learning domains in Grade 7. Students should understand the concept of equity, subtract and add algebraic expressions, and solve first-order one-variable equations. In Grade 8, algebra occupies a more significant part of the curriculum, comprising algebraic expressions, linear equations, and inequalities. Students are expected to recognize algebraic expressions and algebraic factor expressions. The foundations of algebra are laid in Grades 6 and 7. Fundamental concepts, such as equations and variables, have a more prominent place in the middle grades. In Grade 9, students are expected to solve first-order one-variable equations. Every ninth grader in Turkey has followed the same curriculum, with specific learning outcomes.

Participants and Data Analysis

I collected data in an urban school in Turkey's capital from a class of 32 students. In comparison to other ninth graders, the students' success rates were average. The participants in this case study were ninth grade, 15-year-old, high school students and their teacher, Mrs. Seda (a pseudonym). I selected Mrs. Seda's class for our study because she was willing to participate in the research, and she communicated her own and her students' experiences expressively and reflectively to facilitate purposeful sampling and rich, in-depth data collection (Patton, 2002). The data for the study was collected through two classroom observations, each lasting 45 minutes. I focused on the students' and teachers' utterances and actions while transcribing the classroom observations. Analyzing the participants' utterances and actions enabled us to investigate communication failures in the classroom discourse (Emre-Akdoğan et al., 2018). The observed classes were conducted in the participants' native language and then translated from Turkish to English. The transcripts of the classroom observations include participants' utterances and actions. Data regarding participants' routines, specifically ritual and explorative routines in the classroom context, were analyzed (Sfard, 2008).

As indicated in the Theoretical Framework section, we used Nachlieli and Tabach's (2019) methodological lens regarding ritual-enabling and exploration-requiring learning opportunities. We defined exploration-requiring learning opportunities as explorative routines and studied data on how (procedure) and when (initiation and closure) explorative routines occurred. We defined ritual-enabling learning opportunities as ritual routines and studied data on how (procedure)and when (initiation and closure) ritual routines occurred. A routine comprises three parts: initiation, procedure, and closure. The conditions under which a procedure is invoked and by whom, as well as the conditions under which a procedure is regarded as complete, are referred to as initiation and closure, respectively (Nachlieli & Tabach, 2019). Details regarding Nachlieli and Tabach's (2019) methodological lens can be found in Table 1.

According to Miles and Huberman (1994), intercoder reliability enables researchers to give "a clear, unified picture of what the codes mean" (p. 64). First, I independently coded all transcripts, and then a researcher with competence in cognition theory was invited to code the classroom observation transcripts. The number of agreements and discrepancies between the author and the researcher was counted during coding cross-checks. As a measure of intercoder reliability, the ratio of the number of agreements to the number of agreements plus disagreements was utilized (Miles & Huberman, 1994). We achieved 90% intercoder reliability.

Table 1

Ritual and Explorative Routines (Nachlieli & Tabach, 2019)

		Ritual routines	Explorative routines
Initiation (Task)	What question does the teacher pose (raise)?	How do I proceed? How can I enact a specific procedure?	What is it I want to get?
Procedure	How is the routine procedure determined?	Students are expected to follow a certain procedure that others in similar circumstances have previously used. They are not expected to make independent decisions.	Students are expected to choose from a set of procedural options. They are expected to make independent decisions.
Closure	What type of answer does the teacher expect?	A final solution	An indication of the newly produced narrative
	Who determines the end condition?	The teacher	The student (based on mathematical reasoning)

Results

In this study, I focused on the teacher's and students' discourses on variables and equations in the classroom context at the high school level. During the two classroom observations, the teacher and students worked on 16 tasks, and the students clarified questions on the tasks and the topic of first-order, one-variable equations. I analyzed the participants' routines according to initiations, procedures, and closures (Sfard, 2008). I observed that the teacher primarily asked the students to perform tasks in the classroom. I investigated the routines based on the tasks the teacher assigned and the students' clarification of the questions regarding their challenges in the classroom. I observed five routines during the classroom observations on equations and variables within the tasks that the teacher assigned in the classroom (Table 2).

