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Utilizing Cognitive Load Theory and Bruner's Levels of Developmental Learning to Address Students' Struggles Related to Area of Polygons: A Pedagogical Action Research Study

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ABSTRACT

In this pedagogical action research study, we, as post-secondary mathematics teacher educators, built on an existing effort to improve pre-service teachers' mathematical vocabulary understandings by intentionally addressing their struggles related to polygonal area formulas. Utilizing cognitive load theory and Bruner's levels of developmental learning, we adapted and refined an existing "Area of Polygons" lesson to eliminate extraneous elements and scaffold the introduction of essential elements in the context of a cognitively engaging activity. Comparing our resulting lesson components to existing literature on polygonal area, we found two main approaches towards exploring area of polygons. Both approaches emphasized conservation of polygonal area with one focused on the details of attributes and square units and the other focused on comparisons of areas of figures. We discuss the implications of these approaches and the use of cognitive load theory in tandem with Bruner's levels for future curriculum redesign.

Keywords: geometry, pedagogical action research, pre-service teacher education

Introduction

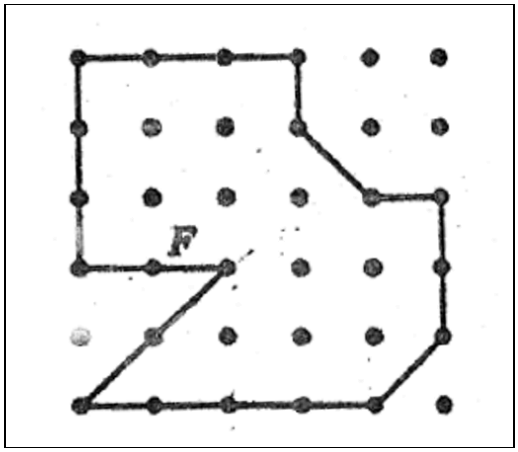
We, as post-secondary mathematics teacher educators, often collaboratively consider the content that our students, elementary pre-service teachers (PSTs), find difficult across the sequence of mathematics courses we teach. As part of a larger study focused on instructor collaboration and pedagogical improvements across the course sequence, we found that our PSTs routinely struggle with mathematical vocabulary (Bullock et al., 2021). Building off this work and connecting to concepts of cognitive load theory (Ayres, 2006; Pass et al., 2003) and Bruner's levels of developmental learning (Bruner, 1986; Reys et al., 2012), the current pedagogical action research study explores our response to students' struggle with polygonal area formulas. The impetus for this study, noticings about students' misconceptions about polygonal area and area formulas, is described below.

In Math 1385: Foundations of Mathematics II, the second mathematics content course in the sequence focused on geometry and measurement, one important topic that students explore is polygonal area formulas. In the original version of the "Area of Polygons" lesson, the first author spent about an hour carefully guiding students through interactive activities with patty paper to help them understand the derivations of the rectangle, parallelogram, triangle, and trapezoid area formulas.

At the end of the lesson, she had the students apply their new knowledge by finding the areas of "Crazy Shapes," an activity adapted from Aichele and Wolf (2008, p. 18). An example figure is provided in Figure 1.

Figure 1

Crazy Shape (Aichele & Wolf, 2008, p. 18).



She anticipated that students would divide the large crazy shape into smaller squares, rectangles, parallelograms, triangle, or trapezoids, find their areas, then add these areas together to find the area of the large crazy shape. However, as she walked around the room, she noticed one student was partially dividing the crazy shape into triangles, using the formula $b \times h$ to find the triangles' areas. Interested in this student's thought process, the first author initiated the following discussion:

> Cory: Why $b \times h$ for the area of a triangle?
> Student: Because that's how we find the area of triangle!
> Cory: How does the area of a triangle relate to the area of a parallelogram?
> [Extended silence.]
> Cory: Do you remember that a triangle's area is always half a parallelogram's area?
> [Extended silence.]
> Cory: So, what is the area formula for a triangle?
> Student: It's always base times height.

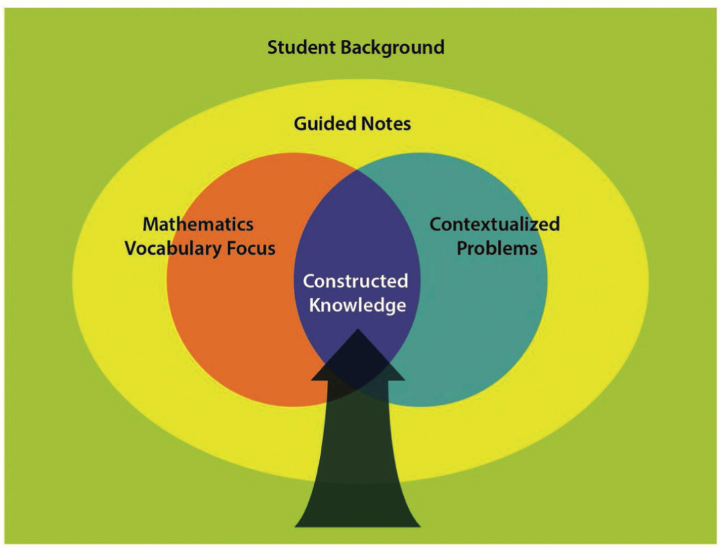
After teaching the "Area of Polygons" lesson a few more times in other sections of the course, the first author noticed similar responses from a number of students. These students seemed to be associating area with "base times height" or "length times width" no matter the type of shape. The first author's experiences initiated a series of lesson revisions intended to better enable students to grasp and retain an understanding of the connections between polygonal area formulas, specifically targeting the $\frac{1}{2}$ in the triangle area formula.

