ELECTRONIC JOURNAL FOR RESEARCH
IN SCIENCE & MATHEMATICS EDUCATION
VOL. 30, NO. 2, 137-164

PCK Grain Size in the 21st Century

William Veal
The University of Charleston

Jan Sermeus
KU Leuven & Royal Observatory of Belgium

ABSTRACT

Pedagogical content knowledge (PCK) is specific to a given discipline, such as science. For example, the knowledge that a physics teacher has is different than the knowledge that a chemistry teacher has. This concept was refined with the introduction of different grain sizes of PCK that include domain-specific and topic-specific PCK. However, as was discussed during the 2024 Lorentz workshop on 21st-century science education, science teaching, learning, and understanding are becoming focused on broader issues that require knowledge and understanding outside of traditional disciplines. Furthermore, the focus on 21st-century skills (e.g., digital literacy, critical thinking, creativity, and collaboration and communication) necessitates students adapting to how they learn and embracing inter- and transdisciplinary approaches to solving societal issues. The new skills and nature of scientific knowledge will impact the PCK of 21st-century science teachers in that science teachers must be prepared to adapt their discipline-specific content knowledge and teaching methods to include inter- and transdisciplinary approaches. The purpose of this paper is to present toy cases as a tool to illustrate the impact of inter- and transdisciplinary science education on teachers' PCK, to address 21st-century skills, and to refine the grain size approach to PCK.

Keywords: pedagogical content knowledge, grain size, science teacher preparation

Introduction

Given the complexity and wickedness of the global and local scientific challenges that society faces in the 21st-century (e.g., climate change, energy, water, AI, and global health), there are calls for more collaboration between natural science disciplines (e.g., Weart, 2013) and for collaboration of the natural sciences with the humanities (e.g., Marcoci et al., 2023). Moreover, the complex societal challenges and the inter- and transdisciplinary approach needed to address them require that scientists develop and utilize collaboration and communication skills, critical thinking, creativity and flexibility, digital and data literacy, and problem-solving skills (i.e., 21st-century skills). The complex societal challenges, the inter- and transdisciplinary approach to (natural) sciences, and the required development of 21st-century skills are not just limited to science but also impact science education. The purpose of this paper is to utilize *toy cases*, an adaptation from toy-models and gedankenexperiments from physics (Redish, 2021), to highlight the knowledge bases required by teachers when teaching inter- and transdisciplinary topics, including socio-scientific issues, and to develop students' 21st-century skills.

Indeed, natural sciences are increasingly going beyond disciplinary boundaries in two ways. On the one hand, there is increasing interdisciplinary collaboration between the natural sciences. For example, there is research in physical chemistry/chemical physics, biochemical engineering, geophysics, computational mathematics, chip-based biosensors, and environmental science. On the other hand, the natural sciences also draw on other fields, such as public health and the humanities, and work in transdisciplinary fields of study. Examples include researchers involved in psychoacoustics, medical nuclear physics, urban studies (combining geography, architecture, history, and sports science), and disaster preparedness and response. Klein (2010) employed a typology from the Organization for Economic Cooperation and Development to define multidisciplinary, interdisciplinary, and transdisciplinary. Multidisciplinary teaching offers distinct methods and perspectives for examining a theme, allowing disciplines to remain separate. Interdisciplinary teaching integrates knowledge, theories, and methods from multiple traditional disciplines (e.g., science) to analyze an issue and synthesize them into a coordinated whole. Transdisciplinary teaching, in contrast, transcends traditional disciplinary boundaries and involves other subject areas (e.g., arts, business, mathematics, and public health) to create innovative solutions to complex, real-world problems.

The increased inter- and transdisciplinary research at universities already translates into inter- and transdisciplinary educational programs offered in university curricula. There is also increased attention in science education for developing 21st-century skills, both for future scientists and for citizens. Indeed, the so-called 21st-century skills are needed for adults in developed and developing nations to enter the workforce and society (Martin, 2018). If students need to develop these skills, it can be expected that their teachers also need to have and be able to transfer these skills.

Teachers can transfer science content and skills to students using diverse and appropriate methods if they have the necessary pedagogical content knowledge (Shulman, 1987). Schulman (1986) introduced the concept of pedagogical content knowledge (PCK) as being “an amalgam of content and pedagogy that is uniquely the province of teachers.” Science can be used as a rich context for developing 21st-century skills in education (Hilton, 2010). The increasingly inter- and transdisciplinary character of science offers additional opportunities to develop these skills, while also introducing students to science in the 21st-century.

In this paper, the impact of inter- and transdisciplinary science education and how it relates to 21st-century skills on the PCK of science teachers will be described. First, 21st-century skills, interdisciplinary and transdisciplinary science, and PCK are defined, including the original description of PCK grain sizes. Based on these definitions and the importance of socio-scientific issues, toy cases are presented that illustrate the impact of inter- and transdisciplinarity on science teachers’ PCK. Next, the relationship between PCK and 21st-century skills is highlighted, resulting in an updated PCK model with additional grain sizes. Finally, the significance of the new model is discussed in the context of science teacher education.

Problem Statement

At the 2024 workshop on 21st-century Science Education (Chan et al., 2024), science education researchers learned about and discussed how the 21st-century brings about several challenges in science teaching and its impact on science teacher training. In short, science knowledge is growing exponentially (Bornmann et al., 2021), scientific information is spreading very quickly through digital information sources (but, unfortunately, so is misinformation), scientists are increasingly specialized, and science is increasingly inter- and transdisciplinary (e.g., Barthel & Seidl, 2017; Van Noorden, 2015).

Inter- and transdisciplinary research brings about specific challenges. The scientists working together ideally have some understanding of the other field and know which questions they can ask their colleagues. Additionally, they must find common ground across disciplines in the theoretical foundations of their work, research methodologies (i.e., What are common research practices?), epistemological aspects (i.e., What constitutes knowledge or evidence in a field?), and communication (How are journal articles written, which journal to choose, and where to disperse information?). Moreover, as collaboration is a relatively new phenomenon, it is unclear how it manifests itself. For example, Gleason (2020) argues that this could be anything from ignoring each other to collaborating as equal partners to one serving the other. All approaches have advantages and pitfalls.

Given that science is increasingly inter- and transdisciplinary, it can be expected that science education will follow suit. Indeed, there exist inter- and transdisciplinary master's programs (e.g., Science and Math for Education, Environmental Studies, Public Health, Biophysics, Biochemistry, Biotechnology, and Sustainability). The challenges presented earlier also find their way into science education and the classrooms of the 21st century, including the K-12 level. Indeed, there is an increasing call to teach 'STEM' in an integrated way and move beyond the siloed approach that was common in the early years of STEM education (Widya et al., 2019). The question thus arises of how the inter- and transdisciplinary nature of 21st-century science (teaching) impacts the PCK of science teachers.

Background

Teaching science content in the 21st-century requires a diverse set of knowledge and skills that can be associated with PCK. Shulman's (1986) initial concept of PCK focused on students, content, teaching strategies, and context as a knowledge base for teaching that was unique to the profession. At first, PCK was "the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations and demonstrations – in a word the ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p. 9). It soon became one part of a teachers' knowledge base comprised of seven categories of knowledge: content knowledge; pedagogical knowledge; curriculum knowledge; knowledge of learners; knowledge of context; knowledge of educational ends, purposes and values; and pedagogical content knowledge (Shulman, 1987). The PCK of teachers in the 21st-century must change to adapt to the integrated approach needed to solve societal issues.

21st-century Skills

There have been many frameworks to conceptualize "21st-century skills." Different literature reviews of papers published in the last 25 years have resulted in lists of emergent categories of skills (e.g., Chen, 2021; González-Perez & Ramirez-Montoya, 2022; Kotsiou et al., 2022). Chen's review of the literature identified seven domains of 21st-century or future skills, including higher-order thinking skills; dialogue skills; digital and STEM literacy skills; values; self-management; lifelong learning; and enterprise, leadership, and flexibility skills (2021). González-Perez and Ramirez-Montoya (2022) created a list of Education 4.0 core competencies. The transversal competencies include critical thinking, cooperation, collaboration, communication, and creativity. The disciplinary competencies include developing technical knowledge and using emerging technologies. Karaca-Atik et al. (2023) developed a list of 21st-century skills for social science students that also included communication, problem solving, critical thinking, initiative and self-direction, and social and cross-cultural skills. For the purposes of this paper, 21st-century skills encompass a composite list of the literature reviews cited, and they include digital literacy, critical thinking and problem solving, creativity and flexibility, and collaboration and communication.

Skills are extremely important for changing the economic, scientific, and technological landscape. One of the most crucial 21st-century skills is digital literacy. With the rapid advancement of technology, educators and learners need to be proficient in using digital tools and platforms. This includes basic computer skills and the ability to navigate and utilize various educational technologies effectively to distinguish fact from fiction. Digital literacy encompasses information, technology, and media literacies (Bliss, 2019), all of which require epistemic vigilance (Osborne, 2024). Epistemic vigilance is the ability of students to evaluate a digital media source, and "the expertise level of the source and to evaluate the claim itself, using scientific standards and appropriate argumentation heuristics" (Bielik & Krell, 2025, p. 869). Digital literacy enables students to access a wealth of information and resources online, while teachers can enhance their teaching methods and engage students in more interactive and innovative ways.

Another vital 21st-century skill is critical thinking and problem-solving. In today's complex and fast-paced world, the ability to analyze information, think critically, and solve problems creatively is more important than ever (Jin et al., 2023). Critical thinking was defined as the "purposeful, self-regulatory judgment which results in interpretation, analysis, evaluation, and inference, as well as an explanation of the evidential, conceptual, methodological, criteriological, or contextual considerations upon which that judgment is based" (Facione, 1990, p. 3). Science teachers need to foster an environment that encourages students to question assumptions, evaluate evidence, and develop well-reasoned arguments like the tenets of the nature of science (Galili, 2019). This skill set is essential for students to navigate the vast amount of information online and make informed decisions about the science content. Moreover, critical thinking and problem-solving skills are highly valued in the workforce, making them indispensable for students' future careers (Humphreys & Kelly, 2014).

