

## Is There Something Useful In Students' Mistakes? : A Cognitive Resources-Based Approach<sup>5</sup>

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### Abstract

The study reported analyzes the mistakes made by university physics students when solving two problems on geometrical optics and two on magnetism. It also offers other teaching contexts in which the same reasoning leading to these mistakes could lead to correct answers. Instructional implications are discussed on the basis of the results. The study is carried out using the concept of cognitive resources proposed by Redish (2004), Hammer, Elby, Scherr & Redish (2005), and Hammer (2004) in their theoretical framework. Results show that this construct is useful to characterize different kinds of “mistakes” made by students, and also that these mistakes can be regarded as a means of probing what students *do* know which in turn can be used to direct the design of useful learning environments.

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### Introduction

Two kinds of approaches can be found to support strategies proposed to teach physics problem solving. One of the approaches, arising in the late 70's, is theoretically based on Cognitive Psychology and is represented by studies of “expert-novice” differences (Maloney, 1994). The other source, based on Scientific Epistemology has its main referent (at least within the Spanish-speaking community) in the “Model for problem solving according to scientific methodology”, developed by Gil Pérez and collaborators, in the early 80's (Gil Perez & Martínez Torregosa, 1983).

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Studies on expert-novice differences are based on the distinct characteristics observed between these two groups of problem solvers (usually, experts and novices are represented by physics teachers and students, respectively). The differences reported are basically related to subjects' knowledge structure on a particular domain in physics and to the strategies these subjects use to address problems (Chi, Feltovich & Glaser, 1981, Chi, Feltovich & Rees, 1982, de Larkin, McDermott, Simon & Simon, 1980, Maloney, 1994).

The teaching strategies proposed within the expert-novice approach aim at fostering the development of expert-like behavior. They emphasize the results desired rather than students' previous knowledge. They focus on the expert-like behavior desired in students and not on the cognitive process by means of which students can build this behavior. (Foster, 2000, Heller & Heller, 1995, Huffman, 1994, Leonard, Gerace & Dufresne, 2002, Mestre, Dufresne, Gerace, Hardiman, & Tonger, 1993). Although these strategies differ from each other, they share one common trait which is to generate constraints that lead students to mimic expert behavior.

The Model for problem solving according to scientific methodology (Gil Pérez & Martínez Torregosa, 1983, Gil Pérez, 2003) proposes a way of teaching to solve physics problems based on the (simplified) characteristics of the way in which the scientific community produces and validates knowledge. The method proposes a parallelism between the student and a novel researcher, between the teacher and an experimented researcher, and between the classroom and the scientific community. The usefulness of traditional end-of-chapter problems is dismissed in light of this parallelism, since these are considered to favor a methodology of superficiality (rote application of formulas). At the same time, Gil Pérez & Martínez Torregosa (1983) and Gil Pérez (2003) propose teaching strategies which include qualitative analysis, hypothesis formulation, solution planning, analysis of partial results, thus mimicking the behavior of scientists solving actual research problems. The appeal of this proposal lies in the fact that it opens a different perspective on the problem solving task, as compared to the way it has been traditionally addressed in educational environments. Nevertheless, as will be pointed out shortly, it still dismisses what students already know.

Gil Pérez's approach has produced various studies on scientific (and particularly physics) problem solving, many of which aim at pointing out how much students' behavior differs from that of experts when they are instructed using traditional problems. (Becerra Labra, Gras-Martí & Martínez-Torregosa, 2004, Guisasola, Furió, Ceberio & Zubimendi, 2003). Their results show the inappropriate cognitive habits and usual procedures of students which, according to the authors, would be reverted if students were taught following the guidelines deriving from the problem solving model proposed by Gil Pérez & Martínez Torregosa (1983). Once more, the actual starting point is not what students know or know how to do (this is characterized as inappropriate knowledge), but rather the knowledge or capacities teachers want their students to have.

Beyond the many differences between Gil Pérez approach and expert-novice differences approach, the point we wish to highlight is that the proposed teaching strategies underestimate the relevance of what students already know and/or are capable

of doing for future learning. Many of these proposals are prescriptive. The basic idea underlying them is “what does a teacher or researcher consider that a student should do in order to solve problems as similarly as possible to the way a teacher or researcher does, *regardless* of what the student already knows and/or is capable of doing”. Even though some of these teaching strategies have aimed at promoting a cognitive conflict -between the student’s ideas and those of the teacher’s or a textbook- to challenge students’ previous conceptions, this has shown to be insufficient to promote a dynamical construction of knowledge (Chi, 2005, Pozo, 1996, Redish, 2004). These conflicts often end up being a unsurpassable barrier between what students really believe and the “correct” answer they must provide to their teachers.

