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Investigating Large-Scale, High School Mathematics Achievement Through the Lens of the Cognitive Domains

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ABSTRACT

A key component of student mathematics achievement relates to the cognitive domains. This paper examines student mathematics achievement in three cognitive domains (knowledge, comprehension, reasoning) as per different achievement grades, across four years (2015, 2016, 2018, 2019), and by gender. This study used the Caribbean Secondary Education Certificate (CSEC) mathematics results across four years from the Caribbean Examinations Council (CXC), which is the main public examination board in the Caribbean. The sample constituted 69,945 public school students from 161 secondary schools in Jamaica. The study found a regular pattern of cognitive domain performance at all grade levels and in each of the four years under consideration. Students performed best in the knowledge domain, followed by comprehension, and then reasoning. Students with the highest overall achievement demonstrated the highest achievement across the three domains and there was also a strong, significant, positive correlation between students' overall grades, that related to knowledge, comprehension, reasoning, and the cognitive domains. Another key finding was that for the knowledge and comprehension domains there was a significant difference in the performance of males and females in favour of females, but the related effect sizes were minimal. Practical implications and potential directions for future research are discussed.

Keywords: mathematics achievement, cognitive domains, gender

Introduction

In present-day societies, proficiency in mathematics is considered to be a central scholastic imperative (Mullis et al., 2012). It is a prerequisite to attaining educational and vocational success, especially in careers related to science, technology, mathematics, and engineering (STEM), and in navigating daily living (Hefty, 2015; Siegler et al., 2012). At the end of high school, evidence of students' mathematics proficiency is often the attainment of a qualification that indicates a passing grade in some standardized, high stakes, exit examination. This qualification is significant since it serves as the main matriculation requirement for further studies and employment.

Performance outcomes in the form of scores, or grades students receive from completing these mathematics examinations, reflect their knowledge and skills of content-related subject matter such as that related to specific strands in mathematics like Geometry or Algebra. For some examination bodies or examining boards, these scores also reflect cognitive dimensions or domains, such as knowing, applying, and reasoning which relate to the thinking processes that students are expected to utilize as they engage with different mathematical topics and tasks (Harks et al., 2014; Mullis et al., 2020). In regard to the cognitive domains in particular, which is a key focus of this paper, Nilsen et al. (2016) stated that students "do not just need knowledge in mathematics, but must also be able to apply knowledge and conceptual understanding in different contexts, and to analyze and reason to solve problems" (p. 7). This is highlighted as a key outcome of mathematics teaching and learning in frameworks in mathematics (e.g., Schoenfeld & Kilpatrick, 2008), mathematics curricular/policy documents (Caribbean Examinations Council, 2015; Department for Education, 2013; National Council of Teachers of Mathematics, 2014) and international assessment frameworks in mathematics such as the Trends in International Mathematics and Science Study (TIMMS) and Programme for International Student Assessment (PISA) (Mullis et al., 2020; *Programme for International Student Assessment*, 2021).

While students' scores capture multiple elements of their knowledge and skills, Steiner and Ashcraft (2012) note that this achievement is not uniform. In other words, two students who obtain the same grade in an assessment may differ widely in their cognitive dimensions and/or content knowledge. Dogan and Tatsuoka (2008) also note that assessment reports that provide only total scores of examinees are limited in that they do not offer insights into areas of students' mathematics difficulties. Consequently, the present author asserts that investigating achievement from the perspective of overall grades provides just one of several possible dimensions related to student achievement. It is important that research focus on a multiplicity of perspectives in order to gain a more in-depth and comprehensive portrait of student achievement. Further, unpacking students' achievement in high stakes examinations also provides insightful observations to countries, in particular "educational policymakers, administrators, teachers, and researchers …in understanding educational outcomes more fully, which is a core concern of effective educational planning and reform" (Mullis et al., 2012, p. 1). An example of this is seen in research undertaken by Shalem et al. (2013) who reported that as South African teachers participated in curriculum mapping of large-scale assessments, they gained an understanding of both the specific, grade-level content that was assessed and the related cognitive demand. Teachers were then better able to evaluate their classroom practice and understand the curriculum.