Creativity and flexibility are also necessary in the 21st-century for both teachers and students. For example, when teachers make decisions, they are aided by the ability to creatively develop new pedagogical tools to enhance the transferability of the content to the students. To achieve this, teachers must learn to be flexible in the classroom so that students can learn the content in appropriate formats (Al-Abdali & Al-Balushi, 2016). Knowing the students' context, culture, and backgrounds and the different structures and representations of the content topics affords the teacher the flexibility to create a rich environment for learning science content. It is the interplay between PCK and creativity in teachers that enables the development of 21st-century skills (Veal & Kind, 2025).

Collaboration and communication are key 21st-century skills that teachers and students must develop. The modern world is becoming increasingly interconnected, and the ability to work effectively with others is becoming increasingly crucial. Teachers should promote collaborative learning experiences where students can collaborate on projects, share ideas, and learn from one another (Rodriguez et al., 2019). Effective written and verbal communication skills are essential for expressing ideas clearly and persuasively. By developing these skills, students can better prepare for the collaborative nature of the modern workplace, while teachers can create a more dynamic and inclusive classroom environment.

Interdisciplinary Science Education

In addition to the development of 21st-century skills by students, which are necessary for living in a complex society, the world's problems can no longer be evaluated and solved through a single lens. Schools need to prepare these students for the real world through progressive pedagogical approaches that highlight the interconnectedness of different subjects and domains of knowledge. The 21st-century classroom thus necessitates an interdisciplinary and transdisciplinary approach to science content (e.g., Hubber et al., 2022). This, in turn, requires that science teachers adjust how and why they teach science content so that their students are prepared for the changing societal demands.

One method is to develop a perspective on science teaching that integrates other science disciplines and other subject areas.

A prominent pedagogical approach that has gained favor in the last 20 years is the integration of science, technology, engineering, and mathematics into STEM. The research literature contains examples of the implementation of integrated science lessons using different pedagogical approaches (e.g., problem-based learning and inquiry) (e.g., Deák et al., 2021). Integration has been synonymous with an interdisciplinary approach. Most research studies have focused on the integration of two disciplines (e.g., mathematics and technology) (e.g., Furner, 2024) or a cross-disciplinary approach (e.g., Chang & Chen, 2020). This body of research also includes many studies on STEM in which integrating four disciplines has often been recommended to support students in developing 21st-century skills. However, many STEM studies have focused on a multidisciplinary, rather than an interdisciplinary, approach to teaching. For example, most studies focus on science, mathematics, and technology as separate content areas that can contribute independently to a central theme (e.g., English, 2023). In contrast, an interdisciplinary approach requires all content areas to be seamlessly used to solve the theme/problem (e.g., Sikand et al., 2021).

In other words, science teachers still need to be experts in their own content discipline but also will need to be able to integrate science topics from other disciplines and more topics from other subject areas. This implies that science teachers will have to expand their content and PCK to include the application of interdisciplinary and transdisciplinary knowledge.

Pedagogical Content Knowledge

Since its inception, PCK has been considered and researched as its own knowledge base (e.g., Magnusson et al., 1999; Veal & MaKinster, 1999). In the brief history of PCK, many researchers have defined PCK as having multiple knowledge bases (e.g., Grossman, 1990; Marks, 1990; Cochran et al., 1993; Magnusson et al., 1999; and Veal & MaKinster, 1999). Indeed, research has shown that there are multiple knowledge bases that a teacher must have. However, there is no consensus on the relationship among the components (Friedrichsen et al., 2011).

There have been a variety of models describing the attributes or knowledge bases related to and describing PCK. For example, Grossman (1990) stated that a teacher with PCK had knowledge of representations, students' understanding, curriculum, and assessment. These were like Shulman's but focused specifically on effective teaching practices. Other researchers (e.g., Magnusson et al., 1999; Marks, 1990) similarly described PCK using the same knowledge bases and expanded some to include the subject matter and general pedagogy. Veal and MaKinster (1990) elaborated on the initial idea that PCK was its own knowledge base and included many other knowledge bases. All the knowledge bases described by other researchers were included in the PCK taxonomy developed by Veal and MaKinster (1999). Table 1 shows the initial development of the definition of PCK to include multiple knowledge bases.

Table 1

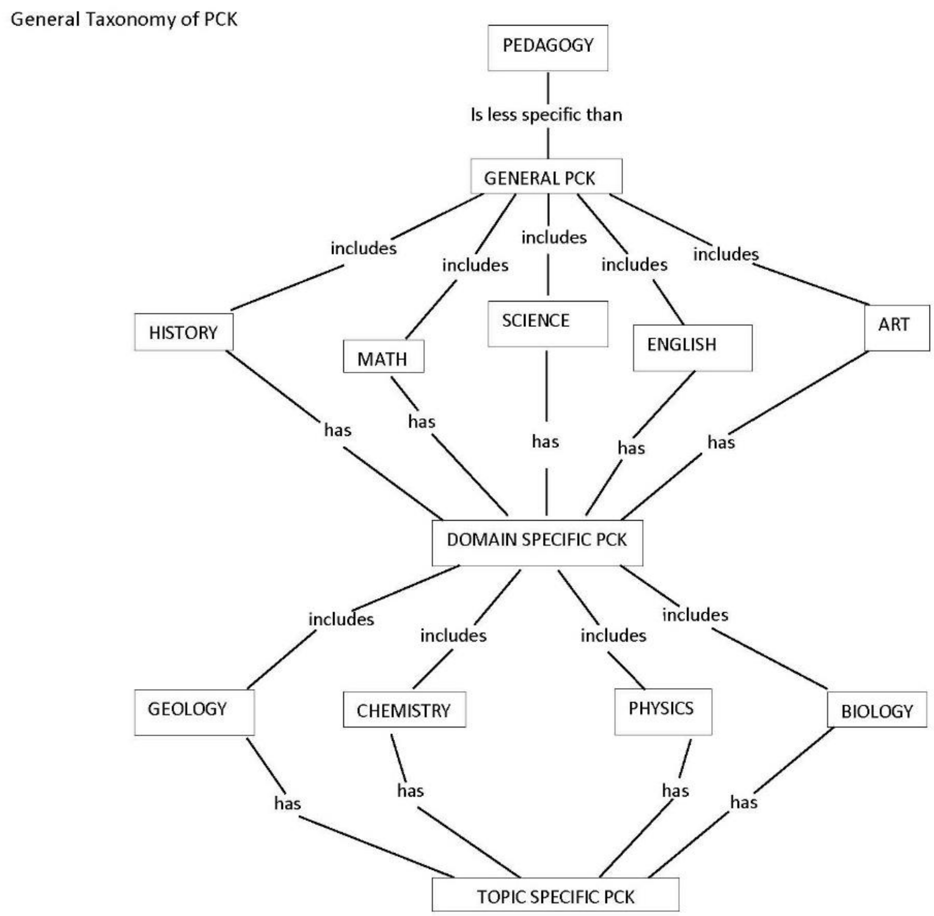
Pedagogical Content Knowledge and its Knowledge Bases

Knowledge of	Grossman (1990)	Marks (1990)	Magnusson et al. (1999)	Veal & MaKinster (1999)
Representations	X	X	X	X
Students' Understanding	X	X	X	X
Subject Matter		X		X
Curriculum	X	X	X	X
Assessment	X		X	X
Environment/Context				X
Socio-cultural issues				X
General Pedagogy				X
Classroom Management				X

Veal and MaKinster (1999) introduced an elaboration to understanding how teachers learn to teach science by determining that there are different grain sizes to PCK. Figure 1 shows the original conceptualization of the grain sizes related to PCK. All teachers possess general pedagogical skills, which include the ability to design a lesson plan, deliver a lecture, form students into groups, and administer assessments. General PCK is common across all disciplines and includes such skills as providing an advanced organizer for students before instruction and facilitating transitions between concepts and activities. However, teachers also have more specific forms of PCK. There is PCK specific to teaching a discipline of 'science' (e.g., introducing the Nature of Science to their students), as well as PCK specific to a domain of science (e.g., physics teachers introducing their students to model-based thinking) and PCK specific to the topics within a domain of science (e.g., a physics teacher working with their students on a lesson on mechanics to recognize and avoid the weight-mass confusion). The cascading and refined knowledge base of PCK begins with general PCK, which is shared by all teachers, and progresses to Domain-specific PCK (DS-PCK) and ultimately to Topic-specific PCK (TS-PCK). Figure 1 represents the flow of grain size originally established by Veal and MaKinster (1999). This hierarchy defines the grain size of PCK, which has been evaluated and applied to science teaching research studies since its inception (e.g., Rollnick & Mavhunga, 2015). It remains an integral part of the Consensus Model (Gess-Newsome, 2015) and the Reformed Consensus Model (Carlson et al., 2019) of PCK.

Figure 1

Grain Size for PCK (presented with permission from the author)



Aims and Methods

This work aims to discuss the impact of an approach to science education that embraces inter- and transdisciplinarity and that develops 21st-century skills in science teachers' PCK by presenting several toy cases. This methodology is inspired by toy models or gedankenexperiments (Luczak, 2017), a known methodological tool in theoretical physics. These are deliberately simplistic and idealized models and experiments used to explore and explain the core mechanisms of a more complex physical system (Redish, 2021). By intentionally stripping away complicating factors, physicists can gain insight into a fundamental principle without getting bogged down in real-world complexity. The model is not meant to be a precise description of reality, but a "caricature" that is easy to analyze and helps build intuition. In an analogous way, "toy cases" are simplified hypothetical situations used to think through complex educational situations.

Cases have been used in various educational settings for decades, including education, social science, business, law, and medicine. Shulman (1986) stated, "While cases themselves are reports of events, the knowledge they represent is what makes them cases" (p. 22). They focus on collecting qualitative and quantitative data to explore complex issues in real-life contexts. By examining specific cases, researchers can identify patterns, relationships, and outcomes that contribute to a deeper understanding of broader phenomena. Toy cases in this paper serve as idealized, simple, and focused representations of how to teach specific inter- and transdisciplinary concepts, including socio-scientific issues, that reflect the need to use 21st-century skills.