In short, physics problem solving is in itself an activity by means of which learning can take place. This learning occurs on the basis of what students already know and are already capable of doing. It is thus quite difficult to explain how students could learn to solve problems in an expert-like fashion from the starting point of knowledge that is either wrong or inexistent.

Redish (2004), Hammer (2004) and Hammer et. al (2005), partly based on the work of diSessa (1993) and diSessa & Sherin (1998), propose a theoretical frame that allows one to approach problem solving from a perspective based on what students *do* know. Instead of focusing on the flaws of students’ previous knowledge (their misconceptions), Hammer et. al (2005), propose to favor learning from the cognitive resources that students do possess, and to take advantage of them during this process. The following section presents the basic ideas of these authors. They will be used in the present study to analyze the verbal protocols of introductory (algebra-based) physics students at the university level, solving two geometrical optics and two electromagnetism (E&M) problems..

### Theoretical frame

The resources framework is a framework still under development. The framework was first presented as such by Redish (2004), Hammer (2004) and Hammer et. al (2005). The development of the framework has continued since then, as accounted for by further publications (for example Tuminaro, 2004, Tuminaro & Redish, 2007, Russ, Scherr, Hammer & Mikesa, 2008, Bing & Redish, in press). Nevertheless, within the present study the concepts of the framework that will be central are those published in Hammer et. al (2005) since 1) these concepts are sufficient for the analysis of the data and 2) these concepts have not changed since the work of 2005.

Hammer et. al. (2005) propose that people possess a collection of *cognitive resources* which they activate contextually when confronted to a cognitive task. Thus, reasoning about any particular situation involves tacitly or explicitly selecting a subset from a collection of available resources. All resources are useful in some context, otherwise they would not exist. In any case, resources can be either fruitful or not to address a given situation. This means that the activation of a particular resource in a given situation can lead to either a physically correct or incorrect statement. Hammer et. al (2005) consider *conceptual* and *epistemological* resources.

Conceptual resources are those that enable people to reason about physical situations. Although they are not themselves “wrong” or “right”, they can be mapped on the particular situation in a way that can lead to “correct” or “incorrect” physical statements. From this perspective, a physically “wrong” answer could arise from a cognitive resource that in another context, or mapped in another way, can give rise to a correct statement.

Hammer et al. (2005, p. 95) pose an illustrative example that shows the usefulness of conceptual resources to understand students’ reasoning (cited from diSessa, 1993). In tests to probe conceptual understanding in physics it is common to ask students about the forces acting on a body thrown vertically upwards. Many answer that there are two forces involved: the weight, pointing down, and another force that points up which decreases as the object reaches its highest position. When asked explicitly about the forces in this highest point, they answer that the downward and upward forces are equal. In order to explain students’ response, the authors interpret that two different conceptual resources are activated. The first, called *maintaining agency*, is the need for an agent to persist in order for the corresponding effect to be observed. In this case, the agent must continue to act for the body to keep moving upwards. When asked about forces, students map *agent* on *force*. However, when thinking about the highest point in the trajectory, the same students activate *balancing* (something directed upwards that must be balanced by something directed downwards). Asked about forces, they answer that it is the upward and downward forces that must be equal. This example is illustrative of how a resources-based approach naturally fits the description of students (context-sensitive) reasoning, and provides a more fruitful theoretical tool than the “movement requires force” misconception. Within the resources framework, a conception is built when, “with reuse, a set of activations can become established to the point that it becomes a kind of cognitive unit, and so a kind of resource in its own right. For instance, an infant comes to think about “objects” in a fairly consistent way across a wide range of situations. The cognitive unit can have its own activation conditions, passive or deliberate. But once activated, the internal coherence in the resource activations is automatic... Its activation continues to depend on context, like any other resource, but its stability does not” (Hammer et. al, 2005, p. 110)

Epistemological cognitive resources operate on people’s prior knowledge and allow them to understand sources of knowledge, forms of knowledge and stances toward knowledge. Epistemological resources tend to become activate in locally coherent sets. This locally coherent set is called a *frame*. In terms of Hammer et. al. (2005): “By a frame we mean, phenomenologically, a set of expectations an individual has about the situation in which she finds herself that affected what she notices and how she thinks to act. An individual’s or group's framing of a situation can have many aspects, including social (Whom do I expect to interact with here and how?), affective (How do I expect to feel about it?), epistemological (What do I expect to use to answer questions and build new knowledge?), and others” (p. 98).

