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Big Ideas of Mathematics: A Construct in Need of a Teacher-, Student-, and Family-Friendly Framework

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ABSTRACT

Across multiple countries the term "big ideas" of mathematics has become a construct advocated as important for teachers' mathematical knowledge. Indeed, several policy or position statement documents about math learning in the United States (U.S.) have stated that "big ideas" of math is a crucial construct for teacher knowledge. With this study we sought to determine if there was consistency about this "big idea" construct that teachers of mathematics in the U.S. are advised to know. To this end, we conducted a content analysis of resources for U.S. teachers of preschool through grade 12. We determined that few resources defined a big idea and those that did lacked agreement with each other. Although most resources delineating big ideas cited Charles (2005) as the basis of their use of the construct, our analysis of the actual big ideas in each resource revealed inconsistent implementation of Charles' theoretical perspective. Thus, to move research and practice forward by more precisely defining and prioritizing this abstract construct we (a) clarified a definition, (b) specified five criteria, and (c) constructed a scholar- and teacher-friendly framework: The Big Ideas Framework. This framework consists of three ordinal levels to distinguish and prioritize the importance of big ideas based on relative size and power: Mighty Mega Math Ideas, Power Math Ideas, and Strong Math Ideas. Moreover, the big ideas construct has focused on Mathematical Knowledge for Teaching (MKT), whereas we urge the field to shift its theoretical perspective to value the entire construct or at least the two most powerful levels of the framework as aspects of Common Content Knowledge. In other words, we urge teachers and mathematics teacher educators to foreground Mighty Mega Math Ideas and Power Math Ideas with P-12 students and families to empower those we serve. Furthermore, given the dearth of peer-reviewed research about big ideas, we encourage a new branch of scholarship to analyze the impact of the practical recommendations we offered here.

Keywords: big ideas, teacher knowledge, math learning, teacher professional development, teacher education

Introduction

The importance of focusing on big ideas is widely advocated to support the teaching and learning of mathematics. The notion of discipline-specific big ideas can be traced back to Bruner's (1960) "fundamental ideas," where he argued that "knowledge . . . acquired without sufficient structure is knowledge that is likely to be forgotten" (p. 31). Similarly, Wiggins and McTighe (2005) argued that, regardless of the discipline, big ideas promote transfer; that is, they apply to future learning both horizontally (across topics) and vertically (across grade levels and courses). Moreover, using big ideas can facilitate "thinning out" an over-crowded curriculum and can "create opportunities to rethink and transform existing approaches to the teaching and learning of mathematics" (Siemon et al., 2012, p. 20).

The challenge is that curriculum standards in the United States (U.S.) are presented to teachers linearly (Hurst & Hurrell, 2014). In the U.S., the first standards documents put forth by the National Council of Teachers of Mathematics (NCTM; 1989) did not constitute a national curriculum as most countries use. These documents separated mathematics into what were called strands of *Geometry*, *Measurement*, *Data Analysis and Probability*, *Number and Operations*, and *Algebra* (NCTM, 1989, 2000). Current mathematics standards in each state are built from the *Common Core State Standards for Mathematics* (hereafter referred to as *CCSSM*; National Governors Association Center for Best Practices [NGA Center] & Council of Chief State School Officers [CCSSO], 2010). In Grades K to eight, standards are divided into four to five domains for each grade, whereas in high school the divisions are overarching "conceptual categories" rather than grade levels or courses. Domain titles and conceptual categories vary in their scope and are organized somewhat differently than the original strands, such as *Geometry*, *Measurement and Data*, *Counting and Cardinality, Number and Operations in Base Ten, Ratios and Proportional Relationships, Building Functions*. Each of these domains is divided further into clusters, which in turn, are divided into standards. These standards are the focus of the teachers' attention when instructing students and by which students and teachers are evaluated on state tests.

The quantity of standards range from a total of 24 in Kindergarten to 33 in eighth grade and 31 in high school Algebra. In spite of the intent of the *CCSSM* having been to "significantly narrow the scope of content in each grade and deepen how much time and energy is spent on major topics in the classroom" (Coleman et al., 2013, p. 3), it is commonly known in practice that the copious number of standards is problematic. Although some efforts have been made to help teachers connect standards vertically and horizontally, such as Achieve the Core (Student Achievement Partners, n.d.), the U.S. curriculum document is a checklist of discrete knowledge. With the focus on these narrowly defined standards separated out by domain within a grade, teachers teach these standards in an unconnected way, and therefore children learn it the same way (Boaler et al., 2017; Hurst & Hurrell, 2014). Siemon and colleagues (2012) suggested using big ideas to further 'thin out' the curriculum (p. 20), and Charles (2005) cautioned not to let the number of big ideas "balloon" (p. 12). The number of big ideas we found in our search ranged from as few as three (Small, 2009) to as many as 80 (Boaler et al., 2017).

To get a sense for how the term "big idea" has been used in prior and current documents related to mathematics teaching and learning, we searched for a variety of publications that have employed the term. Most of the publications we found were teacher educator-focused articles or reports and books aimed to improve teacher's knowledge, only some of which were peer-reviewed. Our search for big ideas yielded reports to government or grant agencies in several countries (Kuntze et al., 2011a; Morgan, 2012; Niemi et al., 2006; Siemon, 2022; Siller & Kuntze, 2011; Tout et al., 2015; YuMi Deadly Mathematics, 2016), popular press books (e.g., Boaler et al., 2021; Early Learning Collaborative, 2014; Schifter & Fosnot, 1993; Siemon et al, 2012; Small, 2009), professional organization books (Toh & Yeo, 2019; Zbiek, 2010, 2011, 2012, 2013, 2014), and proceedings of joint conferences for practitioners and scholars (Skalicky et al., 2007; Siemon, 2013; Watson, A. 2007; Watson, J., 2007; Worsley, 2011). In terms of peer-reviewed publications, we found practitioner articles (Charles, 2005; Clarke et al., 2012; Edwards, 2000; Ritchhart, 1999; Woodbury, 2000), research conference proceedings (Askew, 2015; Hurst, 2014; Kuntze et al., 2011b; Siller et al., 2011; Stehr et al., 2019;), and just five peer-reviewed articles in research journals (Askew, 2013; Greenes, 2009; Hurst, 2014; Hurst & Hurrel, 2014; Siller & Kuntze, 2011).

Although none of the explanations of the term big ideas used the word *power* within formal definitions, some used this word or idea in later paragraphs of the narrative text around big ideas to explain the importance of big ideas. Notice the use of the word *power* in each of the following, which we used italics to emphasize. For example, Carnine (1997) asserted that big ideas "have rich *explanatory and predictive power*, as students can use them in solving many different problems that on the surface appear to be unrelated" (p. 133). Ritchhart (1999) argued that teachers must "look forward to what they can give students *real power*," to "build *mathematical power*, and . . . opportunities. . . for making connections and supporting transfer" (p. 463). Boaler and colleagues (2017) explained that to determine their big ideas, "We also thought carefully about the ideas that get little attention in standards and curriculum, but that are *powerful for mathematical thinkers*" (p. 4). For Tout et al. (2015), "The concept of Big Ideas is *powerful* because it assists teachers in developing a coherent overview of mathematics . . . [and] enables students to develop a deeper understanding of mathematics and its interconnectedness" (p. 19). Wiggins and McTighe (2005) asserted that "a big idea is not 'big' merely by virtue of its intellectual scope. It has to have *pedagogical power*: It must enable the learner to make sense of what has come before; and most notably, be helpful in making new, unfamiliar ideas seem more familiar" (p. 70). Finally, according to Charles (2005), limiting the quantity on which to focus is "what makes the notion of Big Ideas *so powerful*" (p. 11). With our careful reading of these works, we noticed this theme of power as underlying the field's thinking about this construct, however, this has yet to be foregrounded in the meaning of the term itself.

What is a Big Idea and How Big is Big?

Given that the name of the construct of "big" ideas is entirely based on an adjective related to size, we were surprised that few described the size or acknowledged the dilemma of just how big a big idea is. Boaler et al. (2017) did provide visual and narrative indications that not all big ideas are the same size or relative importance. Implicitly the authors communicated varied sizes of big ideas through network diagrams in which the nodes varied in size (Boaler et al., 2017). Askew (2013) described the size of a big idea in subjective terms that might evoke the Goldilocks principle as "big enough . . . but not so big that it is unwieldy" (p. 7). Charles (2005) expressed his dilemma that it was too difficult to articulate how big was sufficiently big to be labeled a big idea, yet like Askew he viewed particular ideas as being important but not "sufficiently robust to qualify as a big idea in mathematics" (p. 9). In a systematic literature review about big ideas, Askew (2013) concluded that there is no standard definition for a big idea of mathematics. A decade later as we sought the definition of "big idea," we found a wide range of specificity in terms of whether the term was used without definition, used with vague explanations around the construct, or with an explicit definition.

Surprisingly, the term "big idea" has been used with the article "the" in policy documents about what teachers must know, yet these documents failed to (a) provide "the" big ideas teachers must know and (b) define or explain what a big idea is (e.g., NCTM, 2000, 2014). Others used broad explanations. For example, Schifter and Fosnot (1993) described big ideas as "central organizing principles of mathematics with which students wrestle as they confront the limitations of their existing conceptions" (p. 24). Some described requisite characteristics or criteria without providing an explicit definition. For instance, Clements and Sarama (2009) identified three criteria which big ideas must meet; specifically, big ideas are "clusters of concepts and skills that are mathematically central and coherent, consistent with children's thinking, and generative of future learning" (p. 1). Alternatively, Kuntze et al. (2011a) asserted that big ideas must possess the following four characteristics: 1) have a high potential for encouraging mathematics learning with understanding of conceptual knowledge, including orientation, linking, and anchoring of knowledge; 2) are relevant for building up knowledge about mathematics as a science; 3) support abilities of communicating meaningfully about mathematics; and 4) encourage reflection processes of teachers connected with designing rich and cognitively activating learning opportunities, as well as accompanying and supporting learning processes of students. These broad characterizations provide some sense of the construct, yet what the field needs is a shared theoretical perspective.