For this research, hypotheticals were developed to reflect a particular form of narrative that examines a particular instance, individual, group, or event. These toy cases are used to elucidate and illuminate professional knowledge bases and attributes of science teachers when teaching interdisciplinary and transdisciplinary science issues. It should be noted that the toy cases are inspired by and describe a realistic classroom situation that incorporates facts, examples, and ideas (Kagan, 1993). The toy cases in this paper are intended to portray scientific issues that span the interdisciplinary and transdisciplinary domains of education. While the science content of the toy cases is critical, it is the way that the inter- and transdisciplinary nature of the content impacts the method of instruction that should be the centerpiece of discussions.

A compact and qualitative notation has been developed to describe the different kinds of PCK a science teacher needs in the 21st-century. The notation described in Table 2 is based on the notation typical in conditional probability. So, in general, $PCK(A|B)$ should be read as the PCK necessary to teach A given B. Throughout the text, the notation uses cap B, cap C, cap P, and cap G to refer to biology, chemistry, physics, and geosciences, respectively. Non-natural science disciplines are written in full or using common abbreviations. If PCK is specific to the intersection between disciplines, this is noted with $\otimes$. Finally, PCK that is specific to a person (or group) is marked as PCK_{T_P} to be read as: the PCK of Teachers of Physics. This notation will only be used if it is needed to differentiate between teachers. The notation does not distinguish between personal and collective PCK, as introduced in the refined consensus model of PCK (Carlson et al., 2019). Rather, in this work, a teacher's personal PCK is simplified to be equivalent to the collective PCK. As an example, in this notation, the last sentence is written as $PCK_{T_P}(topic) = PCK(topic|P)$, assuming that a physics teacher approaches topics from a Physics perspective.

Table 2

Notation of Toy Cases as the Content Applies to PCK Grain Size

PCK level	Notation	To be read as
Transdisciplinary PCK	$PCK(topic\|P \otimes history)$	PCK about teaching a topic at the interface between physics and history.
Interdisciplinary PCK	$PCK(topic\|P \otimes C)$	PCK about teaching a topic at the interface between physics and chemistry
Disciplinary-specific PCK	$PCK(P)$ $PCK(P\|topic)$	PCK about teaching physics (in general) PCK about teaching physics using a specific topic
Topic-specific PCK	$PCK(topic\|P)$	PCK on teaching a topic (within physics)

Case Studies

Toy Case 1 Multidisciplinary DS-PCK: The Optics of the Eye in Biology and Physics

Case

A physics teacher is introducing geometrical optics to his students. He works with his students through the optical paths of a (ideal) convex lens and constructs, on the blackboard, an image similar to that in Figure 2a. Subsequently, he shows an application of this model, the lens of the human eye, by showing Figure 2b. The teacher points out that this is a simplification. The eye is much more complex, but the teacher refers the students to their biology teacher for those details. "For example," he adds, "notice how the image is upside down! The brain can correct this." While the physics teacher also knows that the retina contains rods and cones, that humans are sensitive to red, blue, and green light, and that the optical nerve passes through the retina, creating a blind spot, he does not go into those details, trusting his biology colleague to explain this in a more thorough way.

Next door, his biology colleague is introducing the human eye to her students. She starts with a detailed diagram of the eye (Figure 2b), showing the different anatomical parts and explaining their functions. She points out that humans perceive the world upside down; luckily, the brain can correct this. This is due to the way lenses work, she explains by superimposing a simplified path of the light on top of the anatomical diagram (Figure 2b). "This is just one part of optics, though," she points out to her students. For more details on the physics content of geometric optics, she refers the students to her physics colleague. Like her colleague, the biology teacher knows basic physics. She knows that concave lenses diverge light, how to create basic geometric optical diagrams, and the lens formula. But she, too, knows that her physics colleague will demonstrate the workings of lenses, has lab-work planned for the students, and will derive the lens-formula.

Case Discussion

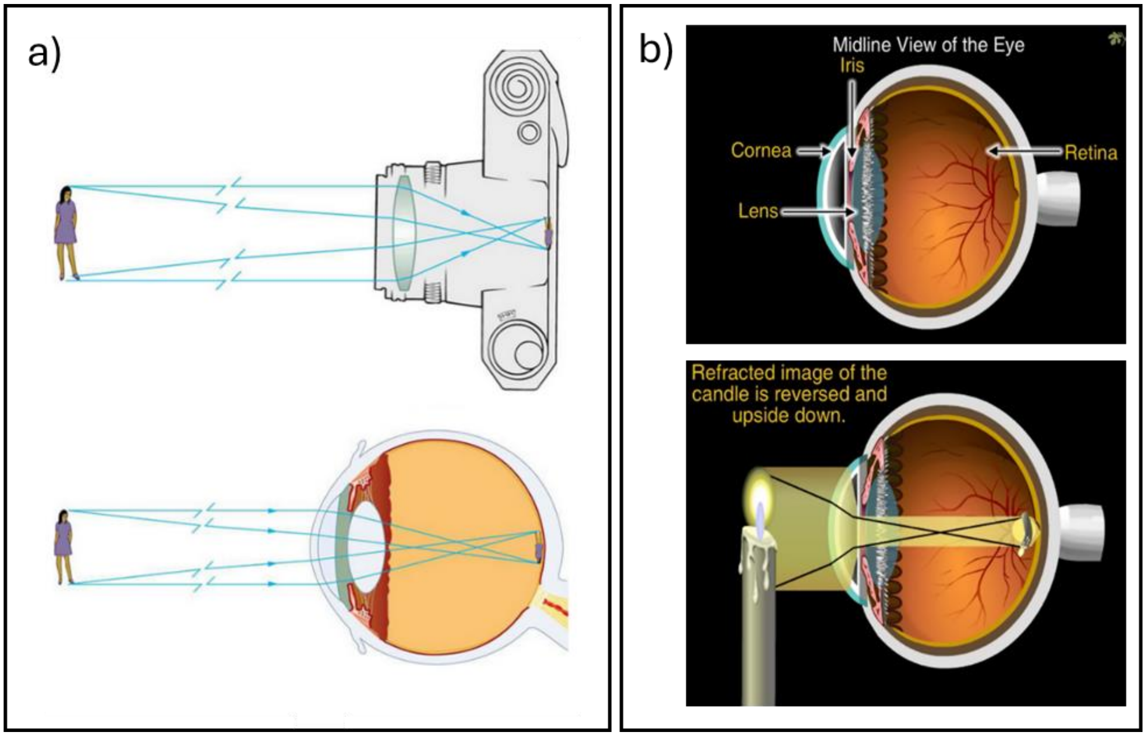
The integration of the different disciplines is limited to a multidisciplinary approach. However, in this first case, there is PCK of the teachers that goes beyond their own discipline. When teaching about the optical path of light in the eye, both physics and biology teachers recognize that students are typically unaware that the image projected in the eye is upside down. Moreover, both teachers know the basic subject matter of the other discipline. They can use this knowledge to guide students through a representation of elements of both disciplines. However, they also know that their knowledge is limited. As such, based on their knowledge of what is planned in the curriculum of their colleague, they can refer to each other.

$$PCK_{T_P}(eye) = PCK(optics|P) + PCK(eye|P \otimes B)$$
$$PCK_{T_B}(eye) = PCK(vision|B) + PCK(eye|B \otimes P)$$

The PCK_{T_P} (eye) represents the physics teacher's multidisciplinary PCK related to the eye. It includes TS-PCK of optics (as it is understood in physics) and knowledge of where physics meets biology for the particular content of the optics of the eye. Likewise, the biology teachers' multidisciplinary PCK consists of their TS-PCK concerning vision and the knowledge of where biology meets physics for the particular content of the optics of the eye. Note that $PCK(eye|B \otimes P) \approx PCK(eye|P \otimes B)$, as the topic-specific PCK of the physics teacher is quite similar to the topic-specific PCK of the biology teacher. Of course, they approach the topic in a different order and include this segment about the optics of the eye in different lessons (the physics teacher focuses on optics, the biology teacher focuses on the biological working of the eye).

Figure 2

Images used by a) a Physics Teacher (adapted from Urone & Hinrichs, 2025) and b) a Biology Teacher (adapted from AniMed, 2017).



Toy Case 2 Interdisciplinary PCK: Health Monitor STEM Project

Case

A school organizes a STEM project. The mathematics teacher (Alice), the physics teacher (Bob), and the Information and Communications Technology (ICT) coordinator (Charly) work together to develop the lessons. Each teacher is responsible for their part. Alice wants to ensure that the mathematics in the STEM project goes beyond calculations, Bob wants to ensure that students develop some conceptual understanding of the science involved, and Charly wants students to learn the basics of coding. The project they develop challenges the students to "measure their physical condition," making them think about energy, building a chronometer using an Arduino to measure how fast they run, and using (discrete) derivatives to calculate velocity and acceleration from positional sensor data.

Students from several classes work on this project in groups under the supervision of Alice, Bob, and Charly. If Alice or Bob gets questions regarding the Arduino, they call on Charly. Charly knows what mistakes beginners typically make in setting up and coding an Arduino (e.g., wiring mistakes, coding errors, syntax errors, powering everything using 5V, etc.). Charly can also provide coding examples that can serve as a starting point for the students. Similarly, Alice and Charly expect Bob's interventions to force students to think about energy, velocity, and acceleration concepts. He knows common misconceptions and can quickly demonstrate how distance traveled, velocity, and acceleration are related. And Bob and Charly know that Alice will introduce the idea of derivatives as the local slope of a curve and set students up to think about what happens if they make the time interval smaller and smaller.

After a while, Alice and Bob also gain the confidence to help students in their coding, especially when the students are struggling with translating the idea of the derivative into code. Likewise, Alice and Charly will, for example, recognize when students failed to take the second derivative to calculate the acceleration. Bob and Charly also grow in their support of the students when they are mathematically processing the data.