This approach implies a shift in the way problem solving is investigated. This shift goes from a researcher-centered view: which are the relevant factors for physics problem solving, as regarded by an expert, to a subject-centered view: what is it that

really occurs when students solve problems and how can we take advantage of that instruction-wise. From this viewpoint, the mistakes students make during problem solving are not mere samples of “incorrect” knowledge, but rather they are envisioned as the result of the activation of their available resources. Therefore, studying these mistakes could give useful information on the productive aspects of their knowledge and on this basis think of possible instructional strategies to take advantage of those aspects.

The goal of the present study is to classify the mistakes made by 8 students on the basis of the idea of cognitive resources. These students pertain to an introductory university-level physics course and the problems they solve are two of geometrical optics and two of E&M. The classification obtained is used to predict contexts in which these mistakes would not occur. The consequences of these results are discussed regarding possible implications for instructional decisions.

### The study

The present is an exploratory study which consists of the interpretation of a few cases. For this reason, transcripts of pieces of the studied protocols are presented as a substantial part of the analysis. The verbalizations for the 8 students solving the task are analyzed following the tradition of case study methodology from qualitative research. The idea is to analyze a small number of students' verbalizations to develop case studies: rich, detailed descriptions of student reasoning in each episode. Although the limited size of the sample only allows to make conclusions regarding the subjects interviewed, they are helpful to improve our understanding of the knowledge that students make use of while solving physics problems, and possible ways to take advantage of it, instruction-wise. The selected episodes are the result of a negotiation between independent interpretations carried out by three researchers (two of them are the authors of this study). The interpretations were about when and where a particular cognitive resource is activated. Given the interpretive nature of the study, the characteristics of the participants' instruction are provided.

#### *Characteristics of the subjects involved.*

The 8 students who participated in the study were freshmen who had just finished the second introductory physics course. They were familiar with basic algebra as well as calculus knowledge, which they had covered in two courses that same year. However, this second introductory Physics course is mainly algebra based. They are students of different careers such as Pharmacy, Biochemistry and Chemistry. The institution is a public university in Argentina and, at the time of the interviews, the students had passed the course with a score of 80% or more, as marked by the school's regulations. Students volunteered to participate in the study. The course which the students had taken covers contents of geometrical and physical optics, electrostatics, electrodynamics, magnetism, and electromagnetism. The students took two 1.5 hour lectures and two 1.5 hour problem-solving sessions every week during 15 weeks.

During the problem solving sessions on the topic of geometrical optics, all students in the course solved typical end-of-chapter problems involving reflection and

refraction, as those found in introductory physics texts. Regarding mirrors, the students usually found problems in which they had to obtain size and position of images of objects placed in front of plane and spherical mirrors, and also to determine the zones in space from which an observer could completely or partially visualize those images. The students spent a total of 4 sessions (6 hr) working on such problems.

During the problem solving sessions on the topic of magnetic forces generated by currents and the Law of Faraday-Lenz, all students in the course also solved typical end-of-chapter problems as those found in introductory physics texts. Frequently, problems requested the calculation of the magnetic field generated by currents in the form of straight lines, coils and solenoids, forces exerted by external fields on conductors carrying currents; values of the total magnetic field on such situations, and the calculation of the magnetic moment of coils with current and the torque on those coils, when placed in an external field. Regarding the Faraday-Lenz law, students calculated electromotive forces generated in coils and solenoids due to variations of magnetic flux and identified the currents thus induced in these conductors. Students spent a total of 4 sessions working on such problems.