Table 2

Types of Routines Identified Through Classroom Observations

Routines	Tasks
Ritual 1: Left-to-right and/or right-to -left directional signal	TASK1, TASK6, TASK7 (closure: unclear for students), TASK8, TASK9, TASK10, TASK15
Ritual 2: Cross-multiplying	TASK6, TASK7, TASK8, TASK9, TASK10, TASK11, TASK13
Ritual 3: Cancelling factors (e.g., cancel x cubed)	TASK 2 (closure: unclear for students), TASK3 (closure: unclear for students)
Ritual 4: Deciding on a solution using an equation (If - 15 = 5, then the solution set is empty; if 0 = 0, then the solution set comprises infinite numbers.)	TASK5 (closure: unclear for students), TASK14 (closure: unclear for students), TASK15, TASK16
Ritual 5: Moving backwards by encircling	TASK10 (closure: unclear for students), TASK11 (closure: unclear for students)

How can we differentiate between ritual routines and explorative routines? Performing a ritual routine usually entails replicating someone else's previous performance; the procedure is strictly followed, and the performer rarely attempts to make individual decisions. In an explorative routine, on the other hand, students engage in discourse to create new narratives or decide amongst various options (Nachlieli & Tabach, 2019). Depending on the initiation and closure of the practiced routine, the same procedure might be regarded as a ritual routine or an exploration routine (Nachlieli & Tabach, 2019). In this study, the routines had an operational structure that I explored in the classroom discourse, based on the processes labeled as rituals. Additionally, participants rigidly followed the procedure the teacher presented in class and did not engage in individual decision making.

During the classroom observations, the teacher's discourse was governed by two rituals (Rituals 1 and 2), namely left-to-right and/or right-to-left directional signal and cross-multiplying. These routines had an operational structure based on the relevant processes. The teacher initiated the task, and as seen in Table 3, the students rigidly performed the procedure. Closure was attained with the provision of the answer to the task problem the teacher assigned. The following are excerpts from the classroom discourses on Ritual 1, left-to-right directional signal, and Ritual 2, cross-multiplying (Table 3).

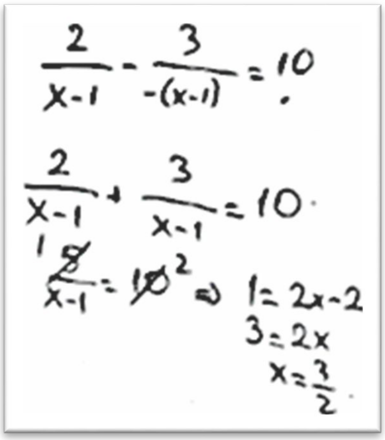
Table 3

Ritual 1 (Left-to-right and/or Right-to-left Directional Signal) and Ritual 2 (Cross-multiplying)

Initiation	Teacher:	Solve this equation $\frac{2}{x-1} - \frac{3}{1-x} = 10$. What should we do?
Procedure	Student 1:	Can we cross-multiply?
	Student 2:	By multiplying.
	Teacher:	There is no multiplication here. Yes, you can speak [*pointing to another student*].
	Student 2:	We can multiply one of them (*indicating one of the fractions*) with a negative sign, then cross-multiply.
	Teacher:	Multiplying one of them with a minus sign—why?
	Student 2:	To equalize the denominators.
	Teacher:	Can I write it like this to equalize the denominators? [*starts doing calculations*] What did I do with this denominator? I bracket the minus sign [*bracketing the minus sign to the fractional* $\frac{3}{1-x}$], right? $1 - x$ means x - 1 with the minus bracket. Multiply minus with minus; what will it be?
	Student 1:	Plus.
	Student 4:	Is the answer 4?
	Teacher:	Now, I can add with the same denominator. Are the denominators equal? Five over x - 1 is equal to 10.
	Student 5:	Cross-multiplying.
Closure	Teacher:	$2x - 2$. We throw this here [*showing the other side of the equation*]. What happened? 3? 3 is equal to $2x$, and x is equal to $\frac{3}{2}$. See Figure 1.