Charles' (2005) Theoretical Perspective of Big Ideas of Mathematics

Although Charles (2005) did not use the terms framework, theory, or theoretical perspective, we view Charles' definition of and criteria for the often-used construct of big ideas of mathematics as an underused theoretical perspective. In terms of a definition of a mathematical big idea, only Charles (2005) provided such a definition: "A Big Idea is a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" [capitalization in original] (p. 10). He then clarified his theoretical perspective by explaining phrases within the definition as well as specifying additional criteria not in the definition. For example, the term "statement" is in the definition and Charles emphasized it is crucial that the idea must be phrased in this way. A statement is a type of sentence that has a verb and a subject; it is also not a question (O'Brien, 2023). Charles was adamant that every big idea must have a name preceding it as a way to refer to it, not just the statement itself. In some work that comes after Charles (2005), the criteria of connections is used without further specification, whereas notice in the definition he quantified this with the adjective "numerous." Although numerous is a vague term, he acknowledged he found it difficult to codify how big or "robust" a big idea must be to be "central to the learning of mathematics" or how many "understandings" would be linked to be "sufficiently robust" (Charles, 2005, p. 9). Charles (2005) and Boaler et al. (2017) are among the few to acknowledge this quandary of how big is big or even that some ideas are bigger than others.

Some have pointed out that it may be impossible to determine a complete canon of ideas upon which professionals could agree (e.g., Boaler et al., 2017; Charles, 2005; Ritchhart, 1999). Some posit that it is important for teachers themselves to determine big ideas within their professional development and unit planning—and it is this process of collaboration and professional growth that is most important (e.g., Boaler et al., 2017; Ritchhart, 1999). Yet, as Boaler et al. (2017) recognized, most teachers and districts do not have the time or structures in place to foster this process approach, which is why they offered eight to ten big ideas in each grade. Moreover, it would behoove us to use the insights of a teacher who explained why it is important to provide big ideas to teachers: "big ideas I think need to come from someone who sees the big, big picture" (Clarke et al., 2012, p. 17). This teacher advocated that rather than teachers developing the big ideas, teachers need the field to provide big ideas as a starting point. This would make it feasible for teacher planning teams to focus on how to implement the ideas for their own students and contexts (Clarke et al., 2012).

In summary, Charles (2005) provided the most specified definition, supporting criteria and 21 big idea statements to form a theoretical perspective for the construct of big ideas (e.g., Askew, 2013; Boaler et al. 2017; Hurst & Hurrell, 2014; Siemon et al. 2012; Small, 2009). In spite of Charles (2005) having provided specificity and being universally cited, there is still little agreement in subsequent literature about what 'big ideas' are (Askew, 2013; Siemon, 2013). This seems to be due to the fact that although most documents discussing big ideas cite Charles (2005), they have not built on the specifics he offered in his theoretical perspective.

Big Ideas Are For Teachers

Many policy documents and articles assert that "big ideas" are crucial to developing teachers' content knowledge and their ability to effectively implement curriculum (Association of Mathematics Teacher Educators [AMTE], 2017; Hurst & Hurrell, 2014; Kuntze et al., 2011a, b; NCTM, 2000; Prawat, 1992; Toh & Yeo, 2019). Almost a quarter century ago, NCTM (2000) published the *Principles* *and Standards for School Mathematics*, which stated that teachers needed to "understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise" (p. 17), without ever defining what a big idea was—except for mentioning "equivalence" and "multiplicative reasoning" as big ideas for Grades 3-5. Note the use of the article "the." A decade later, in the *CCSSM* (NGA Center & CCSSO, 2010), the term "big idea" does not appear anywhere in the document. Most recently in 2017, AMTE published the *Standards for Preparing Teachers of Mathematics*, which explicitly inserted the term into *Program Standard P.2. Opportunities to Learn Mathematics*, which states that

> An effective mathematics teacher preparation program provides candidates with opportunities to learn mathematics and statistics that are purposefully focused on essential big ideas across content and processes that foster a coherent understanding of mathematics for teaching (p. 29).

This document is for those who prepare future teachers, not the teachers who plan P-12 student instruction. This may signal a shift toward valuing and being more explicit about developing mathematics teacher educator understanding of big ideas in the US. Unfortunately, the AMTE (2017) document failed to provide mathematics teacher educators with a definition or criteria for "big ideas." Moreover, the document merely references Charles (2005) in the single grade-band section for upper elementary even while asserting that there must be vertical alignment of big ideas across grade bands: "Teachers preparing to teach in these grade-bands must develop mathematical knowledge that not only spans the grade levels but also provides them opportunities to understand big ideas (Charles, 2005) that unify mathematics across grade-band divides" (AMTE, 2017, p. 90).

There is also inconsistency regarding the terms used in these standards to refer to the same construct (i.e., six terms: *big ideas*, *essential big ideas*, *essential understandings*, *essential ideas, foundational mathematical ideas*, and *key mathematical ideas*) (AMTE, 2017, pp. 29, 47, 48, 50, 94) Although the AMTE document (2017) referenced Charles (2005) once, none of his 21 big ideas were included in the standards. The ideas selected for the AMTE standards (2017) seem to have come primarily from the *Essential Understanding Series* published by the affiliated organization of the National Council of Teachers of Mathematics and the *Mathematical Education of Teachers II (METII)* document (Conference Board of the Mathematical Sciences [CBMS], 2012). Those ideas taken from *METII* were phrased as "essential ideas" and those taken from NCTM were the smaller detailed "essential understandings" rather than the broader statements labeled "big ideas," despite the fact the P2 standard as quoted earlier denotes "big idea" would be the term with "essential" used as a modifier for emphasis of those big ideas as "essential big ideas."

Although the ultimate goal is for students to have a strong and coherent understanding of mathematics, the focus of the writing about this construct has been on teacher knowledge, teacher education, and teacher professional development. In other words, the focus has been on improving Mathematical Knowledge for Teaching (MKT), which includes Common Content Knowledge that all graduates of K-12 schooling are intended to know, but also multiple types of knowledge that is unique to the demands of teaching mathematics (Ball et al., 2008). Moreover, although big ideas of mathematics as a construct is meant to unify ideas within the entire discipline of mathematics (Charles, 2005), the genesis of the construct seems to have focused on the MKT of those who teach the elementary grades (e.g., Charles, 2005; Clements & Sarama, 2009; Ritchhart, 1999; Schifter & Fosnot, 1993).

Another and perhaps more crucial limitation of prior work, is that each of the peer-reviewed articles we found were commentaries or literature reviews that primarily built upon other commentaries or non-peer reviewed publications. Kuntze et al. (2011b) pointed out that peer-reviewed research studies on big ideas were limited, which remains the case more than a decade later. We found only three peer-reviewed publications that analyzed data; that is only three were empirical

(Stehr et al., 2019; Siller et al., 2011; Worsley, 2011). Each of these was a conference proceeding in which the data were university instructor surveys and/or interviews regarding the content they teach. Two of these studies—both at the university level mathematics or mathematics teacher preparation—had methodologies and findings that did not address "big ideas" with the meaning intended in the field, because the interviews or surveys asked for "areas of study" (Worsley, 2011) or referred to an entire domain of high school mathematics and K-12 mathematical practice of "modeling" as though it was a "big idea" (Siller et al., 2011).

The only peer-reviewed conference proceeding that used the term "big idea" in their study design asked instructors of secondary mathematics teacher education courses: "What are the goals or big ideas of this course?" (Stehr et al., 2019). Note the phrasing of the question was "big ideas of *this course*" rather than "big ideas *of mathematics,*" so it makes sense that instructor responses of mathematics pedagogy courses included general pedagogy ideas (e.g., exceptionality, race, gender) and some mathematics-related responses (e.g., integers, proportional reasoning; Stehr et al., 2019). These ideas are all important in such courses. However, even the mathematics-related "big ideas" in the findings of that study were "topics," which Charles (2005) stated are not big ideas.

Although many policy or commentary documents claim that research has demonstrated the use of big ideas for improved teacher and student learning, from our own search and others' systematic literature reviews (e.g., Askew et al., 2013), it seems scholars have staked and built upon claims based on citing articles accepted into *research venues* or popular press outlets, without being based on *research findings* of empirical studies accepted by *peer-review* prior to publication. We have yet to find any empirical studies published in peer-reviewed venues that evaluated the impact of big ideas of mathematics on teacher knowledge or student learning.

Purpose of Study

To be clear, from practical experience we believe just as strongly as those who have published before us that big ideas of mathematics are one of the important ways to facilitate mathematically proficient teachers and citizens. However, as scholars and mathematics teacher educators we need to strive for a standard of evidence, which can only occur if we foster some agreement about the terms and meanings of the construct itself (Leatham, 2019; Spangler & Williams, 2019). To provide a foundation for the field to conduct empirical studies of teacher and student knowledge in the future, we as a field need some clarity and shared understanding regarding the definition, size, and purpose of big ideas of mathematics.

Pepin and Gueudet (2014) define curriculum resources as items teachers use "in their day-to-day teaching, when they decide what to teach, how to teach it, and when they choose the kinds of tasks, exercises, and activities to assign to their students" (p. 132). So we turn to the resources used to inform those who are not academics to uncover what information about big ideas of mathematics are being communicated. Charles (2005) has been repeatedly cited in policy documents about big ideas of mathematics. This work was also the only one we have found to provide an explicit definition along with criteria and supporting examples that could be characterized as a theoretical perspective. So we return to this foundational work as a way to systematically assess resources designed since 2010 — when the *CCSSM* was published— to inform what big ideas are and as a way to promote mathematics as a coherent discipline. Given that teachers are told they must know "the big ideas," the purpose of this empirical study was to use a content analysis methodology to investigate these resources for how big ideas are being treated and portrayed. The specific research questions were:

1) To what audience were the big ideas directed? (RQ1)
2) How did each analyzed resource define or explain the construct of big ideas? (RQ2)
3) Were Charles' (2005) criteria of centrality, naming and grammatical format satisfied? (RQ3)

4) Was there consistency within and across resources about the relative size of big ideas? (RQ4)
5) How well were big ideas used to organize or provide coherency for mathematics? (RQ5)

Methods

We used a content analysis method (Schreier, 2012) to analyze 22 publications designed to inform P-12 stakeholders in the U.S. about big ideas. These 22 publications consisted of 224 big ideas, which we primarily analyzed using Charles' (2005) theoretical perspective. Qualitative content analysis involves a systematic investigation of only selected aspects within a data set based on the particular research questions (Schreier, 2012). Unlike other qualitative methods that include analytic memos and creation of additional data, "QCA reduces data" throughout the process as researchers create and refine categories that support broader interpretations (Schreier, 2021, p. 7). Validity in QCA is viewed as the "extent that your categories adequately represent" the data (Schreier, 2012, p. 7). Sample selection criteria and details of analysis for each research question follow.