#### *The task*

Each of the participants was individually interviewed by the authors during approximately 40 minutes. Students were asked to think aloud as they read each of the sentences in the problem statements (shown in Figure I). Statement sentences subsequently appeared on a computer screen at the students' command (not all together, as in a printed sheet), allowing them to think aloud after each sentence. This technique increased the amount of verbalizations (usually quite scarce in students) and also allowed us to allocate the activation of resources to the different stages of the problem. Interviewers intervened only to ask questions when clarification was needed.

## Problem 1

- S1. There is a table on which a small plane mirror has been placed.
- S2. The lamp lighting the room is on the roof (you may consider the lamp a point)



- S3. Mark the boundaries of a region from which a person could observe the image of that lamp.

## Problem 2

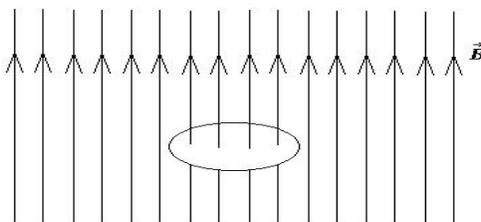
- S1. Consider a person standing in front of a wall on which a plane mirror is to be hung.



- S2. This person is 1.65 m tall and his eyes are 1,55 m above his feet
- S3. Compute the height at which the mirror should be hung and the minimum height it should have in order for the person to see his complete image on it.

## Problem 3

- S1. A conducting coil of area  $A$  and resistance  $R$  is placed in a region of space where there is a uniform magnetic field  $B$ .
- S2. The plane of the coil forms a right angle with the direction of the magnetic field.

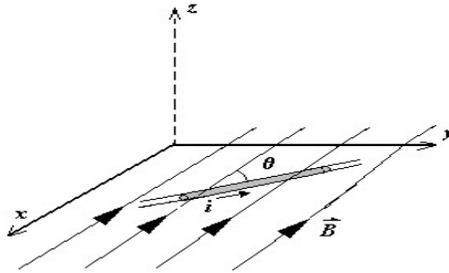


- S3. The intensity of the  $B$  field raises at a rate of 0.1 Tesla per second.
- S4. Compute the intensity of the current induced in the coil, knowing that its area is  $A = 0.01 \text{ m}^2$  and its resistance is  $R = 10 \Omega$

## Problem 4

S1. A conducting rod of length  $l$  and mass  $m$  carries an electric current of intensity  $i$ .

S2. This rod is placed in a region of space where there is a constant uniform magnetic field  $\mathbf{B}$ , also horizontal and which presents an angle  $\theta$  with it.



S3. Knowing that  $i = 0.01$  A,  $B = 0.3$  T,  $l = 0.5$  m,  $g = 9.8$  m/s<sup>2</sup> and  $m = 0.045$  Kg, what is the value of  $\theta$  necessary for the rod to be in equilibrium?

Figure 1. The task given to students

## Results

### Part One: Analysis of mistakes and their potential usefulness.

Mistakes made by students are reported and analyzed in this section. For this purpose, students' productions during the problem solving task are interpreted in terms of the activation of conceptual and frames. Also, other contexts are proposed in which the activation of the same resources could lead to correct answers.

#### *Mistake type 1: inappropriate mapping of a conceptual resource*

As an example, during the solving of problem 2, a conceptual resource was identified which was given the name *container*. This resource, useful in situations in which objects have to fit into containers, has been activated by most students and thus they interpret that the image of the person is contained in the mirror, and therefore the mirror has to be as large as the image to be seen. In the same problem, another resource is activated, which has been named *the farther, the smaller*. Activation of this resource leads students to state that as a person backs away from a mirror, the image is farther away and therefore it looks smaller.

Activation of *the farther, the smaller* together with *container* can account for students' verbalizations in which they state that as they back away from the mirror, a smaller mirror is needed. As an example, student "M" activates these two resources after S3 in problem 2:

"M": ... the mirror, to see himself completely, it should start at the floor, and be at least as tall as the person...