While there are several frameworks for presenting elements of the cognitive domain, arguably, Bloom's taxonomy is one of the most widely used and well-established in education, especially in relation to developing achievement tests and interpreting these test results (Webb, 2020). The original taxonomy, that has since been revised, is organised within a triangle and comprises six levels related to the cognitive domain: knowledge, comprehension, application, analysis, synthesis, and evaluation. The present author notes that although extensively adopted in practice, Bloom's taxonomy has also faced considerable critique. For example, whereas many practitioners perceive the elements of the taxonomy to be hierarchical (Webb, 2012), this notion has been disputed and instead it has been proposed that these components are merely a set of categories (Postlethwaite, 1994). Notwithstanding the aforementioned, the elements of Bloom's taxonomy build on each other (Thomson, 2006). Mullis et al. (2003) also alluded to this and stated that "facility in using mathematics, or reasoning about mathematical situations, depends primarily on mathematical knowledge" (p. 27). Mullis and Martin (2017) add that "without access to this knowledge base … students would find purposeful mathematical thinking impossible" (p. 23). A comprehensive critique of Bloom's taxonomy is beyond the scope of this paper. However, Webb (2020) notes that although not explicitly acknowledged, the influence of this taxonomy on large-scale, high stakes assessments, such as the TIMMS is apparent.

To date, students' cognitive dimensions (e.g., knowing, understanding, reasoning) in examinations have received far less scholastic attention than content-specific components of a given curriculum (Harks et al., 2014). The present author surmises that this may be related to the performance outcome reporting formats that examining bodies use, and/or the type of data these bodies collect on student achievement. For example, very few examining bodies report student outcomes as per cognitive dimensions. Thomson (2006) also notes that

> The content domains are fairly, consistently, and readily found in the curricula of the participating countries, and are the subject of the major international and national reports for

> TIMSS. Developing reliable and valid achievement scales for the cognitive domains is not as straightforward. (p. v)

Notwithstanding the aforementioned, since the development of a range of cognitive skills in mathematics is an educational imperative, and that there is general consensus within mathematics education that assessing and reporting elements of the cognitive domain is important (Harks et al., 2014), research, such as the current one, that focuses on exploring student achievement in this regard is significant. In addition to this, Suurtamm et al. (2016) note that "If the enacted curriculum of the classroom and the assessed curriculum are to inform each other and to enhance student learning in positive and productive ways, then large-scale, external assessments cannot operate in isolation from the classroom" (p. 22). This assertion points to another benefit of this research in relation to its potential to positively impact teachers' classroom practice.

Research Context: Assessing and Reporting Mathematics Achievement

CSEC Mathematics Examination

In the English speaking Caribbean, at the end of high school, most students generally sit for the Caribbean Secondary Education Certificate (CSEC) mathematics examinations to obtain their qualifications in mathematics. These examinations are administered by the examining body, the Caribbean Examinations Council (CXC[1]), and are offered at the General Proficiency level in June and January annually. In terms of international comparison, the CSEC mathematics syllabus states that "the competencies and certification acquired upon completion of this course of study are comparable with the mathematics curricula of high schools world-wide" (Caribbean Examinations Council, 2015, p. 1). Hence, the present author opines that the findings of this study are very likely to be important and applicable beyond the local context.

The CSEC mathematics examination comprises an external and internal assessment component weighted 80% and 20%, respectively. The external component consists of two papers, Paper 01 and 02, that account for 80% of the overall grade. Paper 01 is worth 30% of the final grade and has 60 compulsory, multiple-choice items while Paper 02 includes ten, compulsory, constructed-response items, based on the nine topics and associated objectives covered in the mathematics syllabus. Paper 02 contributes to 50% of the final grade. The internal assessment (Paper 03) is school-based and requires examinees to demonstrate the application of mathematics in real life situations. In this regard it comprises a project on any relevant topic, or combination of topics, that is assessed internally by the teacher and externally by CXC (Caribbean Examinations Council, 2015).