Study Background and Guiding Conceptual Framework

In our work with elementary PSTs, our research team, composed of five mathematics teacher educators (MTEs), embarked on a multi-year effort to improve learning opportunities for our students using guided notes across the sequence of mathematics courses for future elementary teachers. At our Southeastern, public university with a Hispanic Serving Institution (HSI) designation, our student population includes 45% first-generation undergraduates and over 75% of the students are employed while pursuing their degree. Our PSTs take three required elementary foundations mathematics content courses in the mathematics department and one mathematics methods course in the education department. Our team, initially consisting of three mathematics faculty and two education faculty, began our curriculum improvement efforts in response to observations about students' struggles that we routinely observed semester to semester across these courses. These efforts are summarized in the Guided Mathematics Vocabulary (GMaV) Conceptual Framework in which we position students' constructed knowledge at the intersection of an explicit focus on mathematics vocabulary and the use of contextualized problems using the curriculum tool of guided notes. See Figure 2 for this information.

Figure 2

GMaV Conceptual Framework (Bullock et al., 2021).



We acknowledge, seek to understand, and account for the mathematical knowledge that our students bring to our classrooms as we strive to support students' construction of knowledge. Specific insight into these ongoing curriculum and assessment reform efforts can be read in more detail elsewhere (Bullock et al., 2021; Ray et al., 2023).

As a part of our curriculum and assessment reform work, we focused on key topics or lessons that could be refined and adjusted to better support students' mathematical vocabulary understandings and, more broadly, address common areas of misconceptions or difficulty. One specific lesson that stood out to us in MATH 1385 was the "Area of Polygons" lesson (hereafter referred to as the "lesson"). We wanted our students to "look for and make use of structure" (CCSSI, 2010, p. 8) as they make connections between the polygonal area formulas. This Standard for Mathematics Practice (SMP) detailed in the Common Core State Standards for Mathematics (CCSSM), emphasizes the importance of recognizing and analyzing patterns and structures of mathematical objects. Within the context of polygonal area, we often noticed students arriving to MATH 1385 with a memory of area involving the multiplication of attributes but lacking a robust conceptual understanding of polygonal area. Additionally, even after the original lesson, students were not leveraging connections between the polygonal area formulas to complete higher level tasks.

Relevant Research Literature

Cognitive Load Theory

As we worked to develop and refine our lesson, we utilized various aspects of cognitive load theory. According to cognitive load researchers (Ayres, 2006; Pass et al., 2003), learners have a limited working memory, the space in the brain where all conscious cognitive processing occurs. Researchers posit that working memory may only be able to handle two to three new ideas, or elements, at a time. Thus, the instructor's goal is to load as little of the students' working memory space as possible so that novel ideas can be learned efficiently. As a student learns the knowledge elements associated with a particular mathematical concept, the student may incorporate (and perhaps automate) those elements into what is called a *schema*, or web of ideas. Once a schema is formed in a student's long-term memory, their working memory can more easily process related novel ideas and hook them into the already existing schema. Ultimately, a student's schema is so well-learned that it begins to act like a single element, thus vastly expanding the processing capability of working memory.

As a student learns a new mathematics concept, three types of cognitive load may use up the space in working memory: a) intrinsic, b) germane, and c) extraneous. *Intrinsic load* results from the interactivity of elements that are essential or intrinsic to understanding a certain mathematical concept. For example, to understand the triangle area formula, a student might need to understand the elemental definitions of area, triangle, parallelogram, base, height, the multiplication operation, as well as the fractional concept of $\frac{1}{2}$. These elements interact together to create the triangle area formula. The more elements necessary for understanding a concept, the higher the intrinsic load will be. *Germane load* refers to the cognitive activity necessary for a student to form a schema from the necessary knowledge elements. An instructor might influence germane load by creating, for example, activities that involve productive struggle, thus providing students opportunities to engage deeply with the new elements involved in the mathematics concept. *Extraneous load* involves unnecessary cognitive activity resulting from the way the instructor or textbook presents the information. For example, instructional materials may misdirect attention to nonessential aspects of the concept or needlessly require learners to search for relevant information.

According to cognitive load theory, the instructor's goal is to eliminate extraneous load and to decrease intrinsic load through carefully timing and scaffolding the introduction of essential elements. This in turn frees up a student's working memory for a higher germane load so that a robust schema can be developed.

Bruner's Levels of Developmental Learning

We posit that one way to scaffold the introduction of intrinsic knowledge elements so as not to overload working memory, and also guide the instructor in creating activities that boost an appropriate level of germane load, is to incorporate Bruner's levels of developmental learning. These three progressive levels are 1) enactive, 2) iconic, and 3) symbolic (Bruner, 1986; Reys et al., 2012). At the enactive level, students build initial connections between new knowledge elements by participating in activities that involve manipulating, constructing, and arranging real-world objects related to the concept. At the iconic level, students strengthen the previously formed connections and build further connections by participating in activities involving using pictures, images, or other representations of the concept. By the time students reach the symbolic level, a schema has been formed and students are ready to take part in activities that help them connect their work at the enactive and iconic levels to abstract symbolic representations of the concept. With time, students are able to manipulate and use the symbolic representations flexibly and efficiently without referring to their enactive or iconic counterparts. This may indicate that their schema has been encapsulated as a single knowledge element.