Figure 1

Solving the equation $\frac{2}{x-1} - \frac{3}{1-x} = 10$



The students became accustomed to working on left-to-right and/or right-to-left directional signal and cross-multiplying rituals during their elementary grades. However, they had no previous experience with the other rituals (Rituals 3–5). Hence, their struggles with Rituals 3–5 were revealed during the classroom observations. The genesis of the routines may have an explorative structure; however, the teacher's discourse included operational explanations of the routines based on the rules. By observing the routines' closure, I found that the students' discourse was unclear for these routines.

The canceling ritual (Ritual 3) includes a process-based interpretation of variables' canceling factors, that is, a particular way of defining the parameters of first-order differential equations to simplify terms. As seen in Table 4, closure was reached with the provision of the answer to the task problem the teacher assigned. I will provide a classroom discourse initiated by a teacher-assigned task (Table 4).

Table 4

Ritual 3. Canceling Factors

Initiation	Teacher	If the given equation is a first-order equation dependent on x, find the solution set of the equation [$(a-2)x^3 + (b-3)x^2 + 4x + 2a + 4b = 0$].
Procedure	Student 2:	Do we try to cancel x^3 and x^2?
	Teacher:	Yes, cancel x^3 and x^2. This is the third degree; this is the second degree; then, what will happen to them? [*showing* x^3 *and* x^2] What should it be? $a-2$ should be equal to 0; then, a is equal to 2. $b-3$ should be equal to 0; then, b is equal to 3. Then, let's write them in the equation: $(2-2)x^3 + (2-2)x^2 + 4x + 2.2 + 4.3 = 0$ [*writing the a and b values in the equation*]. Here [*showing* $2-2$], what happened to 0? Here [*showing 3*-3], what happened to 0 multiplied by 0? $4x + 4 + 12 = 0$, and $4x + 16 = 0$ [the *students repeat the same equation*].
	Student 1:	Put 16 on the other side of the equation.
	Student 1:	4.
	Teacher:	$x = 4$, right?
	Student 2:	$x = -4$
Closure	Teacher	So, the solution set is -4. Yes, where do we use the curly brackets? For the solution sets because they are the sets of points.

Ritual 4 includes deciding the solution set of the given first-order equation by interpreting a final equation—for instance, if -15 = 5, the solution set is empty, or if 0 = 0, the solution set is infinite—without discussing the mathematical thinking behind these equations. In Task 14, the teacher asked the students the following question: "If the solution set of this equation $m\ (2\text{-}x)$ = nx + 4 is infinite, then what is n?" This task initiated the classroom discourse, and the student and teacher discourses are given in Table 5. I labeled this routine as a ritual because the procedure was rigidly performed without independent decisions.

Table 5

Ritual 4. Finding the Solution Using an Equation

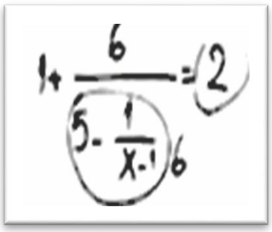
Initiation	Teacher:	If the solution set of this equation $m(2 - x) = nx + 4$ is infinite, then what is n ? Now, what does it mean to have infinite elements?
Procedure	Teacher:	So, we write a is equal to 0 and b is equal to 0 for the equation ax + b. If 0 is equal to 0, then what can we say? $a = 0\ b = 0$ $ax + b = 0$ $0 = 0$ Infinite elements, so the solution set for this equation is real numbers [*writing this Ç.K. = R*], right? Okay, let's organize this: 2m minus mx minus nx minus 4 equals 0. Let's have the bracket of x, -m minus n plus 2m minus 4 equals 0 [x (- m - n) + 2m – 4 = 0]. Here is the expression with x [*showing the coefficient of expression with x*]. What will be the coefficient of the term with x? [*There is no answer from the students*]. It needs to be 0 : 0 multiplied by x plus 0 equals 0. Okay. What will this be [*showing* 2m – 4]? Here, it will be 0. So, - m - n is equal to 0; from here, if I take n to the other side of the equation, m is equal to - n [- m – n = 0 => m = - n]. 2m minus 4 is equal to 0, and then m is equal to 2, right? [2m – 4 = 0 => m = 2]. If m is equal to 2, then what is n?
Closure	Teacher:	$-2.\ [\left.\begin{matrix}-m-n=0\Rightarrow m=-n\\ 2m-4=0\Rightarrow m=2\end{matrix}\right)n=-2]$ $[m(2-x)=nx+4$ $a=0\ \ b=0$ $ax+b=0$ $0=0$ $Ç.K.=R$ $2m-mx-nx-4=0$ $x(-m-n)+2m-4=0$ $\left.\begin{matrix}-m-n=0\Rightarrow m=-n\\ 2m-4=0\Rightarrow m=2\end{matrix}\right)n=-2]$