CSEC Grade Reporting

For the CSEC mathematics qualification, student achievement is reported as grades denoted by Roman numerals ranging from I-VI with I to III designated as passing grades. Each grade band within each grading system has a cut-off point, however, CXC does not make its cut-scores public (McPherson, 2020). Therefore, the range of raw scores within each grade is not known. Also, the

[1] The Caribbean Examinations Council (CXC) is an examination board that conducts examinations, and awards certificates and diplomas based on the results of these examinations in 16 Caribbean countries. The Caribbean Secondary Education Certification (CSEC) is one of several qualifications offered for a range of subjects that individuals would enrol in in high school. The CSEC typically and currently serves as the main matriculation qualification for entry into postsecondary education in the Caribbean.

percentage of the test that students would need to get correct to be considered passing is also not made public. Table 1 presents the grades and the associated descriptors of performance.

Table 1

Descriptors of CSEC Performance Outcomes

Overall Grade	Performance Descriptor	Profile Grade	Performance Descriptor
I	Outstanding	A	Outstanding
II	Good	B	Good
III	Fairly good	C	Fairly good
IV	Moderate	D	Moderate
V	Limited	E	Weak
VI	Very limited	F	Poor

For the CXC, beyond reporting grades as Roman numerals, it reports achievement relating to the cognitive domain as three *profile dimensions* on a scale of A-F (see Table 1). The profile dimensions include knowledge (P1), comprehension (P2), and reasoning (P3), and specify the cognitive demand of the items and questions in the CSEC mathematics examination (Caribbean Examinations Council, 2015). They also capture the thinking processes expected of students as they engage with the mathematics content and are similar to some of the categories in the cognitive domain included in Bloom's Taxonomy (Bloom et al., 1956). For the CSEC mathematics examination, 30% of the items are allocated to knowledge and reasoning, respectively, while 40% are aligned to the comprehension dimension. Table 2 provides a breakdown of marks and percentage weightings in the examination components by profiles.

Table 2

Assessment Items by Profiles and Item Format on the CSEC Mathematics Examinations (Caribbean Examinations Council, 2015, p. 5)

Profiles	No. of Marks in Examination Components			Total
	Paper 01 Multiple Choice	Paper 02 Constructed Response	Paper 03 School Based	
Knowledge	18	30	12	60 (30%)
Comprehension	24	40	16	80 (40%)
Reasoning	18	30	12	60 (30%)
Total	60 (30%)	100 (50%)	40 (20%)	200 (100%)

The Caribbean Examinations Council (2015) informs that the profile '*Knowledge*' requires examinees to recall rules, procedures, definitions, and facts. Items that align to this profile are characterised by rote memory, simple computations, and constructions, while '*Comprehension*' necessitates algorithmic thinking, whereby algorithms are used and applied to familiar problems. In this context, therefore, comprehension is not considered to be the same as reasoning and solving non-routine problems. It further adds that the '*Reasoning*' profile dimension encapsulates several competencies. These include the (i) translation of non-routine problems into mathematical symbols and then choosing suitable algorithms to solve the problems; (ii) combination of two or more algorithms to solve problems; (iii) use of an algorithm or part of an algorithm, in a reverse order, to solve a problem; (iv) making of inferences and generalisations from given data; (v) justification of results or statements; and (vi) analysis and synthesis of mathematical data. Two exemplars of specimen CSEC mathematics examination questions, the related solutions, and related profile dimension allocation are presented in Figures 1-4 in the Appendix.

The profile dimensions of the CXC align very closely with the notion of cognitive domains or dimensions used by the TIMSS, which is an international assessment of mathematics and science at Grades 4 and 8. TIMMS has been conducted quadrennially since 1995. Similar to the CXC, the TIMMS is large-scaled and reports student achievement using three cognitive domains, namely, knowing, applying and reasoning. Mullis and Martin (2017) inform that:

> knowing, covers the facts, concepts, and procedures required by students, applying, focuses on the ability of students to apply knowledge and conceptual understanding to solve problems or answer questions, and reasoning, goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multistep problems. (p. 22)

Table 3 shows the distribution of assessment items by cognitive domain and item format.