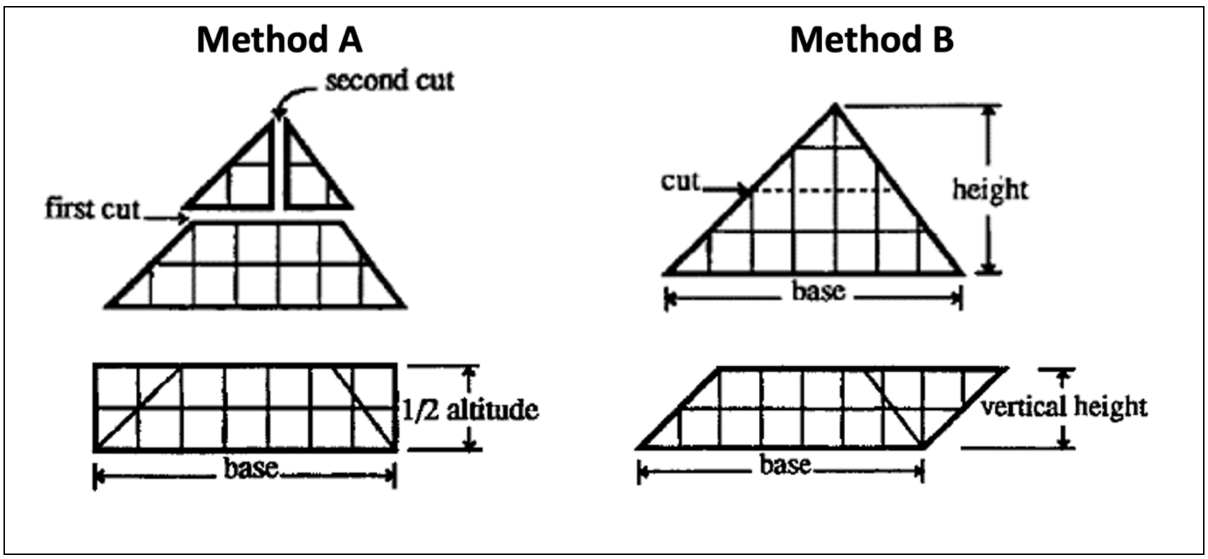
Area of Polygons

In our quest to develop a more effective lesson, we began by consulting the literature. Our search revealed an article by Neatrour (1991), which catalogued his methods for demonstrating various polygonal area formulas. We were especially interested in his two methods for the area of a triangle.

For his first method, he began by making a cut parallel to one of the bases of the triangle through the midpoint of the triangle's height (or altitude). See Figure 3 for an illustration.

Figure 3

Neatrour's (1991) Methods for the Area of a Triangle.



He then cut the resulting smaller triangle on top into two smaller right triangles. He rotated each smaller right triangle by 180º and translated them to the right and left of the bottom piece of the original triangle to create a rectangle. In Figure 3, we see that the bases of the original triangle and the rectangle are the same, but the height of the rectangle is now half the height of the original triangle. Thus,

$$A_{original\ triangle}=A_{rectangle}=base \times \frac{1}{2} height = \frac{1}{2} \times b \times h$$

For his second method, he began by making the same parallel cut as before. However, this time, he simply rotated the resulting smaller top triangle 180º and translated it down to create a parallelogram. This time, if a student knows the parallelogram area formula, the triangle area formula is forthcoming:

$$A_{original\ triangle} = A_{parallelogram} = base \times \frac{1}{2} height = \frac{1}{2} \times b \times h$$

We noticed that Neatrour's (1991) methods focused both on conserving area and halving attributes while at the same time, involved grid squares. While we appreciated his approach, we felt our students might become overloaded if these intrinsic knowledge elements were introduced initially and simultaneously. Instead, we wanted students to have plenty of room in working memory to grapple with the intrinsic idea that a triangle is half a parallelogram. This was the overall relationship we wanted them to experience and remember.

Methodology

This study builds on an existing study utilizing a qualitative, grounded theory, pedagogical action research design (Norton, 2018), in which MTE faculty collaborated to revise the guided notes for mathematics content courses for PSTs. The MTEs in the original study utilized this approach as we explored the pedagogical issue of students' struggles with mathematics vocabulary and methodically developed steps to address these issues. This work resulted in the development of the GMaV Framework detailed earlier (Bullock et al., 2021, see Figure 2) and led to the development of assessment tools for mathematics vocabulary (Ray et al., 2023).

Research Question

In our current study, the authors, two MTEs from the original research team, extended the existing pedagogical action research efforts, to address the pedagogical issue of Math 1385 students' struggles related to polygonal area. Building on the larger group's curriculum revision efforts, we sought to find research-based ways to refine and adjust our existing polygonal area lesson to address these struggles. Thus, we asked the following research question: How can we utilize cognitive load theory and Bruner's levels of developmental learning (Ayres, 2006; Pass et al., 2003) to adapt an existing lesson to address students' struggles specifically related to triangular area?

Data Collection and Analysis

As part of the larger study, during the Spring 2022 semester, we began reviewing the 24 sets of guided notes from Math 1385, along with another MTE from the research team. From this, we decided the guided notes for the "Area of Polygons" lesson would be a beneficial candidate for revisions. The lesson revisions would be our main source of data. This decision came in response to observed students' struggles with polygonal area lingering after the original lesson was taught.

To analyze the lesson revisions, we conducted iterative, thematic analyses and case comparison (Corbin & Strauss, 2008; Glesne, 2006) during the Fall 2022 semester. Here, we analyzed the iterations of the lessons based on the content and nature of the lessons. Then, we compared these iterations to one another and to the polygonal area approaches found in the research literature (Neatrour, 1991). To summarize our findings, we visually represented the approaches of the polygonal area lessons in comparison to one another to explore how these approaches could be viewed through the lenses of cognitive load theory and Bruner's levels of developmental learning.