In the given classroom discourse, the teacher's discourse led the students' discourse on Ritual 4, which had a procedural structure comprising the following: i) If you find one type of equation (-3 = 5), then the solution is empty; ii) if you find another type of equation (2 = 2), then the solution is infinite. However, the relationship between an equation and a solution set was not so thoroughly discussed in the classroom discourse as to make the discourse transparent for students. Questions such as "Why is there a relationship between an equation and a solution set? "What is a solution set?" and "What is an empty or infinite solution set?" need to be clarified in the classroom discourse.

In Ritual 5, moving backwards by encircling entails solving an equation starting from the back and moving toward the front. While performing the procedure, participants encircled some portion of the whole equation that the teacher presented. For instance, when the teacher presented Figure 2 as the given equation, the participants encircled the denominator of the equation.

Figure 2

Encircling the Denominator of the Equation



In Ritual 5, the teacher initiated the classroom discourse by assigning Task 10: Find the value that satisfies x in the equation:

$$1+\frac{6}{5-\frac{1}{x-1}}=2$$

See Figure 3 for the procedure and solution and Table 6 for the discourse.

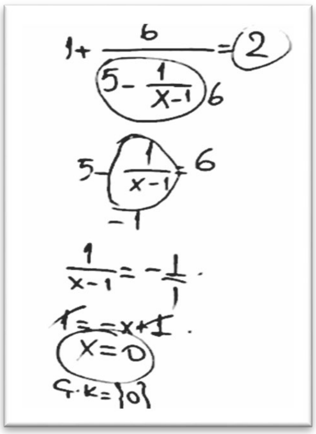
Table 6

Ritual 5. Moving Backwards by Encircling

Initiation	Teacher:	Find the value that satisfies *x* in the equation. $1+\frac{6}{5-\frac{1}{x-1}}=2$ (Task 10)
Procedure	Student 1:	Balloon.
	Teacher:	Balloon. Okay, you named this a balloon.
	Student 2:	Balloon. We call this an equation.
	Class:	This is a balloon.
	Student 3:	We call this an unsolvable equation.
	Teacher:	In this type of task, we start solving at the end [*indicating the denominator* $5-\frac{1}{x-1}$]. However, I did not solve this task like this [*indicating the denominator* $5-\frac{1}{x-1}$]. So, what is the product? [*indicating* 2].
	Class:	2.
	Teacher:	2. Okay. What should I add to 1 to get 2? [*indicating* 1]
	Student 1:	1.
	Student 2:	1.
	Teacher:	Okay, [*closing 1*], cancel this; what will be here? [*indicating the fraction* $\frac{6}{5-\frac{1}{x-1}}$]
	Student 1:	1.
	Teacher:	To have 1 here, what will be the value of here [*indicating* $5-\frac{1}{x-1}$]?
	Student 1:	6.
	Teacher:	6 [*writing* $5-\frac{1}{x-1}=6$]. 5 minus 1 over *x* - 1 equals?
	Student 1:	6.
	Teacher:	6. So, what is the value here? - 1? [*encircling the minus* 1 *over x minus* 1]
	Student 1:	Yes.
	Student 2:	Yes.
	Teacher:	- 1 and 5, what is at the front? a minus sign [*encircling* $\frac{1}{x-1}$]. 5 minus 1 is 6 [*indicating the equation*]. So, what is the value of $\frac{1}{x-1}$? [*writing* $\frac{1}{x-1}=-1$].
	Student:	- 1.
	Teacher:	- 1. Now, by cross-multiplying, 1 is equal to negative *x* plus 1. We put 1 on the other side of the $\frac{1}{x-1}=-\frac{1}{1}$ $1=x+1$ $x=0$ equation, so *x* equals 0. If we write *x* equals 0, what will be the answer?
Closure	Teacher:	The answer is 2 (Figure 3). We write 0, then what happens? Minus 1 minus 1 minus becomes plus 5, plus 1 6, 6 over 6 1, 1 plus 1, 2, so it is true [*by substituting* 0 *for x into the equation and controlling whether the equation is satisfying*], right? By substituting, I am controlling for if I found the correct answer. Thus, the solution set is [*writing on the board Ç.K.* = 0 *(ÇK is the abbreviation of the solution set in Turkish)*]. Is this challenging?
	Student 1:	Yes.
	Student 2:	Yes.
	Class:	Yes, challenging.
	Student 3:	It is not challenging; it is annoying.