Table 3

Distribution of Assessment Items by Cognitive Domain and Item Format in the 2011 TIMMS

Cognitive Domain	Multiple Choice	Constructed Response	% of Score Points
Knowing	43	30	39
Applying	34	41	41
Reasoning	16	21	20
Total	93 (50%)	92 (50%)	100

The TIMMS assessment differs from that of the CSEC in that it is conducted with Grade 8 students, while the CSEC examination is typically completed by students at the end of high school. Additionally, the TIMMS assesses four content domains- Number, Algebra, Geometry, Data and Probability, whereas for the CSEC, nine content domains are assessed. In this paper cognitive domains/dimensions and profiles are used interchangeably.

Overview of CSEC Mathematics Achievement in Jamaica

To provide a context for examining Jamaican public school students' mathematics achievement in each of the cognitive domains, this section provides an overview of their overall mathematics achievement from 2009-2019 (see Table 4) in the CSEC mathematics examination which is the main exit mathematics examination completed by Jamaican students at the end of high school.

Table 4 shows that over the period of 11 years from 2009-2019, on average, slightly over 50% of the students who wrote the examination did not obtain a passing grade. This suggests that after more than 11 years of formal, compulsory mathematics instruction, many students lacked the requisite mathematical knowledge and skills to function effectively in everyday life and to pursue jobs and educational opportunities that required a qualification in mathematics. This trend is consistent with many international jurisdictions, whose learners also struggle with mathematics (Fenanlampir et al., 2019; Nelson & Powell, 2018).

Student achievement as an educational concern and research focus is multi-dimensional and can be explored in varied ways. While the data presented in Table 4 provides a general summary of Jamaican students' mathematics achievement, as previously stated, deeper insights can be gleaned by focusing on different aspects of another layer of this achievement, the profile or cognitive dimensions. In Jamaica, this has not been the focus of previous empirical exploration, but Cato (2020) explored this among a sample of 370 students from the island of St. Vincent in the Caribbean using the 2017 May/June CSEC mathematics examination. This research differs from that of Cato (2020) in that it uses population data and includes multiple examination years. To date, research related to student mathematics achievement in Jamaica have only focused on students' overall grades (e.g., Crossfield & Bourne, 2017; George, 2020; George, 2013; Spencer-Ernandez & George, 2016).

Table 4

CSEC Mathematics Examinations Percentage Pass for Jamaican Public Schools From 2009-2019

Year	No. Sitting	Grades			Pass	
		I	II	III	No	%
2009	19,990	1,623	2,508	4,054	8,185	40.9
2010	20,742	2,029	2,876	4,366	9,271	44.7
2011	20,850	1,652	2,527	4,139	8,318	39.9
2012	23,729	1,909	2,583	4,398	8,890	37.5
2013	22,870	1,764	2,910	4,985	9,659	42.2
2014	23,351	2,955	4,015	5,993	12,963	55.5
2015	23,639	4,203	4,692	5,762	14,657	62.0
2016	23,993	3,063	3,123	5,270	11,456	47.7
2017	23,567	2,751	3,312	5,775	11,838	50.2
2018	22,214	2,793	4,705	5,347	12,845	57.8
2019	21,320	1,748	4,212	5,685	11,645	54.6
Average (%)		**10.8**	**15.2**	**22.6**		**48.5**

Note. Data from the Jamaica Ministry of Education

Another aspect of achievement that this research explores is that relating to students who are near proficiency levels or who are on the border of passing or failing high-stakes tests. This group of students have been given a variety of labels, such as bubble students or kids (McNeil, 2002), cusp children (Bradbury et al., 2021), borderline students (Amrein-Beardsley & Berliner, 2002) or near-passing students (Rothman & Henderson, 2011) and have been given considerable focus in practice (Minarechová, 2012). One reason for this emphasis is the significant impact that these students may have on a school's or district's composite scores on a high-stakes test (Minarechová, 2012). In this regard, Reback (2008) points out that in systems where examination pass rates are prioritised, borderline students would have the greatest impact on a school's performance measure. Another reason is the perception and research finding that enhanced provisions to assist this group in making gains in their achievement would allow them to pass high-stakes examinations, whereas this outcome would be unlikely for lower-performing groups (Hutchings, 2015; Marks, 2014; Reback, 2008).