“I” (interviewer): *is that what happens when you want to see yourself completely?*

“M”:  
*hmm, no, no...*

“I”:  
*could you see yourself completely on a smaller mirror?*

“M”:  
*well, that depends, where you’re standing, I mean, the distance from the mirror... if you move forward, near the mirror, and the mirror doesn’t reach the floor, you can’t see your feet... but if you back away from the mirror, maybe a smaller mirror will be enough...*

Where does this incorrect answer come from? According to the approach described above, this could arise from an inappropriate mapping of the resource *the farther, the smaller* on the mirror situation. In other words, the apparent size of the image (which in fact is smaller when it is farther away) is compared to the actual size of the mirror as if its apparent size did not also change (the mirror is also farther away from a person backing away from it). This conjecture is depicted in Figure IIa. However, mapping this resource onto the image and the mirror simultaneously, can lead to a correct answer. The question that follows this observation is: is there a context in which this resource is spontaneously activated and mapped in such a way that it leads students to give a “correct” physical description? Figure IIb depicts one such situation, in which an observer views the exterior through a window. It seems reasonable to assume that these students have enough everyday experience to decide what can be seen through a window. Therefore, this same resource, but mapped differently, can be useful to reason about seeing objects through a window, as well as to reason about observing one’s own reflection on a plane mirror.

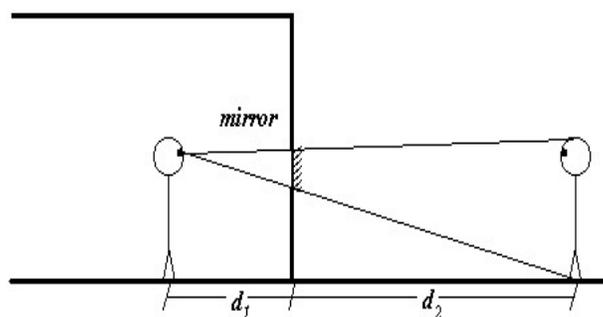


Figure IIa: “The farther, the smaller” inadequately mapped onto the mirror situation.

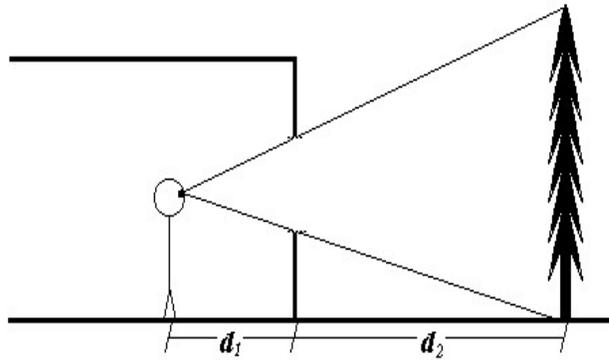


Figure IIb: “The farther, the smaller” mapped onto the window situation.

*Mistake type 2: not productive activation of a conceptual resource*

Another conceptual resource observed is the one named *eye contact*. This resource is useful to reason about two people seeing each other by means of a mirror, and to decide if they are making visual contact (each person can decide if the other one can see his eyes). “E” seems to have activated this resource, and mapped it onto himself and his image to decide about the smallest possible mirror

“E” (after S4): ... they give me the person’s height and how high his eyes are, so, it would have to be, at least, to see all of him, this high, that is as high as his eyes are, if...

“I”: so?

“E”: ... it would have to be this high, the mirror, I mean, at least as high as his eyes, starting on his feet.

“I”: would you like to make any kind of drawing?

“E”: ...no...

Figure IIIa represents the activation of this resource to decide the size of the smallest mirror needed to see one’s whole body image. Figure IIIb depicts the same resource in the context of deciding if a person is able to see another through a mirror.

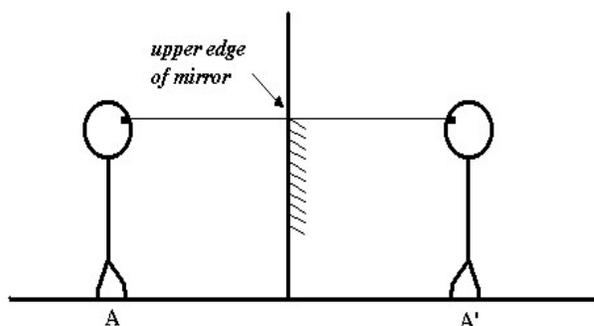


Figure IIIa: eye contact mapped onto a person and his/her image.