Sample Selection Process

Given that the literature review revealed that teachers are supposed to "know the big ideas," and our concern for actual teaching practice, we sought publications for our data analysis whose target audience were teachers or P-12 students, not teacher educators or scholars. For example, neither a special issue of a research journal nor the AMTE (2017) *Standards for Preparing Teachers of Mathematics* would fit our target audiences of readers who are P-12 students or teachers. Of course, teacher educators might use the publications we analyzed and scholars and teacher educators would refer to the sources we cited in the literature review, However, our goal was to find consistency about the construct of big ideas as it is being communicated to teachers and students to be used in P-12 schools. We considered the following as potential publications: P-12 standards documents, K-12 student textbooks, methods textbooks for teaching mathematics, textbooks of mathematics for future teachers, books published for teachers, and popular press books about mathematics.

We attempted to find every possible publication we could with the intent to then reduce the number of data sources to one to three publications for each audience (teacher or students), grade band, content, and publication type. The data we analyzed, however, were the only ones we found that met the criteria. Thus, we reported on all the data sources meeting the following criteria: (a) U.S. audience; (b) Teacher or student as the target audience (not scholars or teacher educators); (c) The term "big ideas" was used in the title or within the text and big ideas were delineated; (d) 2010 or later publication date; (e) Ensure vertical alignment with all grade bands represented: Early childhood, (P-2), Intermediate (3-5), Middle Childhood (6-8) and High School (9-12); (f) For horizontal and vertical alignment, domains (term used in *CCSSM*; NGA Center & CCSSO, 2010) or strands (term used in NCTM, 2000) were represented across grade bands; and (g) Each type of publication was represented: mathematics textbooks for future teachers, methods textbooks for mathematics teaching (hereafter referred to as "methods textbooks"), mathematics textbooks for P-12 students, or professional development resources about mathematics.

Publication types for which we searched but that did not meet the other criteria included P-12 standards, mathematics textbooks for P-12 students, mathematics textbooks used to teach future teachers mathematics content, and methods textbooks for teaching secondary mathematics. Although one methods textbook covered Grades 7-8 of secondary mathematics, we were unable to locate a methods textbook for Grades 9-12 of secondary mathematics. Nor did we find any mathematics textbooks used in mathematics departments to teach mathematics content to future teachers that used the term "big ideas" (even after consulting colleagues who teach such courses).

We also sought textbook series for P-12 students. Although there is a student textbook series called *Big Ideas Math* (Larson Texts, 2013-2018), in the preview copies, we were not able to find any mention of the word big idea within the textbook nor did we receive correspondence after emailing the publisher and author to inquire where the big ideas were located. The omission of actual big ideas from the textbook *Big Ideas Math* was also confirmed by a teacher who had taught from this textbook.

Given the importance of standards and readers' familiarity with such documents, we detail in chronological order more about why each standards document did not meet the criteria. NCTM (2000) standards from a generation ago did not meet the publication date criterion and is no longer used by teachers even though teacher educators and scholars may continue to use that document. *CCSSM* (NGA Center & CCSSO, 2010), which is used by teachers, does not use the term big ideas anywhere in the document. The AMTE standards (2017) is a document designed for mathematics teacher educators and program directors, so it did not fit the target audience of teachers or students. Thus, these standards documents were not considered as data sources of the study, although we used these in the literature review and discussion due to their importance. As is commonly expected in a publication-based content analysis, the sample is listed in alphabetical order in Table 1.

The reputable Erikson Institute's P-2 book (The Early Math Collaborative, 2014) and highly popular Van de Walle et al. (2019) K-8 methods textbook in Table 1 were stand-alone publications in which all covered grades were contained within a single book. In contrast, the NCTM publications (2010-2014) explicitly proclaimed in the titles that these are part of a series. This series offers a set of books for each grade band (P-2, 3-5, 6-8, and 9-12). This is consistent with the definition of series: "a number of things or events of the same class coming one after another in spatial or temporal succession" (https://www.merriam-webster.com/dictionary/series). The Boaler et al. (2017-2021) publications also fit the definition of a series, because the series consists of nine books, one for each grade from Kindergarten to Grade 8. To ensure that a resource was the unit of analysis and reflected vertical and horizontal alignment, we considered each entire series in which it took multiple publications to cover domains and grade bands as a single resource comparable to stand-alone publications that spanned domains and grade bands. Consequently and henceforth, we refer to each stand-alone publication and each series as a resource, resulting in four analyzed resources.

To focus reader attention on the resource type and grade band rather than authors, we refer to *Big Ideas of Early Mathematics* as P-2 Methods Textbook (P2MT), *Teaching Elementary and Middle School Mathematics Developmentally* as K-8 Methods Textbook (K8MT), *Essential Understanding Series* as P-12 Professional Development Resource (P12PD), and *Mindset Mathematics: Visualizing and Investigating Big Ideas* as the K-8 Professional Development Resource (K8PD).

Intended Audience of Selected Resources

Selection of the sample based on the audience is both a process we should report in the methods and also as one of our research questions, so we describe this here—RQ1: To what audience were the big ideas directed? The method for this content analysis was a review of the introduction and preface of each resource to select quotations that related to the intended audience. If no such information was found in those sections, then the table of contents, index, where big ideas were stated, and other pages were examined until the audience was found. All authors discussed until consensus in order to reduce these data to representative quotations. This information is provided in Table 2. We were only able to find one student textbook that used the term "big ideas" in the title. However, no big ideas were stated within the textbook content to speak directly to students or assist teachers to make these connections. Therefore, in every resource in our sample, teachers were the intended audience with the purpose to improve teachers' mathematics knowledge or MKT (RQ1). Table 2 provides evidence for each resource along with the scope of grades addressed.

Table 1

Publications in the Sample

Publication
Barnett-Clarke, C., Fisher, W., Marks, R., & Ross, S. (2010). *Developing essential understanding of rational numbers for teaching mathematics in Grades 3–5* (R. Charles, Ed.). In R. M. Zbiek (Series Ed.), Essential understanding series. National Council of Teachers of Mathematics.
Boaler, J., Munson, J. & Williams, C. (2020). *Mindset mathematics: Visualizing and investigating big ideas: Kindergarten*, Wiley.
Boaler, J., Munson, J. & Williams, C. (2021). *Mindset mathematics: Visualizing and investigating big ideas: Grade 1*, Wiley.
Boaler, J., Munson, J. & Williams, C. (2021). *Mindset mathematics: Visualizing and investigating big ideas: Grade 2*, Wiley.
Boaler, J., Munson, J. & Williams, C. (2018). *Mindset mathematics: Visualizing and investigating big ideas: Grade 3*, Wiley.
Boaler, J., Munson, J. & Williams, C. (2017). *Mindset mathematics: Visualizing and investigating big ideas: Grade 4*, Wiley.
Boaler, J., Munson, J. & Williams, C. (2018). *Mindset mathematics: Visualizing and investigating big ideas: Grade 5*, Wiley.
Boaler, J., Munson, J. & Williams, C. (2019). *Mindset mathematics: Visualizing and investigating big ideas: Grade 6*, Wiley.
Boaler, J., Munson, J. & Williams, C. (2019). *Mindset mathematics: Visualizing and investigating big ideas: Grade 7*, Wiley.
Boaler, J., Munson, J. & Williams, C. (2019). *Mindset mathematics: Visualizing and investigating big ideas: Grade 8*, Wiley.
Caldwell, J. H., Karp, K., & Bay-Williams, J. M. (2011). *Developing essential understanding of addition and subtraction for teaching mathematics in pre-k–grade 2* (E. Rathmell, Ed.). In R. M. Zbiek (Series Ed.), Essential understanding series. National Council of Teachers of Mathematics.
Cooney, T.J., Beckmann, S. & Lloyd, G.M. (2010). *Developing essential understanding of functions for teaching mathematics in grades 9-12* (P. S. Wilson, Ed.). In R. M. Zbiek (Series Ed.), Essential understanding series. National Council of Teachers of Mathematics.
Dougherty, B. J., Flores, A., Louis, E., & Sophian, C. (2010). *Developing essential understanding of number and numeration for teaching mathematics in pre-k–grade 2*. (B. J. Dougherty, Ed.). In R. M. Zbiek (Series Ed.), Essential understanding series. National Council of Teachers of Mathematics.
The Early Math Collaborative. (2014). *Big ideas of early mathematics: What teachers of young children need to know*. Pearson.
Goldenberg. E.P. & Clements, D.H. (2014). *Developing essential understanding of geometry and measurement for teaching mathematics in pre-k-grade 2* (B. J. Dougherty, Ed.). In R. M. Zbiek (Series Ed.), Essential understanding series. National Council of Teachers of Mathematics.
Kader, G.D., Jacobbe, T. (2013). *Developing essential understanding of statistics for teaching mathematics in grades 6-8* (P. S. Wilson, Ed.). In R. M. Zbiek (Series Ed.), Essential understanding series. National Council of Teachers of Mathematics.
Lehrer, R., & Slovin, H. (2014). *Developing essential understanding of geometry and measurement for teaching mathematics in grades 3–5* (B. J. Dougherty, Ed.). In R. M. Zbiek (Series Ed.), Essential understanding series. National Council of Teachers of Mathematics.
Lloyd, G., Herbel-Eisenmann, B., & Star, J. R. (2011). *Developing essential understanding of expressions, equations, and functions for teaching mathematics in grades 6–8*. In R. M. Zbiek (Series Ed.), Essential understanding series. National Council of Teachers of Mathematics.
Lobato, J. & Ellis, A.B. (2010). *Developing essential understanding of ratios, proportions & proportional reasoning for teaching mathematics in grades 6-8* (R. Charles, Ed.). In R. M. Zbiek (Series Ed.), Essential understanding series. National Council of Teachers of Mathematics.
Otto, A. D., Caldwell, J., Lubinski, C. A., & Hancock, S. W. (2011). *Developing essential understanding of multiplication and division for teaching mathematics in grades 3–5* (E. C. Rathmell, Ed.). In R. M. Zbiek (Series Ed.), Essential understanding series. National Council of Teachers of Mathematics.
Sinclair, N., Pimm, D., & Skelin, M. (2012). *Developing essential understanding of geometry for teaching* mathematics in grades 6–8. In R. M. Zbiek (Series Ed.), Essential understanding series. National Council of Teachers of Mathematics.
Sinclair, N., Pimm, D., & Skelin, M. (2012). *Developing essential understanding of geometry for teaching mathematics in grades 9–12*. In R. M. Zbiek (Series Ed.), Essential understanding series. Reston, VA: National Council of Teachers of Mathematics.
Van de Walle, J., Karp, K., Bay-Williams, J. (2019). *Elementary school and middle school mathematics, teaching developmentally* (10th ed.). Pearson.