Considering the deleterious impact of low or no mathematics qualifications on an individual's educational and job prospects, the focus on this borderline group is important. McMahon (2022) notes that there has been limited scholastic attention on the achievement of these students. Additionally, the research context, like many developing countries internationally utilise a minimum competency school accountability system that only include students' test scores via pass rates. Also, the pass rate on mathematics examinations has been historically low and resources are limited, therefore, there would be an interest and emphasis in improving the performance of students who are on the margin of passing by first exploring their examination performance in greater detail. McMahon (2022) adds that a focus on this research domain could provide educational stakeholders with insights that could assist them in more targeted interventions geared towards meeting the needs of this student group in a more targeted way.

In this research, students who obtained Grades III and IV would be borderline students. While the possible grades for the CSEC mathematics examinations range from I-VI, this research focuses on students who received Grades I-III, which are passing grades, and Grade IV, which is a borderline failing grade. Grades V and VI are therefore not included in the current data analysis but could be incorporated in future research. Students who obtained a borderline failing grade have not been the focus of previous research relating to cognitive dimensions in mathematics assessments.

Stemming from the aforementioned, this paper aims to add to the existing literature both globally and locally by investigating the profile dimensions in general and for the years 2015, 2016, 2018, 2019 as per students' examination grades and gender. In particular, this research addresses the following research questions:

1. (a) How does the profile (knowledge, comprehension, reasoning) performance of students with Grades I-IV compare?
   (b) What is the relationship between:
      (i) students' overall mathematics examination grade and each of the three profile grades?
      (ii) the knowledge and comprehension domains; the knowledge and reasoning domains; the comprehension and reasoning domains?
2. How do the profile grades bands (knowledge, comprehension, reasoning) of students with passing grades compare (i) by year (ii) in general?
3. How do the profile grade bands (knowledge, comprehension, reasoning) of students with a borderline failing grade (Grade IV) and a borderline passing grade (Grade III) compare?
4. Is there a statistically significant difference in the profile grades of students with passing grades by gender?
   H_o: There is no significant difference in the profile grades of males and females.

Cognitive Dimensions in General and by Gender

Since research concerning the cognitive dimensions as conceptualised in this research is limited, relevant research related mainly to the TIMMS assessment, which reports student achievement as per the cognitive domain, is reviewed in this section. For the TIMMS assessment data for 2011, 2015 and 2019, there does not appear to be a pattern in achievement related to the three cognitive domains of knowing, applying and reasoning. Based on the TIMMS 2019 mathematics assessment, Mullis et al. (2020) report that more countries had a weakness in the knowing domain than in the applying and reasoning domains. This contrasts with the 2011 assessment in which more countries demonstrated relative strengths in knowing mathematics (i.e., recalling, recognizing, and computing) than in applying mathematical knowledge and reasoning (Mullis et al., 2012). For the 2019 sitting, reasoning was reported to be a relative weakness for approximately 44% (n = 28) of the countries while applying was reported to be a relative strength for about 38% (n = 24). Mullis et al. (2012) informed that generally, the TIMSS 2011 participants with the highest achievement overall also had the highest achievement across the cognitive domains.

Kaleli-Yılmaz and Hanci (2016) investigated the cognitive domain components of student mathematics achievement in general and by gender of 652 eighth grade Turkish students (305 girls and 347 boys). They used items from the TIMMS 2011 mathematics test and reported that students performed best and worst on the applying and reasoning domains, respectively. Thomson (2006) who explored TIMMS 2003 data with a specific focus on Australian student performance found that for both years four and eight, students performed best in the reasoning domain. Furthermore, for Grades four and eight, respectively, achievement relating to the applying and knowledge domains were lower compared to the other domains. As it relates to the relationship between the cognitive domains and mathematics achievement, Pogoy et al. (2015) used TIMMS 2011 data across countries and found a large, significant, positive relationship between each of the three cognitive domains and mathematics achievement.