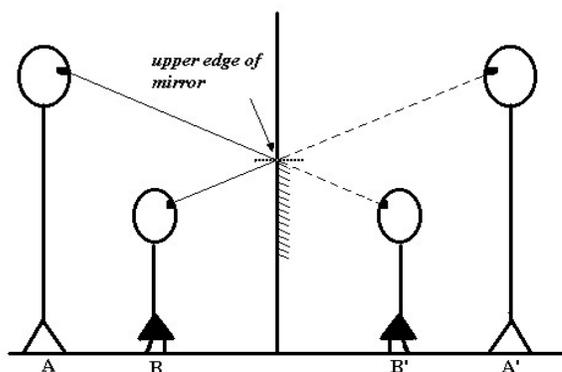


Figure IIIb: eye contact mapped onto two people to see each other.

Unlike the resource *the farther, the smaller*, the resource *eye contact* is not productive for deciding the size of the smallest mirror possible. This kind of mistake does not arise from mapping a useful resource inappropriately, as in the previous example, but rather from the activation of a resource that is not fruitful for the particular situation. Nevertheless, from an instructional point of view, it is fruitful (and will be discussed in the following section) to provide a context in which this resource is useful, such as the one depicted in Figure IIIb. This figure shows the minimum height a mirror should have for two people of different heights make eye contact through a mirror.

Problems 3 and 4 seem to induce the activation of another resource named *alignment*. Mapped on (electric and magnetic) dipolar moments, and (electric and magnetic) fields, respectively, this resource can lead students to provide physically accurate descriptions. This resource is seen to be activated in problem 3, onto the magnetic moment of the current circulating in the coil and the external magnetic field, leading students to give correct answers. However, in problem 4, this resource is mapped onto the conducting rod carrying a current and the magnetic field, which results in a physically incorrect description. Student “J” correctly solved problem 3, stating that the coil would not rotate due to the current induced by the variation of the field, because its dipolar moment was already oriented with the field. However, while solving problem 4, activates the same resource to provide an incorrect answer:

- “I”: *(after the drawing in problem 4) what are you thinking about this?*
- “J”: *that this external field will generate a torque... that makes the magnetic dipole on the conducting rod be aligned with it... and ... that’s it.*
- “I”: *If you had to make up a question to this problem, what would it be?*
- “J”: *Hmm.... to calculate the magnetic moment, of the rod... oh! I don’t have the radius!*
- “I”: *is the rod circular?*
- “J”: *no, it isn’t...*
- “I”: *is there a magnetic moment?*
- “J”: *no*
- “I”: *then... is there anything going on? I mean, does that field have any effect on the rod?*
- “J”: *no... except for aligning it... right?*
- “I”: *you’re saying it will align the rod, with what?*
- “J”: *yeah... well, no, it doesn’t do anything...*
- “I”: *nothing?*
- “J”: *well, no! ‘cause...*
- “I”: *So, you think nothing happens?*
- “J”: *no! I think something does happen, but, hmmm, no... now I’m really not sure that anything actually happens...*
- “I”: *but what do you think does happen?*
- “J”: *Well, the thing about aligning it with the field, but that’s for coils... I don’t know, I’m confused now*
- .....
- “J” (after S3) *... in equilibrium!! But the field doesn’t do anything to it!...It is lying somewhere... but this rod isn’t lying on any surface, is it? ¿WHERE is it?!?! I mean... is it in the air?!?!?!*
- “I”: *yes*
- “J”: *... honestly, I have no idea...*

- “I”: *forget about B for a second... what happens to a rod just placed in mid-air?*
- “J”: *it falls...and for what value will it be in equilibrium... no, I really don't know how to calculate this.... Can it be solved?*

As in the previous examples, this student's mistake comes from the activation of a resource that is not fruitful to address the situation. “J” activates *alignment* and not *balance* which, mapped on the forces acting on the conducting rod, could lead to a physically correct description.

The following is an excerpt from student “C” when she is solving problem 4):

- “C” (after S3): *...I can compute the moment... the moment...*
- “I”: *which moment? Do you mean torque?*
- “C”: *no, torque is the product of the field times a “moment”...*
- “I”: *the dipolar magnetic moment?*
- “C”: *that's it! The dipolar magnetic moment...*
- “I”: *Is there a dipolar magnetic moment there?*
- “C”: *well, if there's a current, I guess there would have to be, right? Cause' if there weren't, there couldn't be a torque that makes the rod rotate (meaning a rotation towards the direction of the field)*
- “I”: *ok... but can we forget about the torque for just a second, and go back to the question of whether there is a magnetic moment?*
- “C”: *the moment was the product of the area times the current?*
- “I”: *yes, for a closed coil.*
- “C”: *...for a closed coil... oh!!! Right! It was for a closed coil... for a conductor... no wonder I couldn't come up with it using my right hand! ... well, no! there is no moment...*
- “I”: *is there anything else there?*
- “C”: *...there has to be a force*
- “I”: *how so?*
- “C”: *a force tending to align the rod with the field.*