Table 2

Target Audience of the Big Ideas Construct in Each of the Sources

Resource	Audience and Who Should Own the Big Ideas	Evidence
P2MT	Teachers Future and Current	In the title: *Big Ideas of Early Mathematics: What Teachers of Young Children Need to Know*
K8MT	Teachers Future and Current	• In preface: "We believe that teachers must…We are hopeful that you will find that this book is a valuable resource for teaching and learning mathematics" (p. xiii). • "Some of you will soon find yourself in front of a class of students; others of you may already be teaching" (p. 1)
K8PD	Teachers	"We hope that our ideas will initiate rich conversations between teachers about the big ideas and the connections that relate them to each other. If you don't have colleagues to discuss the ideas with, (or even if you do) our youcubed Facebook group … is a lovely space for collegial discussions" (p. 10).
P12PD	Teachers	In Preface of each book: "Each volume in the series invites teachers who aim to be not just proficient but outstanding in the-classroom—teachers like you" (p. vii).

Data Analysis of Reported Big Ideas

Based on the inconsistencies we found in our initial literature review, we understood there would not be agreement as to which big ideas should be taught nor that there could be a canon of big ideas (Boaler et al., 2017; Charles, 2005). Thus, we did not seek to develop a coding scheme or an analytical framework that we could use to list or categorize every big idea in mathematics. Instead, the main purpose of this content analysis was to look for consistency and agreement within and across resources about big ideas as a construct, with the resource as the unit of analysis. This unit of analysis at the resource level is to inform the field whether a teacher learning from one resource would have the opportunity to develop similar ideas as a teacher who learned from a different resource. This is so that they would have a shared understanding as the basis for shared instructional planning. Specifically, each resource was analyzed for the meaning and relative size of big ideas, whether these met Charles' (2005) criteria, and how well big ideas were used to organize or provide coherency for mathematics and connections in mathematics.

As in the discipline of mathematics, a way to disprove a conjecture is to look for a single counterexample. Thus, in each analysis our main approach was to look for consistency of adherence to the criteria set forth in Charles' (2005) theoretical perspective and to report counterexamples within and across resources. Although K8PD (Boaler et al., 2017-2021) consisted of separate publications, the same big ideas were published in their summary document, available open source to the public as *What is Mathematical Beauty?* (Boaler et al., 2017), and in each grade-specific publication the authors encourage teachers to access this summary document to understand the big ideas applicable in other grades. Thus, for the purpose of this analysis, we analyzed this summary document as the resource K8PD. In contrast, NCTM did not provide such a summary document that stated all big ideas in P12PD, so we obtained the big ideas from each analyzed book. For our purpose of assessing consistency within a resource, analyzing most of these books adhering to criteria (e) and (f) as described in sample selection was sufficient (n=12). Our purpose was not to code every big idea presented in a resource, but rather to obtain evidence of consistency or inconsistencies.

To address RQ2 and 3, the quotations of explanatory text about the big ideas from each resource were analyzed and compared to determine whether these met Charles' (2005) criteria and how to categorize the character of the explanations when these did not meet Charles' criteria. An iterative process of selecting quotations that might serve as examples and counterexamples of consistency were grouped and discussed until consensus in terms of similarities and differences among the examples and how to name these categories (which in other mathematics education research studies might be called a theme). The first two coauthors then verified that no other categories were used and sought at least one example of each category from each resource. For the resources without examples in any category, those resources were checked again to confirm that nothing was overlooked.

To answer RQ4, the first author selected examples from every resource that might reflect a range of sizes. Then, all coauthors reduced the number of examples to three for each resource and came to consensus on the relative sizes. Finally, for RQ5, coauthors counted the number of big ideas, identified the structural divisions of the resource and discussed the justifications for the case of a big idea that we expected to find in all grade bands across domains (as described in the findings). Next, the first two authors independently read each big idea looking for explicit (exact words used) and implicit (synonyms or ideas used) evidence of the example case. During discussion of the 224 big ideas for RQ5, only one compose/decompose related big idea was missed by one author and the one disagreement was resolved through consensus.

Findings

The audience of each resource we found were teachers (RQ1), which was addressed in the methods section due to its overlap with reporting of the sample selection. Each of the remaining research questions were addressed in the order in which they were numbered: definition (RQ2), criteria (RQ3), how big is a big idea (RQ4) and whether the resources were organized around big ideas as recommended (RQ5).

Definitions of Big Ideas (RQ2)

Table 3 provides the definition or explanation each resource used to inform teachers what a big idea is (RQ2). In column two we summarized our interpretation of how the big idea was defined or explained. As the second column in Table 3 details, one resource defined big ideas, two explained around big ideas without actually explicating a big idea, and one provided neither an explanation nor a definition. Although K8PD stated the overarching reason for big ideas is to share the coherence of mathematics as a subject, it seems that in an effort to meet teacher's needs who must focus on grade-level standards, the immediate goal of the resource was to provide coherence within a grade level because the structure of the K8PD big ideas emphasize connectivity of the big ideas within each grade-level. The definition in P12PD focuses or frames big ideas as topic-based connections. Notice in Table 3 that K8MT implied that big ideas are something other than separate skills or concepts. Yet this resource refers to "lists" almost as if these are sufficient to provide the coherence for teachers' mathematical knowledge. This methods textbook stated the importance of big ideas; however, fails to define or explain what big ideas are. See Table 3; K8MT.

Criteria of Big Ideas Not Always Satisfied (RQ3)

Charles (2005) stated three criteria of big ideas. One criterion was that a big idea should have a name that is not the statement itself. Another criterion was that it needed to be central to mathematics and connect many smaller ideas. The third criterion was a grammatical expectation that a big idea be in the form of a statement and convey an "essential mathematical meaning" (Charles, 2005, p. 10). We next provide the findings related to these criteria (RQ3). The criterion of centrality and connections required several additional analyses to foster reliable and trustworthy interpretations.

Table 3

Definition and Names of Big Ideas in Each Resource

Source	Definition Character	Evidence	How Big Ideas Referenced
P2MT	Explanation around big ideas	• *Mathematically central and coherent.* **Big ideas convey core mathematics concepts and skills** that can serve as **organizing structures** for teaching and learning during early childhood years. • *Consistent with children's thinking.* • *Generative of future learning.* • *Comprehensive.* • *Thoughtful about content.* • *Developmentally organized.* • *Flexible (pp. 4-6).*	Unnamed: Idea Itself
K8MT	No Definition No Explanation	• No definition • Term "Big Idea" not indexed • The only introduction to big ideas occurs in in a callout of a page image: "Much of the research and literature espousing a student-centered approach suggests that teachers plan their instruction around big ideas rather than isolated skills or concepts. At the beginning of each chapter in Part II, you will find a list of the big mathematical ideas associated with the chapter" (p. xviii).	Unnamed: Idea Itself
P12PD	Definition; connects topics	• "The big ideas are mathematical statements of overarching concepts that are **central to a** mathematical **topic** and link numerous smaller mathematical ideas into **coherent** wholes" (p. viii). • "The books call the smaller, more concrete ideas that are associated with each big idea *essential understandings*" (p. viii).	Numbered Locally
K8PD	Explanation around big ideas; connects to other big ideas	• No definition • Explanation stated: "**big ideas are connected to one another within grade levels**, these 'connections give mathematics **coherence** which supports all students in making sense, as students draw on what they know about one big idea to learn about another" (p. 5). • Network diagrams visually communicate which big ideas are connected to each other within a grade.	Unnamed: Idea Itself

Big Ideas Need a Name Criterion

Column four of Table 3 characterized how the resources referenced their big ideas. Had any resource named a big idea prior to the statement of the big idea, we would have used the term *Named*. Resources did not name the big ideas prior to giving the big idea in its entirety. Charles (2005) cautioned against using the "idea itself" as a proxy for a name, so we coded these in Table 3 as *Unnamed: Idea Itself*. P12PD numbered each big idea within the book in which it was written, which we coded as *Numbered Locally*. In other words, even for a topic that was addressed in successive grade-bands (i.e., K-2, 3-5, 6-8 and/or 9-12), the numbering was only a valid reference within that grade-band specific topic publication. For example, even though the topic carried through the resource across grade bands, "Big Idea 3" was a different big idea in each publication. Moreover, potentially similar big ideas in different grade-band publications might be numbered with a number in one grade band, but a different number in another grade band. Thus, use of numbering would interfere with teachers or mathematics coaches being able to vertically align and recognize the coherence and building of big ideas of a topic *across* grade-band barriers within that resource.

Grammatical Format Criterion

According to Charles (2005) a big idea must satisfy the grammatical criteria of being a statement and be meaningful. We reviewed each resource to determine the grammatical structure of ideas labeled as "big ideas" and provide these findings in Table 4. Statements are not questions and they must have a subject and a verb (Obrien, 2023). Column two displays an example from each resource that satisfied the criteria of Charles (2005) and stands alone to provide information about a mathematical idea. Column three satisfies the grammatical criterion of being a statement but lacks the criterion of coherence or ability to stand alone as a resource for understanding. In Charles' (2005) words, his intention was that if written as a statement, it would have "the essential mathematical meaning of that idea" (p. 10), yet each resource had statements that did not do so. In other words, Column two is broad and usefully applicable to many situations, so it has power, whereas Column three is broad to the point of not being mathematically useful (Charles, 2005).

Columns four and five display excerpts from resources in which the stated "big ideas" were neither sentences nor statements. These grammatically consisted of questions (a type of sentence) or phrases. We categorized several of the stated big ideas formatted as phrases to be mathematical topics (e.g., Families of Functions) in Column five, whereas in Column four we documented other stated big ideas that we did not recognize as topics, but rather phrases with the grammatical structure of gerunds or present participles (e.g., "being flexible with numbers"; see Table 4). We also found that the two methods textbooks (P2MT and K8MT) provided some big ideas that consisted of at least two sentences or entire paragraphs rather than satisfying the criterion of being a statement.