Figure 3

Solution of TASK10

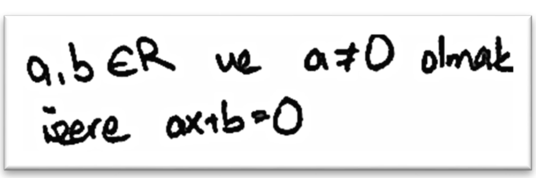


In the given classroom discourse, the students encountered challenges completing the task, the teacher's discourse was dominant, as shown in the excerpt, and the students mostly tried to imitate the teacher’s discourse. One of the significant points of this classroom discourse is the closure of students’ discourses, which included “This is challenging” and “annoying and long.” Such statements indicate the struggles that the students encountered. Moreover, the students’ discourse on variables revealed struggles. Below is a classroom discourse on variables.

Teacher:	When I say first-order equation, there needs to be 1 [*indicating the exponent of x*]. Figure 4 is a first-order equation. How many variables are there?
Student 1:	3.
Student 2:	2.
Student 3:	1.
Student 4:	3: *a, x*, and *b*.
Student 5:	No, *a* and *b* are natural numbers, so there is just one variable. The others [*a, b*] represent numbers.
Teacher:	She said just one. So, here is just one variable; what should I call this? A first-order one-variable equation. Okay? So, what are *a* and *b*? They are real numbers.

Figure 4

Example of First-order Equation



In the classroom discourse, the students considered *a, x*, and *b* as variables because they are unknown in the given equation. The students also considered all unknowns as variables. The teacher stressed that *a* and *b* are real numbers but did not mention the definition of a variable. A variable can also be a real number that can be changed at once in a range of numbers. Thus, the teacher's discourse on variables was not apparent to the students. After implicit classroom discussions about variables, I found a communication failure regarding the difference between letters as variables and unknowns (*x*, *a*).

Teacher: … If the value that satisfies x for the equation $\frac{1}{x+1} - \frac{2}{x+a} = 1$ is -2, what is *a*? How can I ask this question another way?
[*There is no answer from the students.*]
Teacher: If the root of the given equation is -2, what is *a*?
Student 1: But it needs to be by the *x* variable, right?
Teacher: It is by the *x* variable, but there are no other variables in the equation.
Student 1: There is *a*.
Student 2: *a* is a number.
Student 1: When the root of an equation is mentioned, it needs to say that *x* is a variable.
Teacher: There is no variable except *x*.
Student 3: *a* is a number; *a* is a number!
Student 1: Ha, okay!

In the given classroom discourse, the teacher considered *x* to be a variable and *a* to be an unknown. However, differentiating between variables and unknowns was still challenging for the students, so they considered both *x* and *a* to be variables. The questions "What is an unknown?" and "What is a variable?" had to be clarified for the students. At that point, realization of a mathematical definition and statement comes to the forefront.

Another challenge in the classroom discourse was the "root of an equation." A student asked about understanding the root of an equation. The classroom discourse on this topic is given below.

Student 1: What is the meaning of the root of an equation?
Teacher: What is the root of an equation? What is the root?
Student 2: *x*.
Student 3: *x*.
Student 4: Variable.
Teacher: So, is it *x*? If the root of an equation is - 1?
Student 5: Result.
Teacher: So, what is the value of *x*? If it is equal to *x*?
Student 6: Solution set.
Teacher: What is the solution set? - 1. Let's look at the solution sets in the other tasks.
Student 1: *a* is 5, right?
Student 3: *a* is 5.
Teacher: Okay, listen to me. We found that *x* is equal to 3, okay? [*indicating the solution of the empty set*] All of these [*indicating the solution* $x = \frac{3}{2}$]. The root of these equations [*indicating the answers to the three tasks*].
Student 1: Root.
Teacher: So, satisfying this value [*indicating the equation*]. Please take note if you do not know this. Satisfying the value means the root of an equation. What do we call the value that satisfies *x*? The root of the equation.