The present research aims to add to this existing empirical data by exploring assessment data related to cognitive domains across four years instead of quadrennially like the TIMMS or cross-sectionally (e.g., Kaleli-Yılmaz & Hanci, 2016) in order to investigate whether there are patterns of performance. This study also explores the achievement of students with passing and borderline fail grades to investigate the cognitive domain outcomes of students with different levels of mathematics achievement to glean insights from this analysis.

As it relates to gender differences related to the cognitive domain components of student mathematics achievement, Mullis et al. (2016) found, based on the 2015 TIMMS assessment, that the assessment results "show an advantage for girls in the Reasoning domain" (p. 125). In the 2019 assessment, "boys had higher average achievement than girls in many countries in the cognitive domains—31 countries in the knowing domain, 15 in the applying domain, and 28 in the reasoning domain. Girls had higher average achievement than boys in all three domains in Oman, the Philippines and South Africa (fifth grade)" (Mullis et al., 2020, p. 77). Mullis and her associates, however, did not indicate whether these differences were significant. Kaleli-Yılmaz and Hanci (2016) also explored the cognitive domain components of student mathematics achievement by gender and found that although girls outperformed their male peers on all cognitive domains (knowing, applying and reasoning), these differences were not statistically significant. This finding from Kaleli-Yilmaz and her associate aligns with research on general mathematics achievement and gender among high school students which have found no significant gender differences (Cimpian et al., 2016; George, 2022; Lindberg et al., 2010; Lubienski & Pinheiro, 2020). Exploring achievement as it relates to gender continues to be an important component of educational research considering the variability of findings to date (Else-Quest et al., 2010; Forgasz, 2012; Leder, 2012). The sample from Kaleli-Yılmaz and Hanci's (2016) study is from Turkey, and Jamaica has not yet participated in the TIMSS. Therefore, this research endeavours to add findings from another jurisdiction, the Caribbean, related to the important research focus of the cognitive domains.

Research Design and Methods

Data for the Study

The author obtained data on Jamaican public-school students' performance on the June sitting of the CSEC mathematics examination from 2015 to 2019 from the Ministry of Education [Jamaica]. However, the profile dimension data were not available for 2017 and so were not included in the analysis reported in this paper. The deidentified data for the June offering of the CSEC mathematics examinations were chosen for analysis because most candidates in high school complete their examinations within this period. Jamaica was chosen as the focus of this study because in the Anglo-Caribbean, it has the largest number of candidates participating in CSEC mathematics examinations annually which accounts for approximately 50% of the candidates examined. This substantial sample would be adequate in providing answers to the study's research questions and could also form the basis for generalising the findings to Jamaica and the wider Caribbean. Additionally, this study builds on recent previous research conducted in Jamaica relating to mathematics achievement (George, 2022; Spencer-Ernandez & George, 2016) in order to establish a research base on this critical research topic.

Public-school student data were used because it accounts for most of the students who sit for the CSEC mathematics examinations at the end of high school and was accessible to the author through the Ministry of Education [Jamaica]. Data from private institutions, however, were not available to the author. As per the data received, 168 schools entered candidates to sit for the CSEC mathematics examination. Of the 168 schools, seven schools did not have data for all of the years

(2015, 2016, 2018, 2019) that were relevant to this study. The data for these seven schools, which corresponded to 786 students, were removed. Therefore, the data analysis centred on the data from 161 schools. The 161 schools accounted for 95.8% of the performance data for public school students for four years 2015, 2016, 2018 and 2019.

The Performance Criterion

As previously stated, the CXC reports grades from the CSEC mathematics examinations on a scale of I to VI. However, grades I – III are considered to be passing grades. The CXC also reports profile dimensions (knowledge, comprehension, and reasoning) on a scale of A-F (see Table 1).