Findings: Iterations of the "Area of Polygons" Lesson

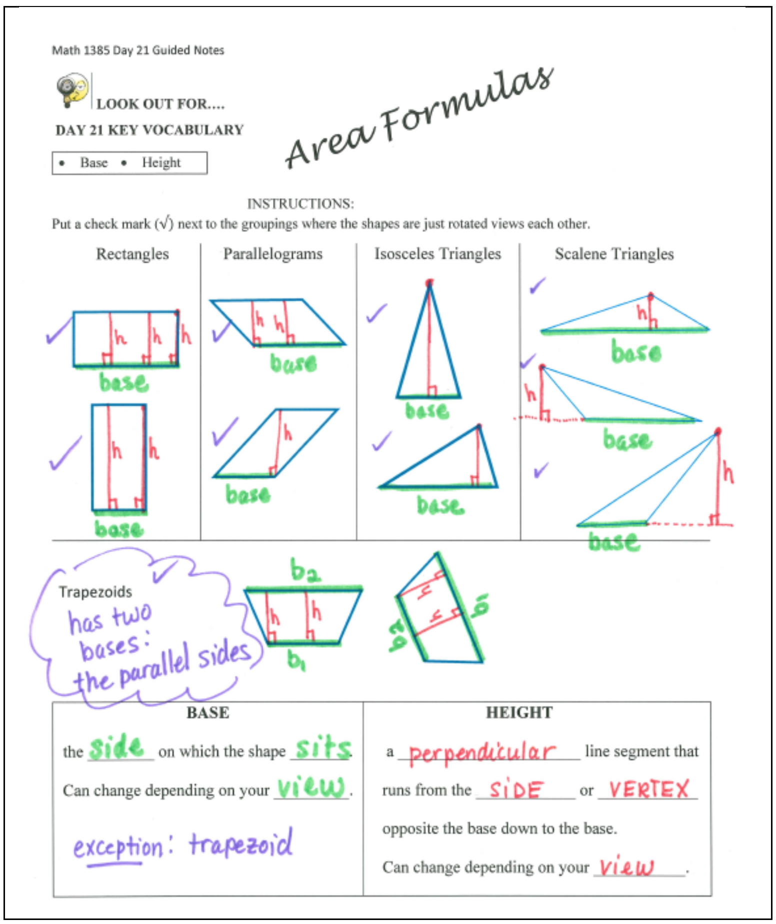
In our findings, we detail our adjustments and refinements to the lesson, specifically focused on the lesson portion involving triangular area. We outline what we found when analyzing a) the lesson portion leading up to triangular area, b) the original lesson portion involving triangular area, and c) the iterative refinements of the triangular area portion, as informed by cognitive load theory and Bruner's levels of developmental learning.

Background: Leading Up to Triangles

In this study, our focus is on our revisions of the triangular area portion of the lesson. However, during the lesson's initial portions leading up to triangle area, students explored pertinent vocabulary and other area formulas intrinsic to the triangle area formula. Students began by exploring the definitions of base and height and labeling the bases and heights of various sets of congruent shapes in different orientations. See Figure 4 for this information.

Figure 4

Exploring Bases and Heights in Different Orientations.



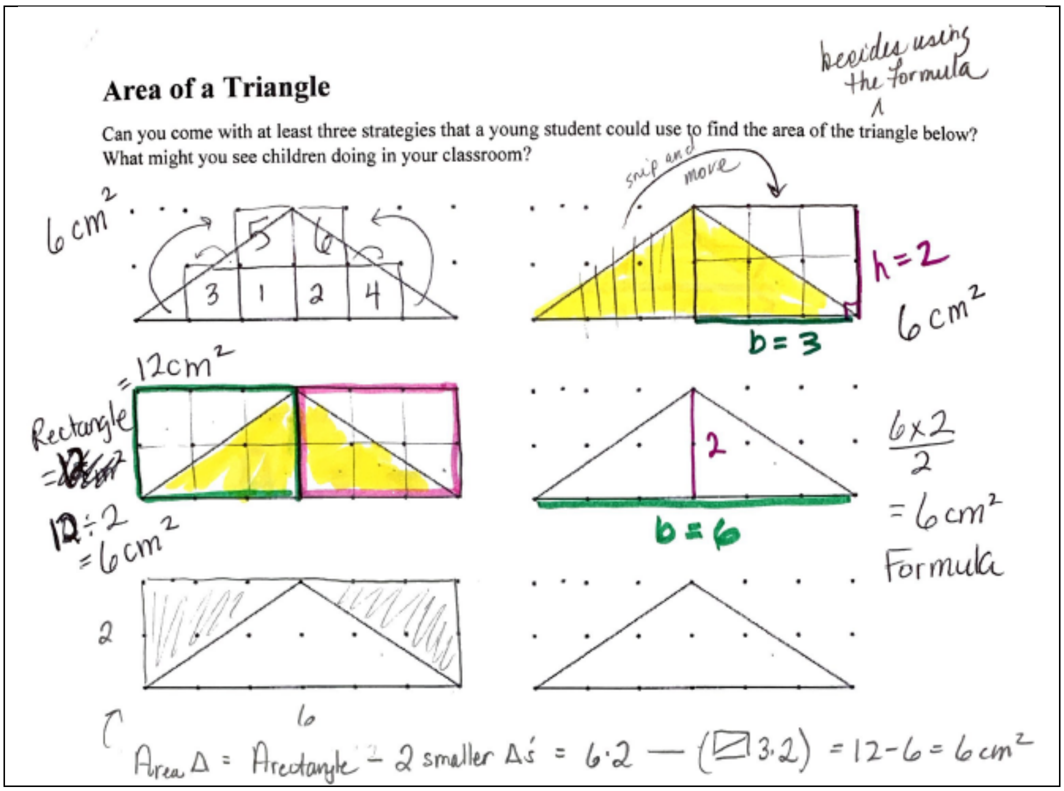
By providing both visual and verbal representations of this vocabulary at the beginning, we were carefully timing the introduction of knowledge elements intrinsic to the triangle area formula so that the terms would be available for students to use throughout the lesson. Next, the class studied the rectangle, dividing it into an array of smaller squares to better understand why its area formula is base times height. From there, the lesson moved to parallelograms where students cut parallelograms into two parts along their heights, rearranged the resulting pieces to make a rectangle, and thus, showed that area is still just base times height. Here again, we were scaffolding the recall of the intrinsic elements of area and parallelogram in preparation for the triangle area formula.

Cory's Original Triangle Area Approach (Version 1.0)

In the original triangle area portion of the lesson, Cory began by giving students six congruent isosceles triangles in their guided notes. She asked student to come up with at least three strategies a young child might use to find the area of the triangle. In the larger class discussion, students shared multiple different approaches. See Figure 5 for details on these approaches.

Figure 5

Student Strategies for the Area of a Triangle (Version 1.0).