Towards the end of the project, a student asks Alice a question about energy usage in the body: "How is the energy in my sugary drink converted into the motion of my body?" Alice recognizes this as a science question and calls Bob. Bob recognizes that this is indeed a biology question, but also that this is not his expertise. As such, he first had to ask a biology colleague (not involved in the project) and could only get back to the student a week later. After the project is completed, the teachers discuss the possibility of including the biology teacher next year.

Case Discussion

In this toy case, the teacher initially approached the STEM project in a siloed way. Each teacher was responsible for their own subject. Of course, each teacher is expected to have PCK regarding their own subject. It enables them to distinguish aspects of the project that are within their own subject. Hence, initially, the combined PCK of the teacher team TT for this project $STEM$ can be written without any PCK at the intersection as

$$PCK_{TT,i}(STEM) = PCK_{T_M}(STEM|M) + PCK_{T_P}(STEM|P) + PCK_{T_{ICT}}(STEM|ICT)$$

The initial PCK of the teacher team related to STEM is equal to the combined PCK of the math, physics, and ICT teachers who approach the STEM project from their own disciplines. However, as is typical in a STEM project (and in the reality of S, T, E, and M beyond the school walls), the different subjects are not independent. Rather, they are interwoven. Velocity and acceleration, concepts from physics, rely heavily on mathematics. The time interval in which data are gathered is the denominator in the discrete derivatives but also depends on the technological capabilities of the sensor and the Arduino. Hence, as the teachers grew throughout the project, they recognized that they also needed to know where their subject touches upon neighboring subjects, and that they needed to know elements of those neighboring subjects. They also needed to know which aspects of the project are beyond their own expertise and they need to know when and whose expertise to call on. As such, the integration goes from multidisciplinary to interdisciplinary. Moreover, the PCK of the teacher team expands, in this notation, to

$$PCK_{TT,f}(STEM) = PCK_{T_M}(STEM) + PCK_{T_P}(STEM) + PCK_{T_{ICT}}(STEM) + 0 * PCK_{T_{P,M,ICT}}(B|STEM)$$

where

$$PCK_{T_P}(STEM) = PCK(STEM|P) + PCK(STEM|P \otimes M) + PCK(STEM|P \otimes ICT)$$

$PCK_{T_M}(STEM)$, $PCK_{T_P}(STEM)$, and $PCK_{T_{ICT}}(STEM)$ are analogous. Note that the final PCK of the physics teacher now includes not only their PCK on the STEM project as they would approach it from their own field (physics), expressed as $PCK(STEM|P)$, but also their knowledge of the boundaries of physics and where it meets mathematics and ICT, expressed as $PCK(STEM|P \otimes M) + PCK(STEM|P \otimes ICT)$. Also note that in the combined interdisciplinary PCK of the teacher team the $0 * PCK_{T_{P,M,ICT}}(B|STEM)$ refers to the inability of the team to recognize the biological elements in the project. The PCK needed by the team to teach this STEM project is dependent upon the collective and integrative knowledge of physics, math, and ICT teachers. This case reflects an interdisciplinary situation, even though it lacks a key component of a biology teacher.

Toy Case 3a Interdisciplinary PCK: Climate Change

Case

Climate change is a topic that is inherently not bound to a single discipline. Ms. Rodriguez is an Earth sciences teacher who calls on her biology, chemistry, physics, and engineering colleagues to spend time on climate change. She sets up a meeting to discuss who can take up which aspects of climate change in their lessons. The notes of this meeting are given in Figure 3a.

Case Discussion

By coming together and delineating who is best suited to discuss which aspect of climate change, the teacher team has taken a multidisciplinary, interdisciplinary approach to teaching climate change. After the meeting, all the teachers involved are aware of which aspects are covered in the other disciplines and of potential student difficulties (that may also arise in their own lessons). Combined, the teacher team (TT) has clarified and expanded their PCK for teaching the topic of climate change. It now, as a team, encompasses all individual PCKs and a shared understanding of the topic.

$$PCK_{TT}(CC) = PCK_{BT}(CC|B) + PCK_{CT}(CC|C) + PCK_{GT}(CC|G) + PCK_{ET}(CC|E) + PCK_{PT}(CC|P) + PCK(CC|B \otimes C \otimes E \otimes G \otimes P)$$

Where E refers to engineering and

$$PCK_{PT}(CC) = PCK(CC|P) + PCK(CC|P \otimes B) + PCK(CC|P \otimes C) + PCK(CC|P \otimes E) + PCK(CC|P \otimes G)$$

Note that the interdisciplinary PCK of the teacher team not only depends on the interdisciplinary PCK of each teacher (which, in turn, consists of TS-PCK from the discipline and knowledge of the intersections with other disciplines), but also on the whole team knowing where all disciplines come together. The latter is expressed as $PCK(CC|B \otimes C \otimes E \otimes G \otimes P)$.

Toy Case 3b Transdisciplinary PCK: Climate Change

Case

Ms. Rodriguez also teaches science to non-science majors. For this group, she calls on her colleagues from all other disciplines to explore how they can integrate climate change into their courses. She sets up a meeting to discuss who can take up which aspects of climate change in their lessons. The notes of this meeting are given in Figure 3b. Students are challenged to create a proposal to build a climate-proof house within a specific budget and present it to the Architectural Review Board of the local city council.

Figure 3a

Discussions and Assignments of Content for Teachers

Biology

- Data and Examples: Discuss the impact of climate change on biodiversity, ecosystems, and species migration patterns. Use data on changing habitats and species extinction rates.
- Graphics: Include charts showing the decline in polar bear populations and maps of shifting biomes.
- Expected student difficulties: Climate change is a natural change.

Chemistry

- Data and Examples: Explain the chemical processes behind greenhouse gas emissions and their effects on the atmosphere. Use data on CO_2 levels (Keeling curve) and their correlation with global temperatures.
- Graphics: Provide molecular diagrams of greenhouse gases and graphs showing historical CO_2 concentrations.
- Expected student difficulties: The ozone hole is responsible for climate change, and only CO_2 is a greenhouse gas.

Physics

- Data and Examples: Describe the physical principles of energy transfer and the greenhouse effect. Use data on solar radiation and heat absorption.
- Graphics: Include diagrams of the greenhouse effect and charts showing the energy balance in the Earth's atmosphere.
- Expected student difficulties: Changing distance to the sun and changing solar activity.

Geology

- Data and Examples: Explore the geological evidence of past climate changes and their causes. Use data from ice cores and sediment layers.
- Graphics: Provide images of ice core samples and graphs showing historical temperature variations.
- Expected student difficulties: The Earth’s climate has always changed.

Engineering

- Data and Examples: Discuss engineering solutions to mitigate climate change, such as renewable energy technologies and carbon capture. Use data on the efficiency and adoption rates of these technologies.
- Graphics: Include schematics of solar panels and wind turbines, and charts showing the growth of renewable energy capacity.
- Expected student difficulties: Technology (e.g., carbon capture) alone is enough to mitigate climate change, and nuclear power emits CO_2.

Figure 3b

Meeting Notes

Science

- Data and Examples: Use data on rising global temperatures, melting ice caps, and increasing CO_2 levels. Include graphics such as the "hockey stick" graph showing temperature rise over the past century.
- Experiments: Conduct simple experiments to demonstrate the greenhouse effect and the impact of CO_2 on temperature.
- Resilient Design: Explore how infrastructure can be designed to withstand the impacts of climate change. Use examples of green buildings and sustainable urban planning.

Sociology and Public Health

- Social Impact: Discuss how climate change affects different communities, particularly vulnerable populations. Explore the health effects of climate change, such as the spread of diseases, heat-related illnesses, and respiratory issues due to air pollution. Use case studies of climate refugees and the social dynamics of communities affected by extreme weather events.
- Class Discussions: Facilitate discussions on environmental justice and the role of social movements in addressing climate change.
- Media Literacy: Teach students to critically evaluate climate change information in the media and identify misinformation.

Mathematics and Economics

- Data Analysis: Teach students how to analyze climate data using statistical methods. Include exercises on calculating trends and making predictions based on historical data. Use data on the cost of natural disasters and the economic benefits of sustainable practices.
- Graphing: Use graphs to illustrate changes in temperature, sea levels, and CO_2 concentrations over time. Teach students how to perform cost-benefit analyses for energy savings.

English and Literature

- Literary Analysis: Analyze literary works that address environmental themes, such as "Silent Spring" by Rachel Carson, "The Road" by Cormac McCarthy, or “Janouk zoekt een duurzame wereld” by Janouk Kelderman (Janouk is Looking for a Sustainable World).
- Creative Writing: Encourage students to write the project proposal, essays explaining the science behind the structure, and short stories about their perspectives on climate change.

History

- Historical Context: Provide a historical overview of climate change, including the Industrial Revolution's impact on the environment and key milestones in climate science.
- Timeline: Create a timeline of significant events in the history of climate change awareness and policy.

Case Discussion

Climate change is a wicked problem; and the solutions are all interconnected. There is also a huge societal component to climate change as it is human-induced. The efforts of Ms. Rodriguez to bring together a teacher team and to approach climate change from many angles and disciplines allow students to get both a broader and a deeper understanding of climate change. The project allows teachers to use their depth of content knowledge to guide students in the project and through other activities. The interconnectedness of the project variables reflects the myriad methods that the teachers must use to present new content and relate it to the real world. The guiding questions of the content should focus on the socio-cultural issues related to climate change using environmental justice. The project to build a sustainable house allows the students to evaluate online resources, collect data, organize data, critically think about solutions, and create their vision of a sustainable home. The project's culmination is an authentic assessment in which students must problem-solve and communicate their results to an outside body of experts.

$$PCK_{TT}(CC) = PCK_{NST}(CC|NS) + PCK_{historyT}(CC|history) + PCK_{econT}(CC|economy) + \cdots + PCK(CC|NS \otimes history \otimes economy \otimes \ldots)$$

The transdisciplinary PCK of the teacher team is equivalent to the PCK about climate change of natural science teachers and the collective PCK about climate change of the other team members in other subject areas as all subject areas are interfaced with each other. Where NS is short for Natural Science.