As Column two of Table 4 reveals, in every resource we were able to find at least one example of a stated big idea that satisfied the grammatical and stand-alone meaning criteria. Yet, we found counterexamples of consistency for each resource. We found at least one example of a stated big idea in each resource that could not stand on its own to convey mathematical meaning, although it was grammatically a statement (see column three Table 4). The two resources for whom the intended audience was practicing teachers (P12PD and K8PD) also provided big ideas that were phrases, rather than statements (see Table 4 column 4 and 5). In P12PD we found phrases that we characterized as mathematical topics (see Table 4). In K8PD we found mathematical topic phrases as well as two other types of non-statements (see Table 4 Column 4): a) phrases that were not mathematical topics, but rather phrases with participles or gerunds indicating an action, and b) question sentence types. In contrast, the two resources that served as methods textbooks for future teachers as well as for teacher professional development—P2MT and K8MT— consistently satisfied the grammatical requirement of being a statement.

Table 4

Examples of "Big Ideas" that Show Varied Grammatical Structures

Resource	Statements that have stand-alone meaning	Statements without stand-alone meaning	Phrases with gerunds or participles that aren't math topics or sentences that are not statements	Phrases of Math Topics
P2MT[a]	"The same pattern structure can be found in many different forms" (p. v).	"Relationships between objects and places can be represented with mathematical precision" (p. vi).	--	--
K8MT[a]	"Percents are simply hundredths and, as such, are a third way of writing both fractions and decimals" (p. 406).	"Algebra is a useful tool for generalizing arithmetic and representing patterns in our world" (p. 299).	--	--
P12PD[b]	"Any rational number can be represented in infinitely many equivalent symbolic forms" (Barnett-Clarke et al., 2010, p. 8). "A written proof is the endpoint of the process of proving" (Sinclair et al., 2012, p. 8).	"Extending from whole numbers to rational numbers creates a more powerful and complicated number system" (Barnett-Clarke, et al., 2010, p. 7). "Expressions are foundational for algebra; they serve as building blocks for work with equations and functions" (Lloyd et al., 2011, p. 12). "Working with diagrams is central to geometric thinking" (Sinclair et al., 2012, p. 7).	--	"Expressions" (Lloyd et al., 2011, p. 9). "Families of Functions" (Cooney et al., 2010, p. 9)
K8PD[a]	"There are many ways to describe and sort objects" (p. 5). "A ruler is a number line" (p. 6).	"Representations and modeling structures help us see math" (p. 6).	"Being flexible with numbers" (p. 7). "Folding and unfolding objects" (p. 8). "What does it mean to divide fractions?" (p. 8). "What is a decimal?" (p. 7).	"Reasoning with proportions" (p. 8). "Thinking in powers of 10" (p. 8).

Note. [a]Each resource that is a stand-alone publication we referenced as a data source using the code as described in the Methods section (P2MT, K8MT, and K8PD), with the page number in a way to avoid emphasizing authors as the American Psychological Association (APA) would suggest if this were just a citation, yet any reader could still refer to the exact source. [b] Given P12PD consisted of multiple publications each of which had multiple authors, we used APA to cite within the table, even though our intention was not to call attention to authors.

Connections and Centrality Criteria

To determine how central or connected the stated big ideas were in each resource, we looked for evidence or counterevidence that the resources were organized based on the big ideas. Someone familiar with these resources might at first glance believe big ideas were the organizing feature. Table 5, however, clarifies how each resource was structured. No resource provided a chapter or section on big ideas to foreground the centrality of the big ideas to mathematics and how these connect across topics or grades. Some resources explained that a particular big idea was important because of its relevance in other topics. However, this was done as an extension and not as a centralizing feature. For example, P12PD followed a structure of Preface and Introduction that was the same in every book, followed by a chapter on the big ideas, followed by a chapter with explanations of connections to other mathematics —the degree to which these explanations provided connections varied by author. Each of the resources presented the big ideas only after the resource was structured or divided into grade bands or grades, then domains or topics, or some combination of these.

Are Big Ideas Consistently "Big"? (RQ4)

The only resource that showed some ideas were bigger than others was K8PD. This was implied, rather than explicit, by increased node sizes on network diagrams (where each node represented a big idea that was connected to other big ideas within that grade). None of the resources we analyzed provided an explanation of size criteria to clarify the meaning of the adjective "big." Thus, to answer RQ4 we reviewed the stated "big ideas" to look for some indication of size and consistency of size in relation to the terms used. Figure 1 provides a visualization of a continuum of sizes of ideas that were all referred to as "big."

What authors determined to be "big" ranged in size even within the same resource. This was true for every resource we analyzed. With this analysis, our goal was not to quantify or articulate the size of stated big idea ideas within or across resources. Rather, our intention was to determine if what was conceived of as a big idea within a resource seemed to be of similar size throughout that resource. The visualization in Figure 1 provides insight that there was inconsistency within and across resources about how big an idea should be in order to be labeled a "big" idea. When viewing Figure 1, it could be helpful to choose a resource and consider how the size of the quoted excerpts differ from small to large from bottom to top. This demonstrates evidence of inconsistency within a resource.

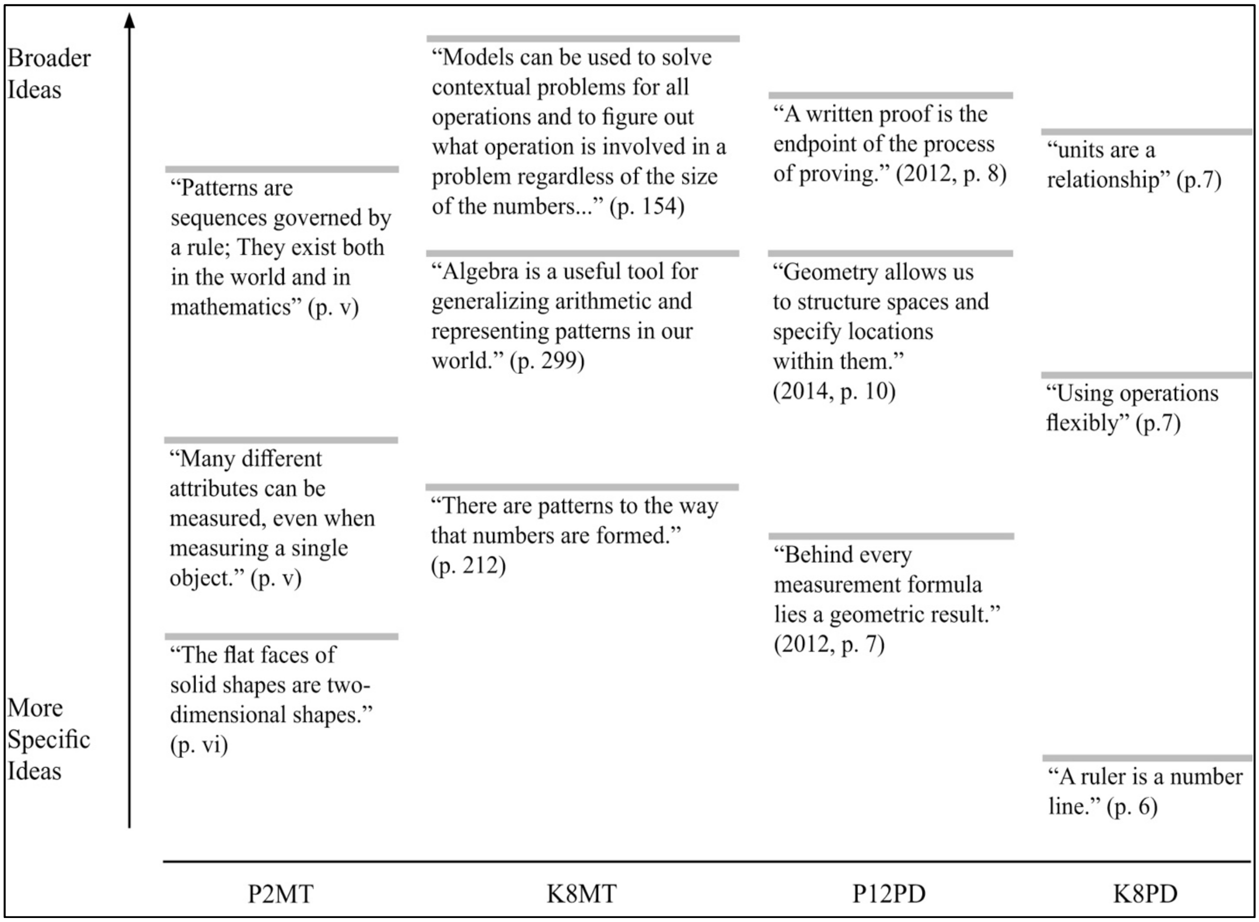
We also attempted to place these chosen excerpts in relation to each other across resources. See Figure 1 for this information. The line segment above each stated big idea indicates the relative position in the vertical dimension. Again, our intention was not to provide measurements for such sizes. We positioned the big ideas by their relative sizes in a visual format to provide a sense that big ideas varied in size and reveal the lack of patterns in the field for this construct.

How Well Have Big Ideas Been Used to Organize or Structure Mathematics? (RQ5)

The literature about big ideas and the analyzed resources explain that big ideas are meant to organize or structure teacher understanding of the mathematics to better understand and more efficiently work with standards and ideas. Thus, we looked for evidence that the resources were structured in a way that fostered this (RQ5). We investigated the overarching structural organization, including indications that redundancy was avoided. We did so in three ways. First, we analyzed how each resource structured the presentation of their big ideas. Second, we reported how many big ideas each resource provided. Third, to look for evidence of redundancies or hierarchical references to big ideas, we chose one example of a big idea, referred to as "bigger than" big by Boaler and colleagues (2017, p. 5). Given the purpose of big ideas is to promote connections (Askew, 2013; Charles, 2005), we chose an idea that was explicitly stated in more than one standard and domain of *CCSSM*: "compose and decompose."

Figure 1

Relative Size Comparisons of Big Ideas in the Analyzed Resources



Resource Organization and Structure Belies Big Ideas are Overarching

Each of the resources presented the big ideas divided into either grade bands, grades, domains, topics, or some combination of these. Three of the four resources used grade bands or grades as the top-level category. No resource provided a chapter or section with big ideas as the top-most level of organization. Narratively, some resources explained that a particular big idea was important because of its relevance in other topics. However, the physical organization of each resource was structured first by traditional divisions of grades or domains/standards/topics, rather than the big idea being the top-most level.