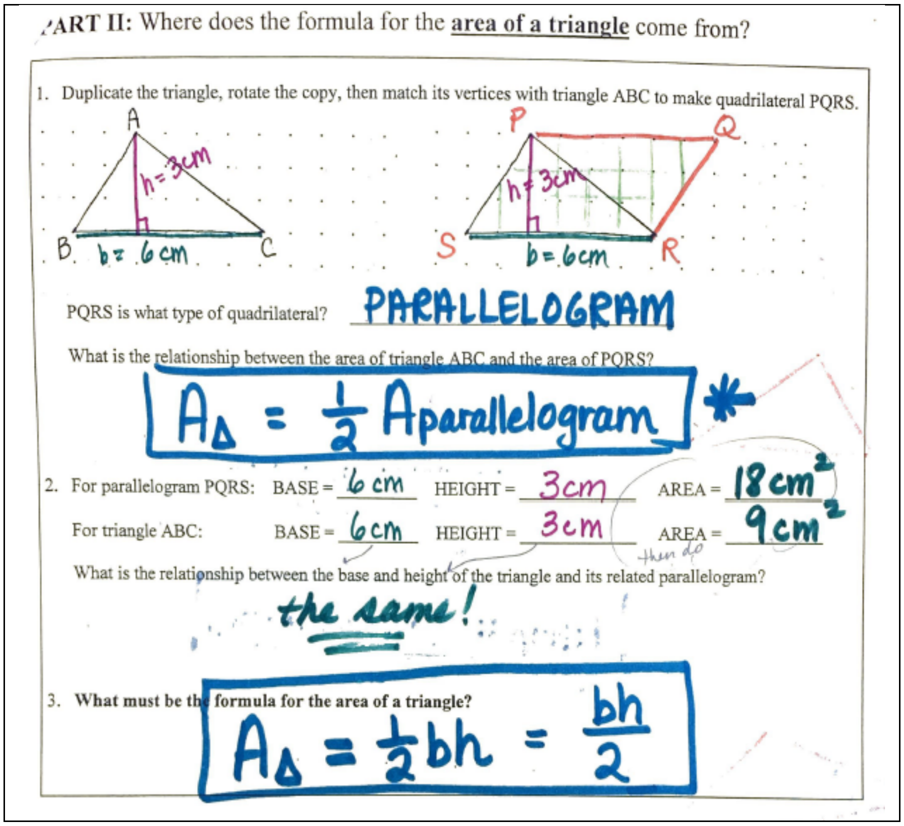


One student split the triangle up into small squares, piecing together left-over parts to make full squares. Another student split the triangle into two smaller right triangles and put the two right triangles together to make a rectangle with the same area as the original triangle. A third student also split the triangle into two right triangles but copied the two right triangles to make a large rectangle with area double the original triangle. Surprisingly, a fourth student found the area of a large rectangle encompassing the triangle. She then combined the extra areas inside the large rectangle but outside the triangle to form a smaller rectangle and subtracted the area of the smaller rectangle from the area of the large one. A fifth student was determined to use the triangle area formula, even though Cory had requested them not to.

After this exploration, Cory narrowed the focus to one specific method for finding the area of a triangle. This method is illustrated in Figure 6.

Figure 6

Original Triangle Area Approach (Version 1.0).



Students copied a given triangle on a patty paper, rotated it upside down, and traced it next to the original triangle to make a parallelogram. The class discussed the fact that the area of the triangle is one-half the area of the parallelogram. Students also noticed that the base and height of each triangle and resulting parallelogram are the same. Therefore, the class concluded that the formula for the area of the triangle must be one-half the formula for the area of a parallelogram (i.e., $A_{Triangle} = \frac{1}{2} \times b \times h$). Cory then had the class practice this method on another triangle. However, as detailed above in Kayla's approach, many students did not seem to retain the importance of the $\frac{1}{2}$ in the area of a triangle formula.

After discussing this original version of the lesson, we realized that if we wanted students to grasp this concept, we needed to eliminate any extraneous cognitive load and decrease any intrinsic load, which we considered unnecessary at this point in the learning process. While not useless, the first activity with the six isosceles triangles, as well as the emphasis on conserving the base and height in the second activity, could be deemed either extraneous or not intrinsic to the overarching importance of the $\frac{1}{2}$ in the triangle area formula. We knew we needed to revise this lesson with the goal in mind.

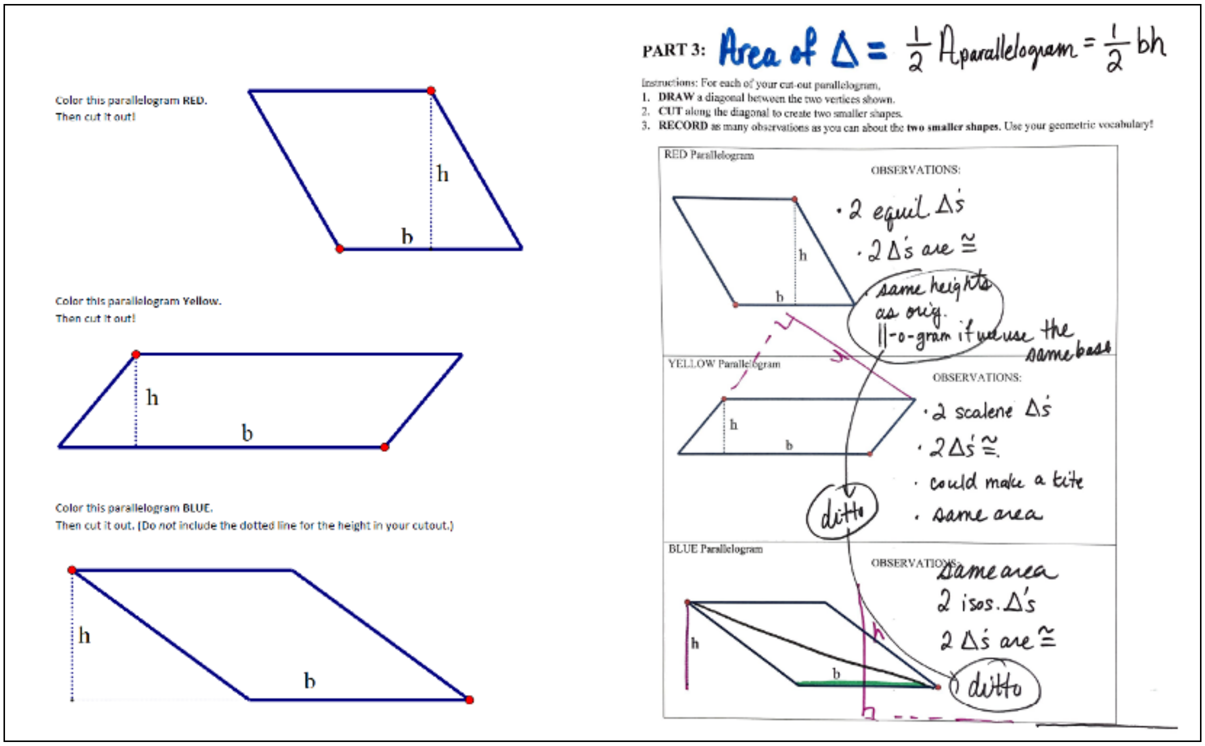
Cory's Approach (Version 2.0)