Toy Case 4a Interdisciplinary PCK: Evolution

Case

In a secondary science classroom, Mr. Smith, a dedicated science teacher, embarks on an innovative journey to teach the topic of evolution. Recognizing the complexity and interconnectedness of scientific disciplines, he adopts an interdisciplinary approach that integrates data, examples, and graphics from biology, geology, chemistry, and physics. This method enhances students' understanding of evolution and demonstrates the relevance of various scientific fields in explaining natural phenomena. The instruction uses multiple hands-on activities and labs to help students overcome traditional misconceptions related to evolution as a theory and its explanation of environmental biodiversity.

Mr. Smith introduces the fundamental concepts of evolution, emphasizing the role of natural selection, genetic variation, and adaptation. He starts by proposing some overarching and guiding questions to orient the students. For example, he can ask, “How do environmental and geological changes, and natural selection drive changes in a population?” He then uses the example of antibiotic resistance in bacteria to illustrate how genetic mutations can lead to advantageous traits that increase an organism's chances of survival and reproduction. This example is grounded in biology but also incorporates chemistry by explaining the molecular interactions that drive genetic mutations.

To provide a broader context, Mr. Smith integrates geology by discussing how the Earth's geological history, including the formation of continents, volcanic activity, and mass extinctions, has influenced the evolution of life. He uses data from geological surveys and paleontological studies to show how different species have evolved over millions of years. Students learn how to interpret the geological record and uncover evidence of ancient life forms by studying rock layers and fossils.

Mr. Smith also incorporates physics by exploring the role of physical forces, such as gravity, radiation, and temperature, in shaping the evolution of life on Earth. He discusses how environmental conditions can drive evolutionary change and uses data from ecological studies and climate models to show how physical factors impact the distribution and behavior of species. This helps students understand the complex interactions between organisms and their environment.

To delve deeper into the mechanisms of evolution, Mr. Smith incorporates chemistry by examining the structure and function of DNA. He explains how chemical reactions and molecular interactions drive genetic mutations and the formation of new traits. By presenting data on the molecular composition of different organisms and using graphics to illustrate the processes of DNA replication, transcription, and translation, students gain a deeper understanding of how genetic information is passed down from one generation to the next.

Throughout the lessons, Mr. Smith uses hands-on activities, such as examining fossil specimens, analyzing DNA sequences, and conducting experiments to observe chemical reactions. He also incorporates graphics, such as phylogenetic trees, geological time scales, and diagrams of ecosystem energy flow, to provide students with tangible evidence of evolutionary processes and help them visualize the dynamic changes that have shaped the Earth's surface and its inhabitants. Assessment of students' content knowledge is accomplished through the completion of graphic organizers, labs, and traditional quizzes. The integration of the different activities, examples, and manipulatives reflects how all science disciplines can be used by one teacher to help students understand an interdisciplinary socio-scientific theme.

Case Discussion

Through this integrated interdisciplinary approach, Mr. Smith engages his students in studying evolution. The activities, labs, and representations used by the teacher help enforce the concepts of evolution. He provides an interdisciplinary, comprehensive, and holistic understanding of the topic by integrating data, examples, and graphics from biology, geology, chemistry, and physics. This method not only enhances students' scientific literacy but also fosters an appreciation for the interconnectedness of the natural world.

Mr. Smith uses subject knowledge from all the science disciplines and knowledge about the different representations (e.g., graphics, illustrative examples, and pictures) that can enhance students' understanding of evolution. He also has knowledge of the environment and context of where the different disciplines intersect with the students' lives, and which examples allow him to get to that intersection. For example, while Mr. Smith could also have chosen to illustrate evolution using examples on much shorter timescales (e.g., the changes in the coloring of the peppered moth in England due to the Industrial Revolution), he did not. By showing this specific example, he could connect to geological times, denoted in the notation as $PCK(evolution|B \otimes G)$.

To develop the students' 21st-century skills, Mr. Smith has students use technology to download data, determine the validity of the data, and analyze the data. During the data analysis, students engage in critical thinking and problem-solving to answer the guiding questions posed by the teacher.

Combined, Mr. Smith's integrated interdisciplinary PCK regarding the topic of evolution can be denoted as

$$PCK(Evolution|B \otimes G \otimes P \otimes C)$$
$$= PCK(evolution|B) + PCK(geological\ time|B \otimes G) + PCK(evolution|G)$$
$$+ PCK(evolution|G \otimes P) + PCK(evolution|P) + PCK(evolution|P \otimes C)$$
$$+ PCK(evolution|C)$$

The teacher's interdisciplinary PCK about evolution is the equivalent of the PCK about evolution of each science domain and the PCK about evolution at the interface of each science domain with another science domain.

Toy Case 4b - Transdisciplinary PCK: Evolution

Case

Mr. Smith has had several students discuss the validity of the theory of evolution by natural selection and is aware of discussions on social media and in school board meetings on evolution. He thinks it is important that students are aware of various arguments in these discussions, their history, and implications. Hence, in the following year, Mr. Smith expanded his integrated approach to additional disciplines beyond the natural sciences. It is now an integrated transdisciplinary approach to teaching the topic of evolution that integrates data, examples, and graphics from various science disciplines (biology, geology, chemistry, and physics) and other academic disciplines (e.g., sociology, English, history, public health, mathematics, and economics).

Mr. Smith now begins the unit by introducing the concept of evolution through a story set in the historical context of Charles Darwin's journey on the HMS Beagle. It is the first introduction to the central ideas in the theory of evolution by natural selection. However, this historical narrative is enriched with excerpts from Darwin's writings, and Mr. Smith dedicates some time to exploring the formal language used.

To illustrate the biological principles of evolution, Mr. Smith uses the example of the peppered moth during the Industrial Revolution. Students analyze genetic data and create phylogenetic trees to understand the relationships between different species. They also examine the fossil record and geological time scale to see how organisms have changed over time. In chemistry, students learn about the molecular basis of evolution, including DNA, RNA, and protein synthesis. These hands-on activities help students understand how mutations in genetic material can lead to evolutionary changes.

To connect evolution to sociology, Ms. Smith examines the social implications of evolutionary theory with case studies. Students are organized into groups with specific roles for each member. They discuss how evolutionary concepts have been used to justify social hierarchies and discrimination, and the ethical considerations of such applications. They also explore the impact of human evolution on social structures and behaviors. Mr. Smith provides guiding questions for exploration and discussion for each role within the groups.

In connection with a public health case study, students learn about pathogens' evolution and antibiotic resistance development. They analyze case studies of disease outbreaks and explore the role of evolution in shaping human health. Students are encouraged to be creative in developing solutions for how public health can prevent disease outbreaks. This transdisciplinary approach helps students understand the practical applications of evolutionary theory in addressing public health challenges.

Mathematics is integral to understanding evolutionary processes. Mr. Smith teaches students how to use mathematical models to simulate population dynamics and predict evolutionary outcomes. They learn about statistical methods for analyzing genetic data and the mathematical principles underlying natural selection and genetic drift. Students must solve problems using data related to real-world scenarios. For example, Mr. Smith explores the economic implications of evolution. Students discuss how evolutionary principles can be applied to understand economic behavior and market dynamics. They examine case studies of industries, such as agriculture and biotechnology, that are influenced by evolutionary processes.

Case Discussion

By using a transdisciplinary approach, Mr. Smith provides his students with a holistic understanding of evolution from multiple perspectives due to the impact this understanding has on the many different aspects of the real world. The PCK components used by Mr. Smith in this transdisciplinary example include using multiple representations in the form of case studies and activities. His depth of knowledge of the topic allowed him to plan the different activities, labs, and case studies. The formation and use of groups with group members playing different roles reflects his general pedagogy and classroom management. Making the case studies parallel to real-life issues shows that the topic of evolution is a socio-cultural issue. This method enhances their knowledge of the subject and helps them develop critical thinking, communication, and problem-solving skills. By integrating multiple disciplines, students gain a deeper appreciation for the complexity and interconnectedness of the natural world.

$$
\begin{aligned}
&PCK(Evolution|all\ disciplines)\\
&\qquad = PCK(evolution|B \otimes G \otimes P \otimes C) + PCK(evolution|history)\\
&\qquad + PCK(evolution|history \otimes language) + \cdots
\end{aligned}
$$

Mr. Smith's transdisciplinary PCK about evolution is equivalent to the PCK about evolution at the interface of all science domains, see discussion of Toy Case 4a, and the PCK about evolution at the interface of all the different and relevant subject areas (e.g., history, language, and public health).

Toy Case 5 Transdisciplinary PCK: Nuclear Bomb

Case

In a physics class, a teacher introduces the working principles of a nuclear bomb at the end of a lesson series on nuclear energy. Nuclear weapons are a particular interest of the teacher, and he thinks it is important to introduce students to more than the physics. The class reads a text on the Hiroshima and Nagasaki nuclear bombs. They read out loud, taking turns, in line with a school-wide effort to increase literacy (the principal is adamant that 'every teacher is a language teacher'). The text introduces the students to mass-to-energy conversion and the historical context leading to the dropping of the bombs. Throughout the reading of the text, the physics teacher probes the students' understanding of the technical and historical terms in the text. After reading the text, the teacher assigns homework where the students look for additional resources online. He assigns specific topics to research to each student based on questions from the students and some additional questions from the teacher. He also provides online resources (e.g., articles, databases, and authors) as a starting point. In the following lesson, the class discusses the effect of the bombs on the cities and the surrounding environment (fauna and flora), ethical considerations, and current global politics.