The students interpreted the root of the equation as a variable and an unknown; thus, it represents *x*, which, for them, can be a variable and an unknown. One of the students explicated *x* as a solution set. Using this approach, the teacher interpreted the root of an equation as a value that satisfies the equation. Another challenge for the students was finding the solution set in different number systems. This result may be attributed to Ritual 4, a conditional process-oriented routine that includes deciding the solution set if 5 = 3 or 2 = 2. Below is a classroom discourse based on the teacher-assigned task: Find the solution of $2x - 11 = 0$ in *N, Q, Z,* and *R*.

Teacher: …What does this mean?
Student 1: Natural number, whole number.
Student 2: Natural number.
Teacher: Find the natural number, whole number, rational number, and real number. Okay, find the solution [*The students are working on the task*].
Teacher: Yes, what is the answer in *N, Z*, and *Q*?
Student 1: 5 in *N*, 5 in *Z*, $\frac{11}{2}$ in *Q*, $\frac{11}{2}$ in R.
Teacher: So, you found 5 in *N*, 5 in *Z*, $\frac{11}{2}$ in *Q* and *R*; any other answers?
Student 2: 6 in *N* and *Z* because it becomes 5, 5, when we round up.
Student 3: I found 6.
Teacher: Congratulations! Any other ideas?
Student 4: $\frac{11}{2}$ in *Q* and *R*. I could not find any solutions in *N* and *Z*.
Student 5: Can we say 5, 5 in real numbers?
Teacher: Yes, you can say it in real numbers. Is 5, 5 not an element of the real number? Or $\frac{11}{2}$. So, what was our question? $2x - 11 = 0$, $x = \frac{11}{2}$. Whose element is $\frac{11}{2}$?
Student 6: Rational and real.
Student 7: Rational and real numbers.
Teacher: So, real numbers and rational numbers. As you know, real numbers subsume rational numbers.
Student 1: Yes.
Teacher: The answer in real and rational numbers is $\frac{11}{2}$. However, if I ask you for the solution set in *N*, the solution set in *N* and *Z* is an empty set. If you say, "It is $\frac{11}{2}$, so I round up and take 5 or 6," this is impossible. You cannot make up; you cannot round up! Can the answer be rounded up? Substitute 6 in the equation; does it satisfy? Substitute 6, 2 multiplied by six minus 11 equals 0. Is this right?
Student 1: Nope, it is equal to 1.
Teacher: So, what happened? I round up and take 5; rounding up and taking 6 is impossible. What do you round up? This is impossible. So, what do we say for the solution set in *N* and *Z*?
Student 1: Empty set
Teacher: Empty set. What is the solution set in rational numbers in *Q*?
Student 1: $\frac{11}{2}$
Teacher: $\frac{11}{2}$. The solution set in *R* is $\frac{11}{2}$. Okay?

According to the classroom discourse, the students' recognition of the solution set depended on the operational view of rituals. When the students encountered different tasks requiring an explorative realization, they struggled to interpret the solution set and navigated based on their discourse. In conclusion, the root of an equation is unknown and variable.