For our first attempt at a new lesson on triangle area, we created three different parallelograms, labeled the base and height in each one, and strategically marked two opposite vertices. See Figure 7 for this information. The students were to color and cut out the three parallelograms before class began. During the lesson, students followed the instructions below in their small groups:

1. **DRAW** a diagonal between the two vertices shown.
2. **CUT** along the diagonal to create two smaller shapes.
3. **RECORD** as many observations as you can about the two smaller shapes. Use your geometric vocabulary!

Figure 7

Cory's Approach (Version 2.0).



Cory then facilitated a whole class discussion about students' observations. For each parallelogram, as anticipated, the students observed that the two smaller shapes were congruent triangles with the same area. They also named the two triangles with the appropriate descriptors (equilateral, scalene, and isosceles). Cory also hoped that the strategic marking of the vertices might help the students easily notice that the base and height of each parallelogram, and its resulting triangles, are the same without causing an intrinsic overload. However, only one student mentioned this. Furthermore, she commented that it was difficult to see the height on the third parallelogram since it had to be drawn outside the shape. The discussion ended by the class highlighting the relationship between the area of a triangle and the area of a parallelogram: No matter the triangle type, a triangle is always half a parallelogram. Therefore, the triangle area formula is:

$$A_{triangle} = \frac{1}{2} A_{parallelogram} = \frac{1}{2} \times base \times height = \frac{1}{2} \times b \times h$$

We felt this lesson was an improvement over the previous version because it did a better job highlighting the structural relationships between the areas of a triangle and a parallelogram without overloading the students with untimely intrinsic elements. However, although the visuals seemed powerful, many students did not appear to deeply engage with the concept. We pondered: Had the students developed robust, meaningful, and lasting connections between triangles and parallelograms? Had their germane load been increased enough for students to begin construction of a triangle area schema in long-term memory?

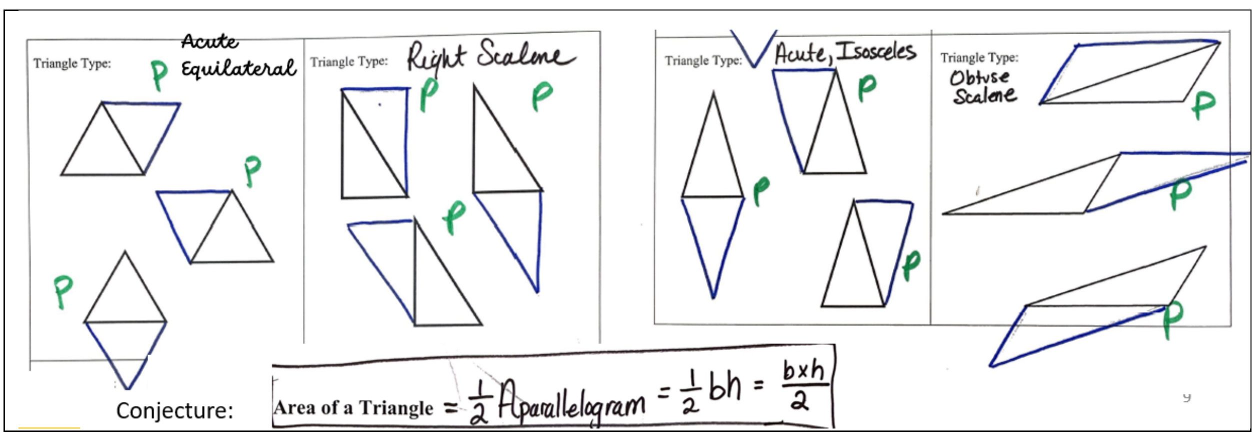
Cory's Approach (Version 3.0)

Our next reimagining of the triangle area lesson began by dividing the students into groups of four and assigning each group member one of the sets of three congruent triangles in their guided notes. Each set contained a different triangle type. Cory asked the students to do the following with their set See Figure 8 for an illustration:

1. **NAME** the triangle with two vocabulary words: Equilateral, Isosceles, Scalene, Acute, Right, Obtuse.
2. **COPY** the triangle onto patty paper.
3. **ROTATE** the patty paper triangle upside down.
4. **TRACE** the upside-down triangle next to the original triangle to a create a 4-sided shape. See if you can do this in 3 different ways!
5. **WRITE** "P" next to the 4-sided shapes that are parallelograms. Share your work with the others in your group.