- “I”: *and what causes that force?*
- “C”: *obviously it has to be the external field... the current and the length of the rod...*
- “I”: *where is that force applied and what characteristics does it have?*
- “C”: *Well, the force is a vector, and  $B$  is also a vector, so it would have to be  $i$  times  $l$  times the sine of the angle, right?*
- “I”: *and where is that force pointing?*
- “C”: *hmmm, well, it would have to be perpendicular to the horizontal plane... upwards, the hmm, the force points up...*
- “I”: *so, how will that affect the rods movement?*
- “C”: *applying the force... it would have to lean towards the direction of the field...*
- “I”: *how?*
- “C”: *This is the rod... I can't do it in the other way with the right hand... the rod would have to go that way, right? ... it would have to move to the left?*
- “I”: *how? Are we looking at the direction where the force is pointing?*
- “C”: *yeah, where the force is pointing... ok, well, I know that the force has to be perpendicular pointing up*
- “I”: *ok, then?*
- “C”: *but no, it's going to move it towards the direction of the field, and the value of theta... well, it's gonna have to be the sine of the angle with the intensity of the field.*

This protocol shows a mistake similar to the previous one. However, it exhibits an activation of the resource *alignment* which is more stable; since this activation persists even after “C” expresses that the force on the rod is perpendicular to the horizontal plane and points up. This mistake comes from the activation of a resource that, though unfruitful in this context, is useful in others.

### *Mistake type 3: not productive activation of a frame*

In what follows a third kind of mistake is analyzed, related to the activation of frames. The excerpt presented corresponds to the protocol of “F”, while solving problem

2. At first, this student activates the *qualitative sense-making frame*, and afterward activates the *quantitative sense-making frame* (Tuminaro, 2004):

“F”:  
*...ok, if he wants to see all his body, it would have to be... hum, I mean, ... it would have to be a large mirror, 1.65m or more... at least 1.65 to see himself completely, I mean, it also depends on the distance he is standing from the mirror... if he is too close, even if the mirror is very large, he will see “less”... and... well, maybe he can come up close to the mirror and look in some way so that he can see his feet... if, well, I mean, I’d place it a bit over his head...*

“I”:  
*Do you think this problem could be solved more... concretely?*

“F”:  
*What do mean “solve”? Make computations? Compute the height? Well, not as it is, I don’t have the distance from the person to the mirror*

“I”:  
*And if you did have that distance?*

“F”:  
*...well, if I have the mirror here, and he is here, well, there I could compute that somehow... looking at the light rays more or less..*

“I”:  
*how?*

“F”:  
*If this is the mirror, and I take out rays from his head here to the end of the mirror, and the other ones to the other end... it’s like... I think... if he’s standing here like the drawing shows... and the mirror... there, (laughs softly) from the eye, I would have to cover his image completely, I mean the reflection, and that way I could come up with the size of the mirror, I think...*

The first part of F’s answer is incorrect and, if the second part were not present, this could be viewed as a mistake due to inadequate mapping of the conceptual resources of *container* and *the farther, the smaller*. However, analyzing the complete protocol allows to understand that the mistake is also related to the activation of the *qualitative sense-making frame*, on the basis of which the answer to the problem does not involve algebraic or graphic computations, and only an argument based on previous (probably everyday) knowledge of mirrors and images. The interviewer’s question regarding a more “concrete” solution seems to induce the activation of the *quantitative sense-making frame*, and thus the solution involves computations and/or graphic considerations. Nevertheless, the resource of qualitative solving is very useful to make qualitative predictions that can be later confirmed and compared to formal computations. Moreover,

it is desirable and often absent in student's problem solving behavior, and its activation should not be disregarded even when it could lead momentarily to "incorrect" answers. Once again, both frames are useful in different contexts, and their activation can therefore lead to correct as well as incorrect answers. This example of F's protocol has been chosen to show how a mistake can come from the unproductive activation of a frame that in other contexts can be very useful.