The Sample

The research sample was taken from candidates (*N* = 89,719) from 161 public educational institutions who sat for the CSEC mathematics examinations in Jamaica in 2015, 2016, 2018 and 2019. For two of the four research questions (Questions two and four), data from students who passed the CSEC mathematics examinations by obtaining Grades I-III were used. This represented 56% of the students who sat for the CSEC mathematics examinations for the four years being considered in this study. For research question three, the sample comprised 21,842 and 19,700 students who obtained a Grade III (borderline pass) and Grade IV (borderline fail), respectively. Table 5 shows the number of students in and gender of the sample for each of the four years under consideration.

Table 5

Sample Demographics

Gender	Year				Total	
	2015	**2016**	**2018**	**2019**	**No**	**%**
Students who obtained grade I-III						
F	8,639	6,684	7,570	6,747	29,640	33.0
M	5,988	4,761	5,182	4,674	20,605	23.0
Total	14,627	11,445	12,752	11,421	50,245	56.0
Students who obtained grade IV						
F	2,032	2,146	3,690	3,474	11,342	12.6
M	1,570	1,604	2,696	2,488	8,358	9.3
Total	3,602	3,750	6,386	5,962	19,700	22.0

Data Analysis

Microsoft EXCEL 2016 and the Statistical Package for the Social Sciences (SPSS 21) were used for the data analysis for this research. Each Roman numeral and letter grade was first converted to a number (see Table 6) and then the requisite statistical analyses, as per the different research questions, were undertaken.

Table 6

Grade and Profile Transformation Summary

Grade	Equivalent	Profile	Equivalent
VI	1	F	1
V	2	E	2
IV	3	D	3
III	4	C	4
II	5	B	5
I	6	A	6

The transformation of Roman numeral grades to a numeric form has been adopted in previous research for investigating similar data from CXC (e.g., George, 2020; Griffith, 2013). While the use of large-grain data (grades/profiles) may be seen as a limitation, the unavailability of students' raw scores made this the most pragmatic approach to the data analysis. In undertaking the analysis, the author considered two options in interpreting the profile grades. The profile data could be considered to be continuous in the same way that data from a Likert scale is widely treated although this is debated (Carifio & Perla, 2008; Wu & Leung, 2017) and so parametric/non-parametric tests could be applied. It could also be conceptualised as data that is ranked, and therefore can be considered to be interval data or nominal data, and so Chi-Square tests would be performed. The author performed the analyses related to each of the previously mentioned considerations and found that for each research question, the conclusion for the statistical analyses done was the same. To answer research questions one(a), two, and three, the current author found frequencies (totals and percentages) and carried out descriptive statistical analyses (means, modes, and standard deviations). For research question one(b), a Spearman's rank-order correlation was performed, while for the fourth research question, an Independent Samples *t*-test was carried out.

In organizing and reporting the results to research questions two and three which focused on profile dimension grade bands, after first examining the profiles individually and across the four years, the number of students who obtained profiles A and B were combined to form a profile grade band for top performance. The number of students who obtained profiles C and D, as well as E and F was combined individually, to form a profile grade band for satisfactory and unsatisfactory performance, respectively. This approach allowed for more efficient reporting of results since there are six profiles and grades.

Results

Research Question 1a: How does the profile (knowledge, comprehension, reasoning) performance of students with Grades I-IV compare?

Table 7 presents summary statistics (mean, mode, and standard deviation) for the individual grades I-IV. An inspection of Table 7 shows that the students with the highest overall achievement (Grades I and II) also demonstrate through the means, the highest achievement across the three profiles.