Table 5

How Each Resource Structured Presentation of Big Ideas and the Quantity Provided

Resource	Top-Level Organizational Structure	Evidence	Number of Big Ideas
P2MT	Book for the specific grade band P-2 then Domains or sometimes topics	Chapters organized by separate domains so no cross-domain connections used (i.e., *sets, number sense, counting, number operations, data analysis, spatial relationships, shape, pattern*)	26
K8MT	Strands or parts of a domain	Chapters are organized as separate strands or parts of domains so no cross-domain connections used. However, given that some domains are grade-band specific, the early grade math topics for number and operations are also organizationally separated from later grade band topics. Big ideas are listed at the beginning of each chapter for Chapters 7 through 22. Unlike the other sources analyzed, the listed big ideas were not emphasized throughout each chapter. Chapters were sometimes strands (e.g., Geometry) and sometimes part of a domain (e.g., Developing Fraction Concepts).	75
K8PD	Grade	Each book in the series is for a separate grade and the overview article separated sections by grade (i.e., K, 1, 2, 3, 4, 5, 6, 7, or 8),	80
P12PD	Grade band, then topic	Separate books by grade bands (i.e., K-2, 3-5, 6-8, 9-12) and Topic (e.g., Functions, Expressions and Equations; Rational Number; Addition and Subtraction)	>43 with >153 Essential Understandings in the 12 books of the series analyzed

Quantity of Big Ideas Obscures Coherence

Refer again to Table 5. Column 4 displays the number of big ideas found. Note that the quantity ranged from 26 big ideas stated in a resource for P-2 teachers, to 80 in a resource spanning P-8. P12PD might seem to be within this range, however, the 12 publications we analyzed were most of the series, so there are more than 43 big ideas in the resource. Moreover, unlike the other resources, the authors specified at least 153 more ideas (i.e., essential understandings) that they stated a teacher must understand in order to comprehend the big ideas themselves. In other words, the big ideas in P12PD were insufficiently independent statements to meet Charles' (2005) criteria for a big idea. Furthermore, in some publications even the "essential understandings" were also insufficiently independent statements such that even 153 is an underestimate of the number of ideas teachers must learn. For example, just one big idea about ratios, proportions, and proportional reasoning was delineated: "When two quantities are related proportionally, the ratio of one quantity to the other is invariant as the numerical values of both quantities change by the same factor (Lobato & Ellis, 2010, p. 11). However, what was labeled as a big idea was really a mathematical definition available in other mathematical texts, such as traditional mathematics textbooks. Consequently, the authors asserted: "Although the big idea of proportionality may at first seem straightforward, developing an understanding of it is a complex process for students. It involves grasping many essential understandings" (Lobato & Ellis, 2010, p. 12). Next the authors proceeded to document 10 essential understandings. Two of these, however, were also insufficient statements that had four bullet points, each of which were really the meanings teachers need. Thus, this purported single big idea was really 16 ideas.

Redundancy and Omission of Big Ideas Inhibits Vertical Alignment: The Case of Composing and Decomposing

Given that big ideas are meant to be overarching and provide coherence, big ideas stated in one grade or topic that are relevant in a later topic or grade should be clearly stated (rather than omitted) in a way that avoids redundancy or repetition. Moreover, K8PD referred to some big ideas as being "even bigger than" big, such as "composing and decomposing with numbers and shapes" (Boaler et al., 2017, p. 5). Thus, we chose this as a case. Table 6 documents each compose/decompose related idea and where this big idea appeared in each of the analyzed resources in terms of grade levels and content domains or topics.

Notice in Table 6 that only the topics of numbers and shapes are acknowledged in these resources as being supported by big ideas of compose/decompose. Also notice the limited number of grades in which even number or shapes was mentioned (see Table 6).

Table 6

Compose and Decompose Relevant Big Ideas Found in Each Resource

Resource	Domain or Topic	Grade	Compose/Decompose Relevant Big Idea (Even If Not Explicitly Stated as Such)
P2MT	Number	P-2	"A quantity can be *decomposed* into equal or unequal parts; The parts can be *composed* to form the whole" (p. v).
	Shapes	P-2	"Shapes can be combined and separated (composed and decomposed) to make new shapes" (p. vi).
K8MT	Base-Ten Number	K-2	"Flexible methods of addition and subtraction computation involve taking apart (decomposing) and combining (composing) numbers in a wide variety of ways. Most of the decomposing of numbers is based on place value or *compatible* numbers-which are number pairs that work easily together, such as 25 and 75" (p. 239).
		1-3	"Multidigit numbers can be built up or taken apart in a variety of ways to make the numbers easier to work with. These parts can be used to estimate answers in calculations rather than using the exact numbers involved. For example, 36 is the same as 30 and 6 or 25 and 10 and 1. Also, 483 can be thought of as 500 – 20 + 3" (p. 239).
K8PD	Number	K	"We can put numbers together" (p. 5).
	Shapes	2	"Partitioning shapes" (p. 6).
	Shapes	6	"Taking apart prisms & polygons" (p. 8).
P12PD	--	P-12	--

P2MT gave two big ideas about composing and decomposing, one for number and one for shapes. K8PD provided three big ideas related to composing/decomposing, one for number (Kindergarten) and two for shapes (Grades 2 and 6). P12PD did not mention composing or decomposing in their stated big ideas. These could only be found by looking at the subcategories of big ideas (i.e., *essential understandings*) and we found this only in a single domain/strand type of Geometry/Measurement in just two non-adjacent grade bands (i.e., K-2 and 6-8). Thus, in P12PD compose/decompose was not aligned vertically across grade bands with a gap between Grades 2 and 6 and then terminates prior to high school. Although K8MT is a methods textbook informing three grade bands, it provided just two composing/decomposing big ideas about base-ten numbers relevant to Grades K to 3. This base-ten number focus, however, was an important application of a compose/decompose big idea that the other resources failed to mention.

It is crucial to point out that only the methods textbooks (P2MT and K8MT) explicitly included the inverse relationship of composing *and* decomposing in each stated big idea. K8PD showed only composing *or* decomposing in any grade-level, which misses the opportunity to emphasize the relationships between putting together *and* taking apart numbers or shapes. In K8PD, we could recognize the big idea of composing/decomposing in the stated big ideas. However, the language differed in each instance (i.e., "put . . . together" in K, "partitioning" in Grade 2 and "taking apart" in Grade 6) such that the overarching connections across grades and domains may not be obvious to a teacher without further explanation.

P2MT's consistent and explicit language in more than one domain and grade would make it easier for teachers to see that this idea connects across domains/strands. Although K8MT used the term compose/decompose, the authors only addressed this big idea in two of the 75 stated big ideas and only for number. Yet, notice the redundancy that these two big ideas were offered on the same page and presented as two separate ideas (i.e., the first about "compatible numbers" and the second about "multi-digit numbers"), rather than a single concise overarching explanation of the value of composing and decomposing quantities that we can apply to calculations with a compatible number strategy or procedures with multi-digit numbers.

Discussion

One consistency we found across resources was that the audience for big ideas was teachers, not students, and the goal was to improve their knowledge for teaching or MKT (RQ1). This purpose found in our analysis was also consistent with the purposes as stated in the field (e.g., Askew, 2013; Charles, 2005; Siemon, 2022). In contrast, across analyzed resources there was inconsistency as to whether a resource stated the importance of big ideas without definition/explanation, talked around big ideas, or offered a definition (RQ2). This should not be surprising given that Askew (2013) found there was not an agreed upon definition in our field.

Almost two decades ago Charles (2005) referenced the dilemma of determining how big a big idea is and what makes an idea robust enough to be considered "big." In light of the recent proliferation of documents asserting that teachers must know "the big ideas" (e.g., NCTM, 2014), we had hoped our analysis of recent resources would provide some clarity to this dilemma. To answer RQ4 regarding the size of the big ideas, we considered what the resources said big ideas were, as well as our analysis of the actual stated big ideas. Both of these approaches revealed inconsistencies *across* resources. The explanations and definition focused to varying degrees on whether the purpose of a big idea was to connect topics, connect ideas within topics, connect big ideas to each other, or something larger (see Table 3). The analysis of the actual stated big ideas also revealed inconsistent size *within* each resource (see Figure 1). Therefore, the analysis could not offer clarity about how big a big idea is (RQ4).

A consistency across resources was that big ideas *should* organize or provide coherency for mathematics, which was consistent with prior claims (e.g., Ritchhart, 1999; Siemon, 2022). However, our analysis for RQ5 revealed that the organizational structures of each resource still used the traditional approach of grade or grade band as the top-level structure, then segmented by topic, and finally by big ideas as a third-level category. In contrast, Charles (2005) who sought content validity from colleagues for his set of big ideas, was told to avoid such artificial divisions and corrected this prior to publication. From a disciplinary perspective of mathematics, we avoid redundancy and seek parsimony. Consider how we value hierarchical categorizations of quadrilaterals to avoid redundantly restating all possible properties of each shape; we value definitions as necessary and sufficient. Yet, our analysis found redundancy and insufficient use of the selected test case of the big idea of composing/decomposing. Therefore, if a resource mentions the importance of compose/decompose for one topic or grade, then as a field we should expect the resource to include many or all instances of this big idea across topics, domains/strands, and grades. However, this was not what we found.

In spite of the composing aspect of shapes explicitly being stated beginning in the Kindergarten standards (K.G.6 *CCSSM*), neither the P-12 Professional Development Series nor the K-8 Methods Textbook included a compose/decompose big idea for shapes and only the decomposing aspect of shapes were included in the K-8 Professional Development resource (see Table 6). Moreover, all resources omitted a composing/decomposing idea about measurement in spite of how crucial the big idea of composing/decomposing is to determining areas or volumes of irregular shapes, linear measurements, elapsed time, and so forth. Identifying a big idea in one instance but omitting it from other relevant instances (grades or applicable topics), reduces the power the big idea could have in a student's mathematical career.

Given that Charles (2005) was the only document we could find that provided a theoretical perspective of big ideas in terms of a definition with criteria, it makes sense that each of the resources and much of the literature used this work as the foundation for their construct of big ideas of mathematics. Yet, as we demonstrated in the findings, Charles' criteria were inconsistently applied in every analyzed resource (RQ3). Recall that in each resource big ideas were found that were topics, questions, paragraphs, or statements that were not in themselves mathematically meaningful (see Table 4). Charles (2005) noted that when asking teachers what a big idea is, they provided ideas such as topics, strands/domains, objectives, or standards. These teacher conceptions are consistent with the variety of big idea formats we found in our analysis of resources designed to inform teachers. Thus, what teachers think big ideas are, is consistent with the resources that informed them. How could the criteria of centrality to mathematics and coherence within mathematics be achieved when 26 to 80 big ideas were given to teachers (see Table 5)? Moreover, these overwhelming quantities in some cases only reflect a narrow set of grade bands (e.g., P2MT) or require 153 additional and sometimes multi-part "essential understandings" in order to comprehend the 43 "big ideas" (P12PD, see Table 5).