Discussion and Conclusions

This study explores the mathematical discourses of high school students and their mathematics teachers on the equal sign and variables in the classroom context. The results indicate that the teacher's teaching was governed by ritual teaching, in which she had the students strictly follow the procedures she performed. Additionally, the teacher enabled students' provision of short, closed questions as the first step to join a new discourse. Ritual teaching may provide a foundation for explorative teaching and is the first step in supporting students' expansion of their mathematical discourse based on previous learning experiences (Nachlieli & Tabach, 2019). Aligning with earlier research (Nachlieli & Tabach, 2019), our findings suggest that ritual instruction is essential for both object- and meta-level learning because it serves as a foundation for explorations and assists students with their first steps into a new discourse. If the teacher assigns tasks from the operational perspective, student discourse can be consistent and dominant in the operational view. For instance, the students imitated the teacher's discourse. In this study, the students used rituals with imitating teacher's discourse to gain their teacher's social approval. Practicing rituals is a first step to get into a discourse of the mathematics classroom, however, students can have challenges interpreting the equal signs and variables independently, on their own, in a flexible way. Besides, when students face tasks that include explorative characteristics, they might have difficulty interpreting the teacher's discourse. Students may struggle when the teacher's discourse is not transparent to them (Emre-Akdoğan et al., 2018). The characteristics of initiations, tasks and closures are among the essential components of routines' genesis. Aligned with Nachieli and Tabach's (2012) findings, I explored the strong relationship between tasks and ritual routines. Additionally, rituals are strongly associated with very restrictive tasks (Sfard, 2008). In this study, the students were accustomed to working on left-to-right and/or right-to-left directional signal and cross-multiplying rituals with which they were familiarized during their elementary grades. However, they had no experience with the other rituals. Thus, I observed their struggles with Rituals 3–5 in the classroom discourse. This may be because of the explorative structure of the routines' genesis that emerged from the tasks. However, the teacher's discourse included operational explanations of the routines based on the rules. This led to challenges for the students and unclear student closure of the routines. Presently, different rituals cannot be seen as interchangeable since they produce the same closures (Sfard, 2008).

In line with the literature, I explored students' perception of the equal sign as a left-to-right and/or right-to-left directional signal (Kilpatrick et al., 2001). Students in this study focused more on operational approaches than algebraic reasoning and mathematical objects while solving equations (Asquit et al., 2007; Berger, 2004; Carpenter & Levi, 2000; Kieran, 2004). According to the Turkish curriculum, the foundations for algebra are laid in Grades 6 and 7. Fundamental concepts, such as equations and variables, are more prominent at the middle-grade level. Subsequently, in Grade 9, students are expected to solve first-order, one-variable equations and consider equations from an operational view. However, the literature stresses that if students do not have a relational understanding of the fundamental concepts, they will struggle to solve equations (Kieran, 2004). Further, suppose students do not thoroughly understand the sign by relating it with mathematical objects. In that case, their use of mathematical signs will be restricted, and they may face challenges while implementing them (Berger, 2004).

Student discourses do not have a clear approach to unknowns, and this may be due to the fact that students tend to use ritual routines as imitating the teacher's operations in the class. This approach to unknowns conveys struggles with basic mathematical constructs, such as variables and the root of an equation. One of the reasons for students' struggles with unknowns may be due to teacher discourse on unknowns and variables, where the teacher implements process-based routines that do not have an explorative structure. As Berger (2004) mentioned, students' use of signs within a social community allows them to develop the sign's meaning, which is compatible with the community. The other reason may be students' preexisting routines on the concept of unknowns. Also, just using ritual routines with unclear closures convey struggles on the concept of unknowns. If, during the elementary grades, students consider unknowns to be a tool to perform operations as ritual routines, and do not have a relational understanding in an explorative way and relate unknowns with mathematical objects, this may lead to challenges. In line with the literature, this study found that the teacher's discourse on using letters as specific unknowns, generalized numbers, or variables was not transparent and explicit for students (Küchemann, 1978). This led to students' difficulty differentiating between variables and unknowns in this study. Küchemann (1978) stressed that mathematics teachers use the blanket term "variable" to refer to any letters in generalized arithmetic. However, to improve algebraic thinking, it is important to be aware of using letters as objects, specific unknowns, generalized numbers, or variables in algebra (Küchemann, 1978).

In conclusion, both ritual and explorative routines should be used in the classroom to create more explicit and transparent discourses for students. To diversify the types of routines implemented in the classroom, task characteristics play a substantial role. Teachers should assign operational and explorative tasks in the classroom. These types of tasks should convey operational and explorative routines. Moreover, it is necessary to conduct longitudinal research that extensively explores at which level, when, and how to implement operational and explorative routines. Moreover, mathematics educators should focus on different performances in a broader discursive context to analyze routines in detail since the difference between ritual and exploration lies in when they are conducted (Sfard, 2008).

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