Case Discussion

A teacher can't be well-versed in all disciplines. However, for a single topic, a teacher is likely to develop enough knowledge (both CK and PCK) to introduce students to (parts of) biology, earth sciences, history, philosophy, political science, etc. The teacher introduces the effects of radiation on biological tissues as an impact from the bombs. The connections between the historical events and the social geography are presented through accounts from the diaries of survivors that students read and share. The teacher extends the discussion about nuclear bombs by presenting current, potential, global, and political case studies. In this toy case, the physics teacher has developed transdisciplinary PCK about nuclear bombs NB.

$$PCK_{T_P}(NB) = PCK(NB|P) + PCK(NB|B) + PCK(NB|G) + PCK(NB|history) + PCK(language|NB)$$

The transdisciplinary PCK about nuclear bombs of the physics teacher is equivalent to the PCK about nuclear bombs in physics, biology, and geosciences plus the PCK about nuclear bombs in other subject areas (e.g., history and language). This transdisciplinary approach also develops students' information and technology literacy by supporting students in looking for information online and their critical thinking about the role of nuclear weapons in global politics.

Discussion

The toy cases were developed to show the impact of an approach to science education that embraces inter- and transdisciplinarity and develops 21st-century skills in science teachers' PCK. The cases show how different topics can be taught, ranging from a multidisciplinary approach to an interdisciplinary approach to a transdisciplinary approach. Three themes emerged from the development of the toy cases. The first theme that emerged early in the development was that inter- and transdisciplinary science education offers great opportunities to develop students' 21st-century skills. It could also be read inversely, i.e., the influence of 21st-century skills forces science teachers to broaden their science background among science domains and be able to apply the new interdisciplinary content knowledge to real-world applications. Either way, the combination of inter- and transdisciplinary science and 21st-century skills forces science teachers' PCK to expand. Adding inter- and transdisciplinary PCK as separate grain sizes of PCK is the second theme. Finally, as a byproduct of this newly expanded PCK, these toy cases reflect how preservice and in-service science teachers should be trained. These three themes are discussed in more detail below.

PCK Components and 21st-century Skills

The toy cases presented introduced inter- and transdisciplinary aspects of science education, and thus also of science teachers' PCK. Moreover, the cases also introduced different ways to include the 21st-century skills in science education. Table 3 summarizes which elements of PCK and which 21st-century skills are present in the different cases. The cases are complementary as each shows a different aspect of introducing inter- and transdisciplinarity into science education. Moreover, the cases are presented in an order that adds layers of increasing complexity.

The first introductory case, where physics and biology teachers introduce students to optics in the eye, shows how classical science education already has elements of multidisciplinarity. The teachers remain firmly within their own discipline but are aware of their colleagues' content. Indeed, a teacher should have and apply curriculum knowledge, both vertically (What content do students encounter in other years?) and horizontally (What content do students see in other courses?). However, it is worthwhile to expand from multidisciplinarity to interdisciplinarity. This happens in the second case by illustrating how a teacher team guides students through a (typical) STEM project.

While they started multidisciplinary, each respecting the boundaries of the disciplines, the project and students pushed them to adopt an interdisciplinary approach. The PCK components and the 21st-century skills used and exemplified in these cases are few. The interaction among the knowledge components and competencies lacks specificity.

The following two cases expand this notion even further to transdisciplinarity. Cases 3 and 4 illustrate how a team could approach topics in science in both an interdisciplinary and transdisciplinary way. For climate change (toy case 3), this is obvious and perhaps even expected. Climate change is, after all, a truly transdisciplinary problem. The increase in the number of PCK components and 21st-century skills increases due to the nature of the cases being interdisciplinary and transdisciplinary. However, the fourth case shows that it is also possible to develop a topic that is typically taught in a unidimensional way, here evolution, to an inter- and even transdisciplinary approach. It is by expanding the approach to be inter- and transdisciplinary that students' 21st-century skills could be developed and expanded. This would necessitate the teacher developing more PCK components.

The fifth case adds a final layer of nuance. Where in all previous cases several teachers work together, the final case shows that it is possible for a single teacher to work in a transdisciplinary way. As argued in the case discussion, it cannot be expected that a single teacher has in-depth CK and PCK for all topics. However, diving deep into a single topic and exploring it from different angles, whilst paying attention to 21st-century skills, could be enriching for both the students and the teacher.

Interdisciplinary and Transdisciplinary PCK

The societal issues and application of science content necessitate teachers to understand science content beyond their specialty and their impact on society. Science teachers at the secondary level should still have a depth of science content knowledge within their own domain. However, they should also be able to expand their science knowledge beyond their specialty area. The nature of societal issues and wicked problems forces citizens to understand and critically think about multiple variables that might influence how society solves problems. Toy cases 3a and b reflect the idea that science content, when related to a social issue, needs to be taught in an interdisciplinary and not a multidisciplinary manner. In many instances, teachers in a school will approach a collaborative project by siloing their respective subject areas and not communicating and sharing knowledge. As a multidisciplinary approach, science content is not understood or learned as relating to each other. If the science teachers in the interdisciplinary evolution toy case collaborate and show how the different science domains provide a holistic approach to an issue, the students would have an easier time applying the science principles to real-world applications. Science teachers and students benefit from the seamless integration and crossover of science content to help contextualize evolution and how it impacts and can be explained using multiple science domains.

In the transdisciplinary toy case for evolution, students take the next step in realizing how evolution applies to their lives. Learning is increased when science is personal or applicable. Applying science to real-world problems or examples solidifies the science content for many students. The transdisciplinary approach to learning science content provides the societal and personal contexts for enhanced learning. Science facts become real, and epistemic knowledge is solidified.

Table 3

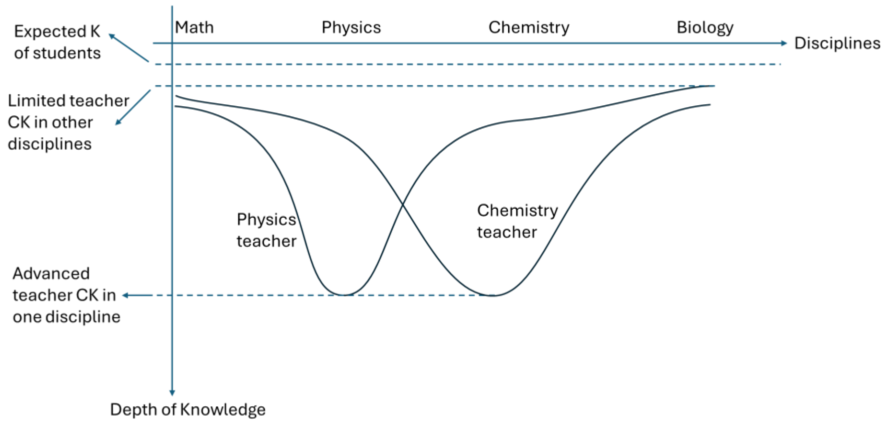
Alignment of PCK components and 21st-century skills with toy cases

	Case 1 Eye inter	Case 2 Health monitor inter	Case 3a CC inter	Case 3b CC trans	Case 4a Evol inter	Case 4b Evol trans	Case 5 Nuclear bomb trans
PCK components							
Representations	X		X	X	X	X	
Students' Understanding		X	X		X		
Subject Matter	X	X	X	X	X	X	X
Curriculum	X		X		X		
Assessment				X	X		
Environment/Context				X	X		X
Socio-cultural issues			X	X		X	X
General Pedagogy						X	X
Classroom Management		X				X	
21st-Century Skills							
Technology literacy		X	X		X		X
Media literacy			X	X	X		
Information literacy				X			X
Critical thinking			X	X	X		X
Problem-solving		X	X	X	X	X	
Creativity & Flexibility			X	X		X	
Collaboration & Communication		X	X	X		X	

So, given the premise that societal challenges require complex solutions, that science is becoming more inter- and transdisciplinary, and assuming that science education should or will follow suit, this evolution has implications for the knowledge of science teachers. As argued above, the teachers' content knowledge will have to expand. Figure 4 describes teacher content knowledge across domains, which is deepest in their own domain and is deeper than what is expected from students for neighboring domains. As teachers are expected to teach in a more interdisciplinary or transdisciplinary way, their content knowledge will need to deepen in multiple domains of science. However, they can and should still maintain the depth of their knowledge in their primary science domain. For example, a physics teacher will maintain deep content knowledge in physics but will also need to build a deeper insight into mathematics, chemistry, and biology without being expected to become a full expert in those neighboring domains. To be clear, the positioning of the disciplines on a single (horizontal) axis is a gross simplification of the relation between the disciplines. As such, the figure is schematic and should not be read as a true two-dimensional graph.

Figure 4

Disciplinary Knowledge of Science Teachers across Domains



Additionally, as the toy-cases illustrate, teachers need knowledge of how their domain, and the topics within it, are related to other domains within (inter) and beyond (trans) science. Moreover, and specifically for teachers, they must know mistakes students make in the secondary domain that are connected to their domain, they need to know how to connect both domains in different ways to support students in their understanding, they need to know which representations (with elements of both domains) allow this connection. In other words, the PCK of science teachers will also need to expand. Specifically, the model of Veal and Makinster (1999) that introduced PCK grain sizes should be expanded to include interdisciplinary (ID-PCK) and transdisciplinary (TD-PCK) PCK. Figure 5 shows a tentative and expanded framework for PCK based on Veal and Makinster (1999). It should be clear that teachers, of course, still need general, domain-specific, and topic-specific PCK. This expanded PCK will allow science teachers to understand and introduce students to how the topics, concepts, and issues relate to other domains of science and impact society and the real world. Science teachers must broaden their scope and understanding of their knowledge base to make learning more accessible and personal, thus providing students with the opportunities to develop 21st-century skills.

Figure 5 presents the remodeled grain size for PCK. This model reflects the external and societal pressures on science teachers due to socio-scientific issues necessitating an interdisciplinary and transdisciplinary approach to solving them. Science teachers must develop ID-PCK and TD-PCK to teach science appropriately while integrating 21st-century skills. Interdisciplinary PCK sits atop the domains of science, as all content and areas of science are blended rather than being individually assembled with discrete knowledge, as in a multidisciplinary approach. Transdisciplinary PCK resides atop the traditional school subjects and domains of knowledge. When solving more complex 21st-century socio-scientific issues, teachers must smoothly integrate other subject areas to prepare effective solutions for these issues. Both new knowledge bases require extending the knowledge and skills of the science teachers. The toy cases serve as a blueprint for understanding how ID-PCK and TD-PCK can be conceptualized in the preparation and development of excellent science teachers.