*Part Two: the knowledge of mistakes and their relation to instruction*

The analysis of mistakes on the basis of cognitive resources can provide suggestive approximations to the problem of instruction. Two questions arise from this analysis: "what is the use of knowing what kind of mistake a student is making when solving a problem?" and "what is the use of knowing in what context the activation of the same resources could lead to correct answers?"

Understanding where students' mistakes come from allows to better knowing what it is that students do know and to be better prepared to work on that basis. For example, a mistake arising from the inadequate mapping of a conceptual resource that is potentially useful for the situation could require a different action than a mistake due to the activation of an unproductive conceptual or frame. An instructor's intervention should be different in each case because the "useful information". If the mistake comes from an inappropriate mapping, as in the case of the resources *container* and *the farther, the smaller* for problem 2, the comparison of this situation with another one such as a person looking out of a window (Figure IIb) could likely induce the activation of the same resources, and therefore the subject could compare the answer given in each situation. Since looking out of a window is a part of everyday experience for (almost) everyone, it is likely that the activation of these resources that naturally takes place in this context could result in an aid to address the mirror problem (it has been studied in more detail in Buteler & Coleoni, 2009).

If the mistake observed comes from the activation of a resource which is unproductive to solve the problem, as in the case of "eye contact" or "alignment", an efficient strategy could be to have induced the comparison with the answers given in contexts where such resources are productive, analyzing similarities and differences. Such a comparison could lead students to "learn" in what contexts those resources are productive and why the characteristics of other contexts make this resource unproductive.

In any case, the comparison is made between the reasoning of one same subject in different contexts, and not between the student's and the teacher's or a textbook. These strategies do not foster a barrier between the student's thinking and the "correct" reasoning, because they allow reinforcing students' ideas in the appropriate contexts. These comparison strategies, however, require certain knowledge of contexts in which students could potentially activate fruitful resources. Teachers' expertise in physics instruction together with studies aimed at testing the effectiveness of such strategies could be of great value.

Finally, mistakes coming from an unproductive activation of a frame could call for more extended actions sustained through time. Comparison strategies between “close” contexts as the ones presented in this paper could be insufficient for this purpose. Close contexts are those sharing the kind of task, the social or physical environment, and that differ only in the physical situation presented. Far contexts are those in which the tasks presented differ more radically from each other (such as problem solving vs. argumentation for or against a thesis) or in which the social environment is different (classroom vs. informal interviews), etc. It is likely that the changes in context needed to foster the activation of productive or unproductive frames be more pronounced than those needed for conceptual resources. The authors intend to address these issues in the future.

### Discussion

The theoretical framework adopted has allowed us to interpret students’ verbalizations. This analysis allows understanding possible mechanisms by which students produce physically “incorrect” answers. The interpretation of protocols in terms of conceptual resources activated and mapped in different contexts can account for two mistakes of a different nature. One of them (mistake type 1) occurs when the resource activated is useful to address the situation, but has been mapped in such a way that leads to contradict a physically correct result. Such is the case of *the farther, the smaller* in problem 2. The other kind of mistake takes place when the resource activated is not fruitful to address the situation (mistake type 2). Such is the case of *alignment* in problem 4, or of *eye contact* in problem 2. This analysis of mistakes enables to imagine contexts in which a productive activation of these same resources could occur. Comparison between these two situations could lead students to learn in a way that is tuned with what they already know. They are not lead to disregard their knowledge when it is incorrect, but rather to refine the way in which they reason with the cognitive tools they do have. These findings also bring up the question of how exposing students to contexts in which the resources activated are useful could foster metacognitive processes that enable them to reorganize what they already know. These questions are being approached by the authors at present.

Regarding frames, a suggestive result is the variability in their activation by one same student. Such is the case of student “F” in problem 2 (mistake type 3). This indicates that students have epistemic capacities potentially useful to address physics problem solving, and opens the question of how to make the best use of these abilities, instruction-wise.

In general terms, the present study aims at showing how a more detailed analysis of students’ mistakes can change the view of teaching and of research. The view of teaching changes because a new meaning is assigned to students’ “incorrect” answers, and they can be regarded as valuable. As for research, new questions arise that lead to hypothesis regarding the efficiency of comparison strategies aimed at fostering students’ learning on the basis of what they know.

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