Hence, to vertically and horizontally align all of mathematics would require even more ideas. Rather than making teaching easier, this expansive set of ideas, in addition to standards, would make teaching more challenging. In contrast, the foundational work of Charles, which was cited by these resources, proposed fewer big ideas than any of the analyzed resources. The way Charles accomplished this was by making the big idea the top-level of organization in two ways: 1) he avoided organizing by grade band and 2) due to colleagues' content validation feedback on a draft, he eliminated the content strand/domain as an organizing feature to instead use it as sub ideas of applications of each big idea. To be clear, each of his sub ideas provided specificity and examples to support the same core overarching idea—which is in sharp contrast to the ways the "essential understandings" with sub-bullets were distinct additional ideas (as noted in the RQ5 findings section). By eliminating this structural redundancy, Charles (2005) was able to reduce the number of stated big ideas by about a third— down to 21 for all of grades K to 8. Thus, as a field we have much work to do to clarify and organize the construct of big ideas and our communication of big ideas to teachers.

Conclusions and Implications

Many people who have invested considerable time with the construct of big ideas specifically state that a canon of agreed upon big ideas is unlikely, perhaps impossible, or even undesirable (Askew, 2013; Boaler et al., 2017; Charles, 2005). In spite of this, U.S. policy statements and documents refer to big ideas with the article "the" as though these are delineated things teachers should have learned during teacher preparation or professional development (AMTE, 2017; NCTM, 2014). To take the next step in the evolution of this construct to be a useful support for teachers, teacher educators, and scholars we offer several suggestions. Philosophically, we ask the field to approach the construct of big ideas the way Charles (2005) and Boaler et al. (2017) do by using the article "a" instead of "the." This is an especially important revision for future editions of those policy documents and pedagogy books that use the term "big ideas" without articulating what they are and often mentioned the construct as though a command to learn an existent list to which teachers should already have access. This shift in recognizing "a set" instead of "the" big ideas would soften the language in such documents to honor teachers' professionalism and better reflect scholarly humility that more accurately reflects the current state of the field that there is much that is unknown and not agreed upon.

The construct of big ideas needs guidelines that are teacher, teacher-educator, and scholar friendly. To advance, we look back to Charles (2005) and then build on his valuable theoretical perspective.

Definition and Criteria for Big Ideas

Let us begin with Charles' definition that encouraged broader connections and implications than simply connections within a topic or between topics as some analyzed resources did: "A *Big Idea* is a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" (Charles, 2005, p. 10). We used and extended his criteria to specify the following five criteria.

Portable and Meaningful Name Criterion

Big ideas must have a name (Charles, 2005). We agree with this naming criterion. However, we suggest better implementation of this criterion than Charles himself implemented or any of the resources we analyzed. These should be named so as to be a concept a person can own and use in varied contexts and situations. In other words, it should be portable. The name should not be a topic, because it would be insufficiently descriptive given that multiple big ideas could relate to a topic. We suggest that a shortened descriptive form of the meaning of the intended statement would be most useful. For scholars it may help to think of this as a "Running Head".

Big ideas should avoid numbering, even if named. Numbering big ideas locally as we referred to them in the findings, restricts their portability to another context. Given that several have claimed that there will not be universal agreement on which big ideas should be used (Askew, 2013, Boaler et al., 2017; Charles, 2005), then numbering them in any resource creates additional barriers and hurdles to using these within districts or at a more macro scholarship level to build knowledge as a field. Moreover, numbering connotes an ordered sequence that violates the intended purpose of promoting connections and implicitly prioritizes the first big idea as most important. Thus, naming rather than numbering is the only way they way will be useful for the lay person and most likely the only way it could be useful for teachers. Naming big ideas, however, could foster a common language about each particular big idea.

Ethical Communication Criterion

To ensure broad accessibility, the simplest lay language possible should be used (Su, 2017). To humanize mathematics as something humans created and do, active voice should be used. Active voice is also easier to understand than passive voice (Schimel, 2012).

Portable and Stand-alone Meaning Criterion

Grammatically, a big idea must be a statement and it must stand-alone. This was the intention of Charles (2005) that it must be a statement with "mathematical meaning" (p. 10) and that it must be "useful to teachers, curriculum developers, test developers, and those responsible for developing state and district standards" (Charles, 2005, p. 11). The big idea statement must convey meaning of an important idea in and of itself. This might seem obvious, however, as Table 3 revealed, each of the analyzed resources presented at least some big ideas that would need to be revised to meet this criterion.

Connection and Example Based Presentation Criterion

A big idea should be the top-level statement with applications of this idea or sub ideas organized into domain/standard and subtopics via bullets (e.g., Charles, 2005), a table, and/or a diagram (e.g., Boaler et al., 2017; Boaler & Williams, 2021). Although Charles did not explicitly state this as a criterion, he modeled this approach when he presented a set of big ideas. We believe this is important to facilitate understanding of the degree to which a big idea applies to and connects mathematical ideas.

Criterion to Categorize and Prioritize Big Ideas by their Size and Power

Big idea statements should be stated at the broadest level of implication possible that still fulfill the criterion of being a mathematically meaningful statement. "Big Ideas need to remain BIG and they need to be the anchors for most everything we do" [capitalization in the original] (Charles, 2005, p. 12). Using this mind-set to determine and state a big idea in this way would eliminate the need to create separate listings of redundant big ideas. This would create an efficient and coherent system that is more manageable for teachers, just as a hierarchical categorization of quadrilaterals or number systems promotes efficiency and coherency in the discipline of mathematics itself (De Villiers, 1994). The details of how and where this big idea applies would be clarified by adherence to the previous criterion about how to present a big idea over the applicable domains and examples.

The construct of what a "big idea" is warrants a more precise definition that indicates relative size and connective power. We see our stance to ask for more precision of this pedagogical construct as analogous to expectations of Mathematical Practice 6 (NGA & CCSSO 2010). That is, we call for more precision about the language of teaching mathematics analogous to precision of the language to do mathematics. For instance, when even very young children use the word "big," the recommendations are to encourage them to refer to specific attributes with words like taller, shorter, longer, heavier, and so forth. In favor of more precise indications of the size of an idea, let us let go of using the term "big idea" to refer to the specific statement. We primarily suggest this due to the lack of agreement on the meaning and size of "big" ideas as well as the issue that many other content areas and pedagogical approaches, such as the International Baccalaureate (2023), use the term "big ideas" in much broader and different ways. We also saw this in instructor responses to the big ideas of secondary methods courses (Stehr et al., 2019). For these reasons the vague and relativistic connotation of "big" would continue to perpetuate confusion among teachers and scholars alike.

Thus, we suggest sets of ideas or the construct can continue to be referred to as "big ideas," however, we advocate for more precise terms about the sizes of big ideas.

Not all big ideas are equally important or central to the learning of mathematics. Clarifying the size of a big idea by the quantity and type of connections will help the field to (1) reduce the number of big ideas expected of teachers and (2) prioritize big idea instruction based on the differing contexts of P-12 classroom learning or teacher education. This would be consistent with the intended purpose of big ideas (Askew, 2013; Hurst, 2014; Siemon, 2022). To this end we next offer the Big Ideas Framework.

Big Ideas Framework

To categorize and prioritize big ideas by their size and power we developed the Big Ideas Framework shown in Table 7.

Table 7

Big Ideas Framework

Name	Size/Applicability Description	Examples
Mighty Mega Math Ideas	Overarching idea *across* **domains/strands**: An idea that spans grades and **unites domains/strands** to em**power** students to look for these ideas in any new concept to **succeed in the discipline of mathematics**. Consistent use of these should potentially be high-leverage practices.	*Compose & Decompose*: We can look for ways to put together and take apart things in math to solve situations.
Power Math Ideas	Overarching idea *within* a **domain/strand**: Spans grades and **unites topics** to em**power** students to succeed in a domain/strand.	*Purpose of Measuring*: We measure to compare the same attribute of two or more objects or groups of data. *How We Classify Shapes*: We identify and classify shapes by their properties.
Strong Math Ideas	Overarching idea *within* a **topic**: Spans grades within a topic to **strengthen** student understanding of a topic.	*How We Write and Think in Base-Ten*: Our number system uses a base of ten, so we use the digits 0 to 9 to write numbers and think in groups of ten (and groups inside groups inside groups…) in special ways so that each bigger or smaller group is a unit with a special name.

Each of these ordinal levels span grades to ensure vertical alignment, which is consistent with prior assertions (Boaler et al., 2017; Charles, 2005; Small, 2019). Further, to be sufficiently central to mathematics we intend that spanning grades also means bridging across grade band(s), consistent with AMTE (2017).

The framework consists of three ordinal levels: Mighty Mega Math Ideas, Power Math Ideas, and Strong Math Ideas. Note that we used the word "math" to reinforce the content area of mathematics within the name of each level. Metaphorically taking the perspective of the ideas, the levels are shown in decreasing order beginning with the greatest power as in it takes greater power to span or connect across a gap, so thinking of ideas that connect across larger barriers as needing to be stronger. From the perspective of the students, these levels have varied strength in the degree to which they might empower students to succeed based on the quantity of ideas that a level could support.

Boaler et al. (2017) stated that "As we wrote these big ideas, it was clear to us that there are some even bigger ideas that pervade all of mathematics" (p. 5). Charles (2005) explained that "many" big ideas span strands, which implies that not all big ideas are that big. Hence, we specified those ideas that empower students in all of mathematics both vertically and horizontally by spanning multiple strands/domains as Mighty Mega Math Ideas. Those ideas that span topics within domains or strands are Power Math Ideas, which is the level consistent with a "generative idea" from the strand of measurement that teachers working with Ritchhart (1999) determined.

The four ideas we selected as examples to illustrate each level in Column 3 of Table 7 have their roots in prior work: *Compose & Decompose* (Boaler et al., 2017; Clarke et a al., 2012; Early Learning Collaborative, 2014; Van de Walle et al., 2019); *How We Classify Shapes* (Charles, 2005; Early Learning Collaborative, 2014; Van de Walle et al., 2019); *Purpose of Measuring* (Kader & Jacobbe, 2013; Ritchhart, 1999; Van de Walle et al., 2019); and *How We Write and Think in Base-Ten*, (Boaler et al., 2017; Charles, 2005; Van de Walle et al., 2019; Yumi Deadly Mathematics, 2016). Each of which we revised in Table 7 to adhere to the five criteria we set forth in this section.