Figure 8

Cory's Approach (Version 3.0)



After students completed the work for their set, Cory displayed a completed worksheet so everyone could see the correct results for all sets. As in the earlier versions, the lesson ended with a discussion emphasizing that, no matter the triangle type, the area of a triangle is half the area of a parallelogram, along with an opportunity for students to connect this concept to the symbolic formula:

$$A_{triangle} = \frac{1}{2}\,A_{parallelogram} = \frac{1}{2} \times base \times height = \frac{1}{2} \times b \times h$$

This lesson iteration provided an opportunity for students think more deeply about the different ways two congruent triangles could form a parallelogram (an increased germane load). Also, we wondered if seeing the numerous Ps covering their worksheets would make a helpful impression (an increased but timely intrinsic load). At the same time, the lesson posed some difficulties. Many students struggled to rotate their patty paper triangles 180º, creating kites rather than parallelograms (an extraneous load). We wondered what we could do reduce any confusion with the patty paper tool, yet still retain the germane problem-solving aspect of the lesson.

Cory's Approach (Version 4.0)

Version 4.0 of the triangle area lesson began by dividing the students into groups of four and giving each group member a pair of congruent laminated triangles, each pair of a different type. Cory asked the students to do the following:

1. Describe your two congruent triangles with two geometric terms: Acute, Right, Obtuse, Equilateral, Isosceles, Scalene.
2. How many different ways can you create a parallelogram with your two congruent triangles?

Before sending them off to work, Cory reviewed the definitions of parallelogram and kite, making sure students recalled the properties of these shapes. Then, after sufficient individual problem-solving time, various students shared their findings with the whole class, using their laminated triangles to demonstrate the different ways they made a parallelogram. Afterward, the students recorded their work using patty paper for their assigned triangle type on a worksheet similar to the one from Version 3.0. Finally, each group member shared their expertise by recording their work on their group members' worksheets for them. As in previous iterations, the lesson concluded with the full class highlighting the fact that a triangle is always half a parallelogram, no matter the triangle type, and by connecting this concept to the algebraic formula.

This fourth lesson iteration seemed to provide four main benefits. First, it scaffolded students' construction of a triangle area schema through Bruner's levels of developmental learning. In our lesson, students began by engaging in a hands-on activity involving laminated triangles, in line with Bruner's level one (enactive). Students next created pictorial views of their findings with patty paper, thus transferring their thinking to the pictorial representations detailed in Bruner's level two (iconic). Our lesson concluded with a discussion aimed at connecting the visual concept of a triangle being half a parallelogram to the symbolic ½ in the triangle area formula, aligned with Bruner's level three (symbolic).

Second, our fourth iteration involved a stronger germane load. Students were required to problem-solve as they thought deeply about the intrinsic definition of a parallelogram and as they utilized their spatial visualization skills to create parallelograms from triangles.

Third, rather than having Cory share the correct answers as in Version 3.0, this iteration made students responsible for their portion of the activity, requiring them to share their expertise with others. This gave them further opportunity to strengthen long-term connections within their triangle area schema.

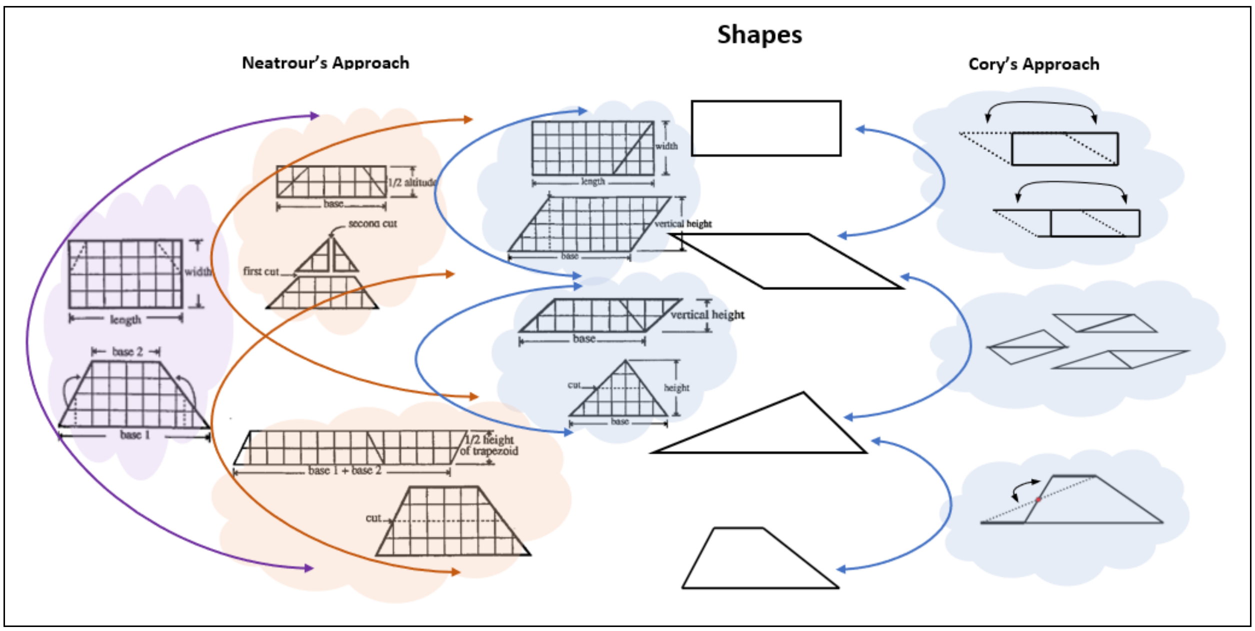
Fourth, the revised lesson focused students on the big idea that the area of the triangle is half the area of a parallelogram rather than diverting students' attention to the details of grid squares and attributes (an untimely intrinsic load), which while important, seemed to prevent some students from grasping the overall concept.

Summarizing Our Findings: Two Approaches Towards Area of Polygons

From the iterations of the triangular portion of the lesson and our comparison to Neatrour's (1991) suggested strategies for students' exploration, two main approaches towards teaching polygonal area emerged. These two approaches emphasized conservation of area of polygons in different ways. Neatrour's approach focused on the details of finding polygonal area, including figure attributes and square units, while Cory's revised approach highlighted the comparisons of areas of figures. These two approaches are summarized in the provided visual comparison show in Figure 9, where we see the strategies suggested by each approach when transforming between pairs of four different polygons – rectangle, parallelogram, triangle, and trapezoid. The arrows in the diagram indicate the two shapes involved in the corresponding transformation. Each arrow includes a visual representation of the corresponding transformation between the two indicated shapes suggested by each approach.