Figure 5

A Revised Framework for PCK based on Veal & MaKinster (1999)

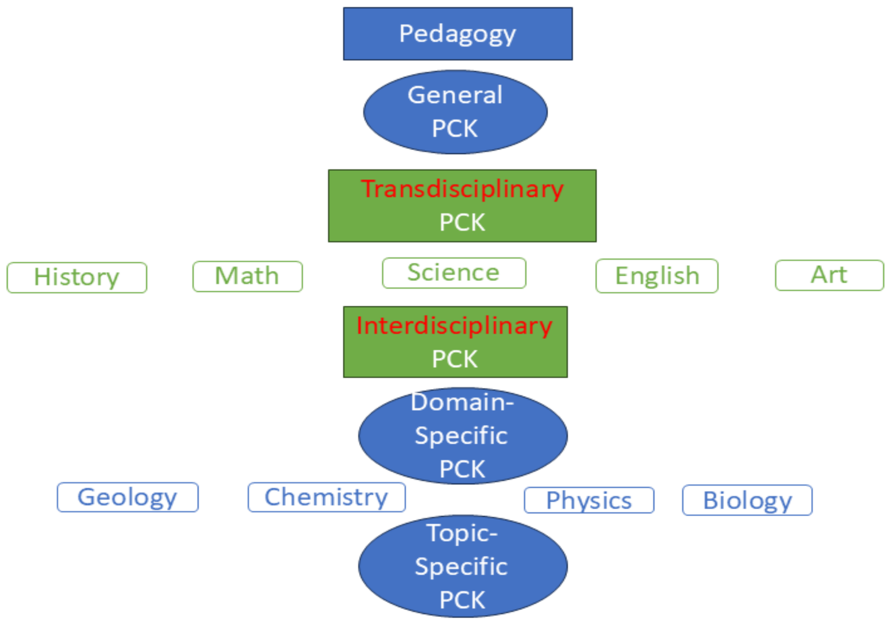
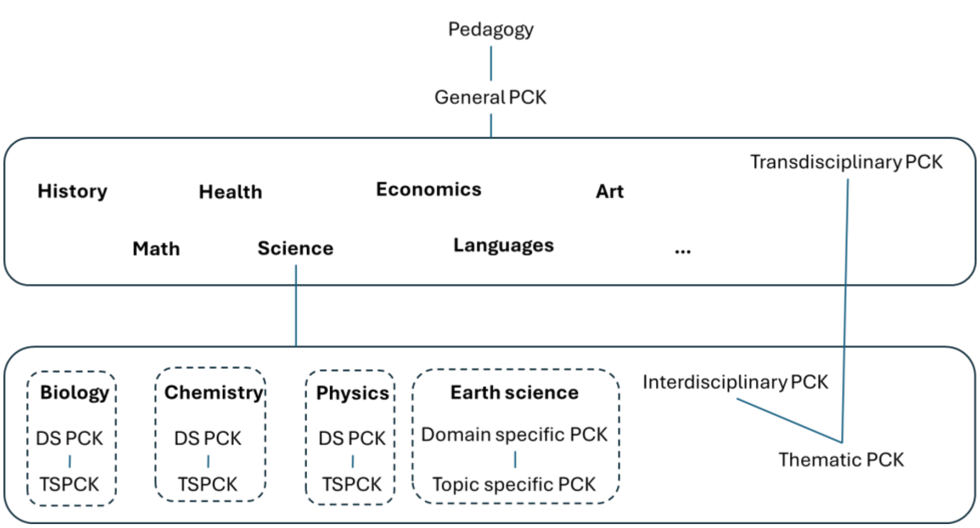


Figure 6 represents an explanatory figure for showing how ID-PCK and TD-PCK interact within and beyond the domains of science and including the knowledge of other subject areas. Interdisciplinary PCK encompasses the domains of science. The boundaries of the domains are depicted as dashed lines to reflect how all science domains contribute to solving the thematic problem or issue. This presents a potentially new idea of Thematic-PCK, which includes a broader vision of a theme that includes facts, concepts, domains, and extensions. Transdisciplinary PCK encompasses all school subjects and domains of knowledge.

Figure 6

An Explanatory Figure for PCK Grain Size



Professional Development

The impact of this new framework dictates that science teachers, new and old, must adapt their knowledge structures so that they can teach science differently. For example, how does a physics teacher present information about climate change? Climate change is a wicked problem that mandates a broad understanding of the science behind it and its potential impacts on different facets of society. For a science teacher to appropriately and logically teach a unit on climate change, this science teacher would have to have interdisciplinary PCK in which science facts and concepts are intertwined to present the scientific case for climate change (Toolin et al., 2022). If the science teacher wants to present the practical application, causes, and effects of climate change, they must develop and show how it influences other aspects of the students' lives with examples and applications to the economy, communication, public health, and history. By designing lesson plans in this transdisciplinary manner, science teachers should model how scientists and society attack wicked problems.

At the preservice level, budding new science teachers must be trained in how science can be positioned with other science domains and academic subjects. This may influence how preservice teachers are certified to teach (Hermann, 2016). For example, in the United States, each state has its own laws on how to certify middle and secondary science teachers. In most states, science teachers can be certified in one science domain (e.g., Chemistry, Biology, Physics, or Earth Sciences) or as a general science teacher. Applying this new framework to PCK necessitates that preservice teachers be certified as science teachers with strong science content knowledge in at least one domain but also having science content knowledge across other science domains. Science teachers will need to be certified with the new standards for teaching, while the idea of just being a chemistry or physics teacher will no longer be recognized. By contrast, in Flanders, Belgium, it is generally accepted that teachers need in-depth domain-specific PCK. Indeed, this paper does not negate the value of having dedicated physics or chemistry teachers, for example. Rather, the toy cases illustrate that it might no longer be sufficient for them to have only deep content knowledge within their own discipline. Instead, teacher training programs should require students to get at least some in-depth science content knowledge in other disciplines. Additionally, the teacher training programs themselves should emphasize developing, in addition to in-depth PCK in a single domain, PCK in other domains as well as inter- and transdisciplinary PCK.

Of course, this argument about preparing science teachers with ID-PCK and TD-PCK extends to the professional development and expectations of science teacher educators. As Abell (1997) stated, science teacher educators must also have PCK and all the associated knowledge bases that Shulman described. The argument extends to science teacher educators at the university, state, and district levels, in that they must also have ID-PCK and TD-PCK to prepare future science teachers.

Conclusion

This paper argues for two distinct and related points that can influence how science teachers use inter- and transdisciplinary topics or themes, including socio-scientific issues, in their classes while developing students' 21st-century skills. First, the grain size of PCK needs to be expanded to include Interdisciplinary PCK (ID-PCK) and Transdisciplinary PCK (TD-PCK). This results from how scientists look at societal issues and real-world problems. Teaching these concepts, known in many instances as wicked problems, necessitates a new teaching method that a science teacher must use. The 21st-century skills that students need influence the movement from isolated science content to an integrated approach. By developing these 21st-century skills, students will be more prepared to understand and solve socio-scientific issues. The interdisciplinarity of science within science domains and with other academic subjects reflects how science should be taught and learned. Silos and isolated science content cannot continue to be the norm if students are prepared to advance society in the 21st-century.

The authors received no financial support for the research, authorship, and/or publication of this manuscript.

William R. Veal (vealw@cofc.edu) is a professor of teacher education and chemistry at the University of Charleston. His research interests focus on reform-based science education and its impact on preservice teacher accreditation. He also researches the intersection of pedagogical content knowledge and creativity in science teaching and learning.

Jan Sermeus (jan.sermeus@kuleuven.be) is an associate professor at KU Leuven and the Brussels Planetarium, researching astronomy education and science education. He is interested in students' understanding of astronomical concepts and their understanding of the nature of science.

References

Abell, S.K. (1997). The professional development of science teacher educators: Is there a missing piece? *The Electronic Journal for Research in Science & Mathematics Education, 1*(4). https://ejrsme.icrsme.com/issue/view/711

Al-Abdali, N. S., & Al-Balushi, S. M. (2016). Teaching for creativity by science teachers in grades 5-10. *International Journal of Science and Mathematics Education*, *14*(2), 251–268. https://doi.org/10.1007/s10763-014-9612-3

AniMed (2017), *How the Eye Works Animation - How Do We See Video - Nearsighted & Farsighted Human Eye Anatomy*. Retrieved March 6, 2025, from https://www.youtube.com/watch?v=YcedXDN6a88

Barthel, R., & Seidl, R. (2017). Interdisciplinary collaboration between natural and social sciences–status and trends exemplified in groundwater research. *PloS one*, *12*(1), https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0170754.

Bielik, T. & Krell, M. (2025). Developing and evaluating the extended epistemic vigilance framework. *Journal of Research in Science Teaching*, *62*(3), 869–895. https://doi.org/10.1002/tea.21983

Bliss, A. C. (2019). Adult Science-Based Learning: The Intersection of Digital, Science, and Information Literacies. *Adult Learning*, *30*(3), 128–137. https://doi.org/10.1177/1045159519829042

Bornmann, L., Haunschild, R., & Mutz, R. (2021). Growth rates of modern science: a latent piecewise growth curve approach to model publication numbers from established and new literature databases. *Humanities and Social Sciences Communications, 8*(1), 1-15. https://doi.org/10.48550/arXiv.2012.07675

Carlson, J., Daehler, K. R., Alonzo, A. C., Barendsen, E., Berry, A., Borowski, A., Carpendale, J., Chan, K. K. H., Cooper, R., Friedrichsen, P., Gess-Newsome, J., Henze-Rietveld, I., Hume, A., Kirschner, S., Liepertz, S., Loughran, J., Mavhunga, E., Neumann, K., Nilsson, P., Park, S., Rollnick, M., Sickel, A., Schneider, R. M., Suh, J. K., van Driel, J., & Wilson, C. D. (2019). The refined consensus model of pedagogical content knowledge in science education. In A. Hume, R. Cooper, & A. Borowski (Eds.), *Repositioning pedagogical content knowledge in teachers' knowledge for teaching science* (*pp. 77–92*). Springer Nature Singapore Pte Ltd. https://doi.org/ 10.1007/978-981-13-5898-2_2

Chan, K. Van Driel, J., Henze, I., Stammes, H., & de Vries, M. (August 24, 2024). *History and Development of PCK* [Meaning of teacher expertise and PCK for 21st century science education]. 21st century science education: What does it mean for teachers? Leiden, The Netherlands.