Notice that we did not restrict Compose & Decompose to numbers and shapes as the analyzed resources did. Compose & Decompose should be one of the high-leverage ideas to prioritize and organize instruction across all domains due to its horizontal and vertical strength. How much power or leverage could this Mighty Mega Math Idea lift? Compose & Decompose, if prioritized, would empower students and teachers to make connections and provide coherence within grade levels, which initial analyses in other in-progress work we found applied to at least 29% of kindergarten standards, 43% of grade one standards, 37% of grade two, 30% of grade three, 32% grade 4, 18% grade 5, 10% in grade 6, and 30% in grade 7. For example, standard 6.G.1 includes the exact phrase of "composing . . . decomposing," whereas 6.G.4 (NGA & CCSSO 2010) does not contain this language nor synonyms. Yet to succeed on this standard about finding surface areas students need to use a composing and decomposing conception of mathematics. In subsequent grades Compose & Decompose is relevant to some standards (albeit with lower impact percentages as the content to be learned emphasizes more proportional reasoning while continuing to require additive reasoning in the problems they solve). Moreover, the purpose is that a Mighty Mega Math Idea would empower students as they move through grades and learn new domains such that even these large within grade-level percentages underestimate the long-term cumulative vertical power of this Mighty Mega Math Idea.

Purpose of Measuring (Ritchhart, 1999) and *How We Classify Shapes* (Charles, 2005; Early Learning Collaborative, 2014; Van de Walle et al., 2019) are central to mathematical ideas and metaphorical heavy lifters within the strands of Measurement and Geometry, respectively. Thus, they are needed to empower students to succeed in these strands. Yet, given that they are only applicable to a single strand, they cannot be as central to mathematics as any other such idea that teachers or scholars determine applies across multiple strands/domains and grade bands. Nevertheless, the *Purpose of Measuring* is a Power Math Idea that could empower citizens to understand the utility of measuring beyond accurate procedures of measuring—the larger purpose of why we measure, what we measure and how to make decisions about measuring is important for physical measurements (Ritchhart, 1999) as well as data. This is the reason the *CCSSM* (NGA Center & CCSSO, 2010) domain connects data and measurement. Moreover, *How We Classify Shapes* could go a long way toward correcting the misconceptions that shapes are classified by memorizing an image or a template, which adults and children harbor (Fujita, 2012; Nurnberger-Haag et al., 2020; Nurnberger-Haag et al., 2021; Ozdemir Erdogan & Dur, 2014). Furthermore, imagine the impact on student learning if this Power Math Idea— that shapes are classified by their properties— was reinforced while following the Property-Based Shape Sequence (Nurnberger-Haag & Thompson, 2022).

Although weaker in its centrality to mathematics overall, disciplinary knowledge cannot be strong without a strongly connected understanding of any given topic. Strong Math Ideas promote this connectivity within a topic that standards implicitly fail to impart due to aspects of topics being separated into different grades and often further separated into discrete bits of knowledge. Indeed, this conceptual understanding or bigger picture of the base-ten number system conveyed in our wording of the Strong Math Idea *How We Write and Think in Base-Ten* communicates essential patterns and concepts of the base-ten number system that are almost non-existent for students and teachers who focus on incrementally building up discrete place value and calculation skills in each grade. This Strong Math Idea would be an important guidepost for students and teachers from the beginning of base-ten number instruction. Rather than *trading*, *bundling* or *regrouping*, thinking of the number system as successive sets that *contain* ten is crucial to a strong sense of number (see Nurnberger-Haag, 2018). Thus, we see Strong Math Ideas as the smallest level of the big ideas construct, yet whose strength is necessary to ensure understanding a topic.

Prior authors have included the idea of power in their thinking about big ideas of mathematics (e.g., Boaler et al., 2017; Carnine, 1997; Tout et al., 2015). However, the idea of power has not appeared in a definition, criteria, or denotation of the size of big ideas. Previously relative size of big ideas was either not attended to, which implied all big ideas were of equal size, or vague indications of size were offered (see section Are Big Ideas Consistently Big?). Whereas we emphasize this power-based connotation of the reason to use big ideas in the definition at each level to denote the relative size within each level of the Big Ideas Framework.

Shift from MKT to CCK

Some frameworks should remain pedagogical guidelines that influence instruction but are never taught to P-12 students. For example, the van Hiele Framework of Geometric Reasoning is important for teachers to understand how to better teach geometry (van Hiele, 1986). It would not make sense, however, to teach children the van Hiele levels even though the goal is to help students progress *through* these levels. The van Hiele levels framework is an example of Mathematical Knowledge for Teaching (MKT; Ball et al. , 2008; Nurnberger-Haag et al., 2021). In the literature cited as well as the resources we analyzed, big ideas were developed for and continue to be intended as MKT.

We argue that big ideas of math need to be Common Content Knowledge. We propose a fundamental shift from the construct of big ideas being theoretically perceived as an aspect of MKT to being understood and implemented as an aspect of Common Content Knowledge. Big ideas are so important that they are crucial to helping students and families understand mathematics as a coherent and logical discipline. Our intention is that from one grade to the next, if students themselves own the most expansive and powerful ideas (i.e., Mighty Mega Math Ideas) as part of their Common Content Knowledge and have been taught to look for these in new topics, even if in a subsequent grade they have a teacher who does not foster these connections, the students could independently feel empowered to do so. In other words, students could have greater agency as mathematical thinkers. Imagine schools where Mighty Mega Math Ideas were posted on classroom and hallway walls (i.e., as an element of environmental math; Nurnberger-Haag et al., 2019). What if these ideas were also shared with families in other ways to destigmatize math, help families feel the power of math, and recognize that it was the disconnected way they were taught that may have fostered math anxiety, not mathematics itself. This could help students and families feel that mathematics makes sense and envision futures that include using math. Studies should investigate these longitudinal hypotheses. If students, themselves, truly own the Mighty Mega Math Idea of Compose & Decompose, for instance, in the U.S. it would support about one-fourth of their K-8 standards (287 standards with at least 70 related to composing and decomposing) as being interconnected and coherent. Imagine if just a few other Mighty Mega Math Ideas were similarly prioritized. Most sources espouse the goal to have students see mathematics in this connected way, which will not happen if big ideas remain hidden as the pedagogy of the teachers. Thus, we adjusted Charles' (2005) definition of a big idea by changing the phrase "central to *the learning of* mathematics" (p. 10) to "central to *understanding* mathematics."

Summary and Future Directions for Big Ideas of Mathematics

Individual studies, constructs, and areas of research in mathematics education must use theoretical framing (Leatham, 2019; Spangler & Williams, 2019). Mathematics education values explicit theoretical frameworks that can be used as analytic frameworks to create a shared language for constructs, provide frames to recognize which aspects of a construct might be the focus of research questions, delineate initial codes for analysis, and build shared understanding through multiple studies reporting about the same phenomenon (Spangler & Williams, 2019). Yet, as others had noted there has been a lack of shared language or meaning related to big ideas (Askew, 2013; Siemon, 2022), which our study confirmed. That is, big ideas were a construct in need of a framework. As Boaler et al. (2017) noted, the big ideas construct "will evolve with our thinking" (p. 5). Thus, we suggested next steps in the evolution of the theoretical framing of the big ideas construct to foster the impact upon which the field has agreed: to help students see mathematics as an interconnected and coherent whole (Askew, 2013; Boaler et al., 2017; Tout et al., 2015).

Just as it is easier for students to learn accurate mathematics the first time it is introduced, rather than to correct or clarify terms later (NRC, 1989; Nurnberger-Haag et al., 2021), in the U.S. it will likely take teacher educators and teachers longer to clarify and advance the big ideas construct than countries where mathematics teacher educators and scholars could use the following seven recommendations to break ground to build a strong foundation from the beginning. The first suggestion is to use our revised definition that is essentially Charles' (2005) definition but with the crucial shift to Common Content Knowledge: "A big idea of mathematics is a statement that is central to understanding mathematics, one that links numerous mathematical understandings into a coherent whole." Second, to foster vertical and horizontal alignment, use the Big Ideas Framework consisting of three ordinal levels that span grades as well as grade bands: Mighty Mega Math Ideas (unite domains/strands), Power Math Ideas (unite topics within a domain/strand), and Strong Math Ideas (unite ideas within a topic). Third, when selecting or using ideas in any of these levels, use the five criteria summarized in the bullets below.

- Portable and Meaningful Name
- Ethical Communication
- Portable and Stand-alone Statement
- Connection and Example Based Presentation
- Categorize and Prioritize Big Ideas by their Size and Power

Fourth, instruct P-12 students and families on selected ideas to develop a society that can see the roots though the leaves when so many mathematical ideas and topics have become camouflaged due to language that obfuscates the underlying ideas that unite them. At a minimum, Mighty Mega Math Ideas would be beneficial candidates to prioritize for vertical alignment within school districts from preschool through Grade 12. Fifth, organize mathematics teacher education courses around a few Mighty Mega Math Ideas and Power Ideas. As mathematics teacher educators we already have such limited course time that we cannot cover every standard for every grade during our methods or content courses, so we already make difficult choices about what to cover. Using the two most powerful levels of big ideas to organize instruction should be a high-leverage practice way of making such choices. Sixth, although, many have advocated organizing instruction around big ideas (Bruner, 1960; Charles, 2005; Ritchhart, 1999; Siemon, 2022), our analysis revealed that resources have yet to be structured in this way. New resources as well as the next editions of existing books about big ideas could use these recommendations to reduce, reorganize, and structure big ideas as the top level in light of the five criteria and the framework.

Finally, we were disheartened that given the growing popularity of the construct, what Kuntze and colleagues (2011b) lamented is still true a decade later: “empirical research on professional knowledge connected with big ideas in mathematics is scarce” (p. 2717). Moreover, we have yet to find peer-reviewed research publications that empirically test the impact of using big ideas. Given how adamantly policy documents in the U.S. and numerous authors have claimed that big ideas are crucial teacher knowledge, as do we from our own practice, such claims have yet to be substantiated with research. As a field of mathematics education scholars, we can do better. The Big Ideas Framework provides three levels of big ideas to focus research designs, specify what was investigated, and communicate results in a way that studies could build upon each other, consistent with the purpose of a framework in mathematics education (e.g., Spangler & Williams, 2019). Research should investigate how using particular levels of big ideas impacts teacher knowledge and P-12 students’ performance on the traditionally accepted aspects of Common Content Knowledge as well as their perception of mathematics as a discipline of coherent and interconnected concepts.

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