Figure 9

Visual Comparison of Neatrour's (1991) and Cory's Approaches.



In the visual summary (see Figure 9), Neatrour offers options for considering the relationships between area of polygons. For example, Neatrour details multiple pathways for transforming trapezoids and triangles into parallelograms or rectangles. Additionally, Neatrour's suggested strategies emphasize figure attributes, such as base and height, and how transforming shapes from one to another changes these attributes or repositions these attributes as components of a transformed shape. For example, the transformation of a triangle into a parallelogram leads to a parallelogram with the same base length as the original triangle but with a vertical height that is half the height of the original parallelogram. Additionally, looking across the suggested strategies, Neatrour's choice to impose polygons on a grid visually emphasizes square units and may encourage students to only view area as determined by the number of boxes, or units, inside the shape.

Like Neatrour, in the beginning portion of Cory's lesson, she highlighted the definitions of base and height and square units inside a rectangle. However, her revised portion of the lesson involving triangles offers an alternative emphasis. As shown in the visual summary (see Figure 9), Cory offers a single pathway for transforming between the four different shapes, from rectangle to parallelogram to triangle to trapezoid and vice versa. Essentially, these strategies build from one area formula to the next without using labeled attributes or grid squares. Thus, unlike Neatrour, Cory's revised approach includes fewer *choices* for transforming between shapes and intentionally highlights area comparison as a way for students to make connections between polygonal area formulas. Also, by refraining from labeling attributes or using grid squares, Cory's approach eliminates extra *chatter* in the form of additional detail that may be useful or meaningful for some students but not necessary to build up and explicitly connect between the polygonal area formulas. In other words, Cory's approach more readily favors the big idea of area comparison, rather than the details of the polygonal figures.

In summary, our evolving approaches towards polygonal area eliminated extra detail and provided students with a single pathway for building from one shape to the next, thus minimizing extraneous load and introducing intrinsic load in a timely fashion, rather than overwhelming working memory with multiple pathways. We posit that this approach helped students develop stronger initial connections between the polygonal area formulas. We are not suggesting that details of attributes and grid squares or the use of multiple and varied approaches are not important. Instead, we propose that more comprehensive approaches could perhaps be explored once students have a clear foundational understanding of the conceptual connections between area of polygon formulas, or that these additional approaches could allow room for differentiation when students exhibit varying levels of understanding.

Discussion and Implications

In this study, we explored iterations of an "Area of Polygons" lesson using the lens of cognitive load theory. In the process, we became aware that a thoughtful consideration of the layout of our lesson in terms of Bruner's levels of developmental learning helped us better leverage the tenets of cognitive load theory. Particularly, we were better able to scaffold students' construction of a triangle area schema by intentionally moving them through the three levels. Moreover, to increase the germane load necessary for creating stronger, stable schemas, we increased the problem-solving necessary at the enactive and iconic levels. Additionally, at each level, we recognized the need to reduce or eliminate extraneous and untimely intrinsic load which was distracting our students from grasping the big idea of the lesson. Thus, combining Bruner's levels with components of cognitive load theory provided us frameworks for revising the lesson in a powerful way.

We posit that these frameworks could be useful for effectively adapting instruction across the grade levels and mathematical content to help students create strong schemas around any mathematical topic. More broadly, we suggest that these frameworks offer flexibility in considering the unique contexts, backgrounds, and needs of learners. For example, if advanced students are ready for additional intrinsic load, they may benefit from explorations involving more detailed information that could be considered "chatter" for other students. Additionally, these frameworks provide explicit language and resources for instructors to improve curriculum materials and learning experiences across a wide range of mathematical content. Our study focused specifically on polygonal area, but we anticipate many other content areas where students traditionally struggle could also benefit from a review using our combined frameworks. In conclusion, cognitive load theory and Bruner's levels of developmental learning proved to be useful lenses for our curriculum redesign efforts.

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References

Aichele, D.B. & Wolfe, J. (2008). *Geometric structures: An inquiry-based approach for prospective elementary and middle school teachers*. Pearson Prentice Hall.

Ayres, P. (2006). Impact of reducing intrinsic cognitive load on learning in a mathematical domain. *Applied Cognitive Psychology, 20*, 287-298. https://doi.org/10.1002/acp.1245

Bruner, J. (1986). The course of cognitive growth. *American Psychologist, 19*(1), 1-15.

Bullock, E., Ray, A., Herron, J., & Swarthout, M. (2021). The GMaV conceptual framework: Constructing elementary pre-service teacher's mathematics vocabulary understanding through contextualized guided notes. *Investigations in Mathematics Learning, 13*(4), 287-302. https://doi.org/10.1080/19477503.2021.1985906.

Common Core State Standards Initiative (CCSSI). (2010). *Common Core State Standards for Mathematics*. National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org

Corbin, J., & Strauss, A. (2008). *Basics of qualitative research, 3rd edition*. Sage.

Glesne, C. (2006). *Becoming qualitative researchers: An introduction, 4th edition*. Pearson.

Neatrour, C. R. (1991). A strategy for discovering the formulas for finding the area of polygons. *School Science and Mathematics*, *91*(8), 362-66.

Norton, L. (2018). *Action research in teaching and learning: A practical guide to conducting pedagogical research in universities.* Routledge.

Pass, F., Renkl, A., & Sweller, J. (2003). Cognitive load theory and instruction design: Recent developments. *Educational Psychologist, 39*(1), 1–4.

Ray, A., Herron, J., & Bullock E. (2023). Exploring mathematics vocabulary alignment in a future elementary teacher's trajectory: A case study. *Journal of College Reading and Learning*, DOI: 10.1080/10790195.2023.2214188

Reys, R., Lindquist, M., Lamdin, D., & Smith, N. (2012). *Helping children learn mathematics*, *10th edition*. John Wiley & Sons.