Chang, C. C., & Chen, Y. (2020). Cognition, attitude, and interest in cross-disciplinary i-STEM robotics curriculum developed by thematic integration approaches of webbed and threaded models: A concurrent embedded mixed methods study. *Journal of Science Education and Technology*, *29*(5), 622-634. https://doi.org/10.1007/s10956-020-09841-9

Chen, D. (2021). Toward an understanding of 21st-century skills: From a systematic review. *International Journal for Educational and Vocational Guidance*, 23, 275-294. https://doi.org/10.1007/s10775-021-09511-1

Cochran, K. F., DeRuiter, J. A., & King, R. A. (1993). Pedagogical content knowing: An integrative model for teacher preparation. *Journal of Teacher Education, 44*(4), 263-272. https://doi.org/10.1177/0022487193044004004

Deák, C., Kumar, B., Szabó, I., Nagy, G., & Szentesi, S. (2021). Evolution of new approaches in pedagogy and STEM with inquiry-based learning and post-pandemic scenarios. *Education Sciences*, *11*(7), 319. https://doi.org/10.3390/educsci11070319

English, L. D. (2023). Ways of thinking in STEM-based problem solving. *ZDM–Mathematics Education*, *55*(7), 1219-1230. https://doi.org/10.1007/s11858-023-01474-7

Facione, P.A. (1990). *Critical Thinking: A Statement of Expert Consensus for Purposes of Educational Assessment and Instruction. Research Findings and Recommendations*. American Philosophical Association: Newark, Delaware. https://eric.ed.gov/?id=ED315423

Friedrichsen, P. J., van Driel, J. H., & Abell, S. K. (2011). Taking a closer look at science teaching orientations. *Science Education, 95*(2), 358-376. https://doi.org/10.1002/sce.20428

Furner, J. M. (2024). The best pedagogical practices for teaching mathematics revisited: Using math manipulatives, children's literature, and GeoGebra to produce math confident young people for a STEM world. *Pedagogical Research*, *9*(2), em0193. https://doi.org/10.29333/pr/14194

Galili, I. (2019). Towards a refined depiction of nature of science: Applications to physics education. *Science & Education, 28*, 503-537. https://doi.org/10.1007/s11191-019-00042-4

Gess-Newsome, J. (2015). A model of teacher professional knowledge and skill including PCK: Results of the thinking from the PCK summit. In A. Berry, P. Friedrichsen, & J. Loughran (Eds.), *Re-Examining pedagogical content knowledge in science education* (pp. 28–42). Routledge. https://www.taylorfrancis.com/chapters/edit/10.4324/9781315735665-4/model-teacher-professional-knowledge-skill-including-pck-gess-newsome-julie

Gleason, D. W. (2020). The humanities meet STEM: Five approaches for humanists. *Arts and Humanities in Higher Education*, *19*(2), 186-206. https://doi.org/10.1177/1474022218806730

González-Pérez, L.I.; & Ramírez-Montoya, M.S. (2022). Components of education 4.0 in 21st century skills frameworks: Systematic review. *Sustainability*, 14, 1493. https://doi.org/10.3390/su14031493

Grossman, P. L. (1990). *The making of a teacher: Teacher knowledge and teacher education.* New York: Teachers College Press.

Hermann, R.S. (2016). Elementary education majors' views on evolution: A comparison of undergraduate majors understanding of natural selection and acceptance of evolution. *The Electronic Journal for Research in Science & Mathematics Education, 20*(6). https://ejrsme.icrsme.com/article/view/16305/10375

Hilton, M. (2010). Exploring the Intersection of Science Education and 21st Century Skills: A Workshop Summary. https://doi.org/https://www.nap.edu/catalog.php?record_id=12771

Hubber, P., Widjaja, W., & Aranda, G. (2022). Assessment of an interdisciplinary project in science and mathematics: Opportunities and challenges. *Teaching Science, 68*(1), 13–25. https://doi.org/https://asta.edu.au/resources/teaching-science-journal/

Humphreys, D., & Kelly, P. (2014). *How Liberal Arts and Science majors Fare in Employment: A Report on Earnings and Long-term Career Paths*. Washington, DC: Association of American Colleges and Universities.

Jin, Q., Kim, M., & Wong, S. (2023). Elementary students' online information problem solving (IPS) in a science classroom. *Alberta Journal of Educational Research, 69*(1), 1–19. https://doi.org/https://journalhosting.ucalgary.ca/index.php/ajer/article/view/71768

Kagan, D. M. (1993). Contexts for the use of classroom cases. *American Educational Research Journal*, *30*(4), 703-723. https://doi.org/10.3102/00028312030004703

Karaca-Atik, A., Meeuwisse, M., Forgievski, M., & Smeets, G. (2023). Uncovering important 21st-century skills for sustainable career development of social sciences graduates: A systematic review. *Educational Research Review, 39*, 1-15. https://doi.org/10.1016/j.edurev.2023.100528

Klein, J.T. (2010). The taxonomy of interdisciplinarity. In Frodeman, R., J.T., Klein & Mitcham, C., *The Oxford Handbook of Interdisciplinarity* (pp. 15-30). Oxford University Press.

- Kotsiou, A., Fajardo-Tovar, D.D., Cowhitt, T., Major, L. & Wegerif, R. (2022). A scoping review of future skills frameworks. *Irish Educational Studies,* 41:1, 171-186, https://doi.org/10.1080/03323315.2021.2022522

Luczak, J. (2017). Talk about toy models. *Studies in History and Philosophy of Science Part B: Studies in history and philosophy of modern physics, 57*, 1-7. https://doi.org/10.1016/j.shpsb.2016.11.002

Magnusson, S., Krajcik, J., & Borko, H. (1999). Nature, sources, and development of pedagogical content knowledge for science teaching. In J. Gess-Newsome & N. G. Lederman (Eds.), *Examining pedagogical content knowledge: The construct and its implications for science education* (pp. 95-132). Kluwer Academic.

Marcoci, A., Thresher, A. C., Martens, N. C., Galison, P., Doeleman, S. S., & Johnson, M. D. (2023). Big STEM collaborations should include humanities and social science. *Nature Human Behaviour*, *7*(8), 1229-1230. DOI: 10.1038/s41562-023-01674-x

Marks, R. (1990). Pedagogical content knowledge: From a mathematical case to a modified conception. *Journal of Teacher Education, 41*(3), 3-11. DOI:10.1177/002248719004100302

Martin, J. (2018). *Skills for the 21st century: Findings and policy lessons from the OECD survey of adult skills*. (Working Paper No. 166). OECD Publishing. https://doi.org/10.1787/96e69229-en.

Osborne, J. (August 19, 2024). *21st Science Education: Desired Student Outcomes* [21st Century Science Education, Student Outcomes, and Consequences for Teachers]. 21st Century Science Education: What does it mean for Teachers?, Leiden, The Netherlands.

Redish, E. (2021). Using math in physics: 4. Toy models. *The Physics Teacher*, 59(9), 683-688. https://doi.org/10.1119/5.0025840

Rodriguez, S., Allen, K., Harron, J., & Qadri, S. A. (2019). Making and the 5E Learning Cycle. *Science Teacher*, *86*(5), 48–55. https://doi.org/https://www.nsta.org/publications/browse_journals.aspx?action=issue&id=116215

Rollnick, M., & Mavhunga, E. (2015). The PCK Summit and its effect on work in South Africa. In *Re-examining pedagogical content knowledge in science education* (pp. 135-146). Routledge.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14. http://dx.doi.org/10.3102/0013189X015002004

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1–23. https://doi.org/10.17763/haer.57.1. j463w79r56455411

Sikand, M., Mazzatenta, C., Wong, K., Bush, J., & Socha, A. M. (2021). Two year community: Sustainability, energy, and the green economy: An interdisciplinary course on environmental sustainability and life cycle analysis. *Journal of College Science Teaching*, *50*(3), 8-16. https://doi.org/10.1080/0047231X.2021.12290502

Toolin, R.E., Jorgenson, S., & Ratmeyer, S. (2022). STEM touchstones for teacher professional learning: An analysis of teacher content and pedagogical content knowledge in a place-based professional development program. *Electronic Journal for Research in Science & Mathematics Education, 26*(2), 32-55. https://ejrsme.icrsme.com/article/view/21986/14614

Urone & Hinrichs (2025), *College Physics 2e*, https://openstax.org/books/college-physics-2e/pages/25-6-image-formation-by-lenses.

Van Noorden, R. (2015). Interdisciplinary research by the numbers. *Nature*, *525*(7569), 306-307. https://doi.org/10.1038/525306a

Veal, W.R., & Kind, V. (2025, August 25-29). *Connecting creativity with PCK: Creating a framework for science teacher education* [Paper presentation]. European Science Education Research Association Biannual Conference, Copenhagen, Denmark.

Veal, W.R., & MaKinster, J.G. (1999). Pedagogical content knowledge taxonomies. *The Electronic Journal for Research in Science & Mathematics Education, 3*(4). https://ejrsme.icrsme.com/article/view/7615

Weart, S. (2013). Rise of interdisciplinary research on climate. *Proceedings of the National Academy of Sciences - PNAS*, *110*(Supplement 1), 3657–3664. https://doi.org/10.1073/pnas.1107482109

Widya, Rifandi, R., & Laila Rahmi, Y. (2019). STEM education to fulfil the 21st century demand: A literature review. *Journal of Physics. Conference Series*, *1317*(1). https://doi.org/10.1088/1742-6596/1317/1/012208