Table 7

Summary Statistics for Students with Grades I-IV

Summary Statistics	Knowledge	Comprehension	Reasoning
Overall for Students with Passing Grades			
N	50,245	50,245	50,245
Mean	5.16	4.70	4.36
Mode	5.00	4.00	4.00
Std. Deviation	0.76	0.90	1.01
Grade IV			
N	19,700	19,700	19,700
Mean	3.70	2.90	2.48
Mode	4.00	3.00	2.00
Std. Deviation	0.61	0.55	0.56
Grade III			
N	21,842	21,842	21,842
Mean	4.53	3.88	3.53
Mode	5.00	4.00	4.00
Std. Deviation	0.54	0.47	0.52
Grade II			
N	16,623	16,623	16,623
Mean	5.40	4.91	4.50
Mode	5.00	5.00	5.00
Std. Deviation	0.52	0.39	0.58
Grade I			
N	11,780	11,780	11,780
Mean	5.99	5.91	5.72
Mode	6.00	6.00	6.00
Std. Deviation	0.10	0.29	0.46

For students who obtain the highest grade possible, which is a Grade I, the most common grade for each of the three profiles is an A (Mode = 6), while for students who obtain a Grade II it is a B (Mode = 5). The result is more nuanced for students who perform less well by obtaining a Grade III or IV. In relation to the former group, for the knowledge profile, the most common profile grade is B, while for comprehension and reasoning it is a C. For students who attain a Grade IV, only the knowledge profile has a mode of 4 (profile grade C) which represents a fairly good performance. For the comprehension and reasoning profiles, the performance is moderate and weak (profile grades D and E), respectively. For students with passing grades (Grades I-III), the modal descriptor for the knowledge profile is good (Mode = 5) while for comprehension and reasoning it is fairly good (Mode = 4). This suggests that students that pass the CSEC mathematics examination generally perform very well as it relates to the three profile or cognitive dimensions.

In addition, for all grade levels (I-IV), the knowledge and reasoning profiles have the highest and lowest means, respectively. This suggests that students generally perform best in the knowledge profile, followed by comprehension and then reasoning, in descending order. For the standard deviation, this observation is reversed. This means that for each of the three profiles, the students who obtain Grades I and IV appeared to be the most and least homogeneous, respectively, in terms of achievement. The standard deviation for knowledge, comprehension, and reasoning, respectively (0.76, 0.90, 1.01) suggest that there is greater homogeneity in profile performance for the knowledge profile in comparison to that of comprehension and reasoning.

Research Question 1bi: What is the relationship between students' overall mathematics examination grade and each of the three profile grades?

For students with passing grades (Grades I- III), a Spearman's rank-order correlation was run to determine the relationship between students' overall grade and their profile grade. Results of the Spearman correlation indicated that there is a strong, positive correlation between the overall and profile grade for knowledge, comprehension, and reasoning, which were all statistically significant ($rs(50{,}243) = .802$, $p <= .001$); ($rs(50{,}243) = .900$, $p <= .001$); ($rs(50{,}243) = .839$, $p <= .001$), respectively.

Research Question 1bii: What is the relationship between the knowledge and comprehension domains; the knowledge and reasoning domains; the comprehension and reasoning domains?

The relationship between profiles was also explored. The analyses found that there was a strong, positive, statistically significant correlation between the profile grades (knowledge and comprehension; comprehension and applying; comprehension and reasoning. The knowledge profile was more strongly correlated to the comprehension profile than the reasoning profile ($rs(50{,}243) = .739$, $p <= .001$); ($rs(50{,}243) = .680$, $p <= .001$), respectively. The comprehension profile was more strongly correlated to the reasoning profile than that of knowledge ($rs(50{,}243) = .764$, $p <= .001$); ($rs(50{,}243) = .739$, $p <= .001$), respectively.

Research Question 2: How do the profile grades bands of students with passing grades compare (i) by year (ii) in general?

Figure 5 presents the profile (knowledge, comprehension, reasoning) grade bands (A-B, C-D, E-F, as applicable) for students who obtained passing grades of I, II, and III across each of the years 2015 - 2019. Every student receives a grade that ranges from A-E for each of the three profile dimensions. The profile grade bands A-B, C-D and E-F indicate top, satisfactory, and unsatisfactory performance, respectively. Therefore, for 2019 for example, for all students who passed the CSEC mathematics examination for the Knowledge profile 66.5% of the cohort obtained a profile grade of A or B, while 33.5% received a grade within the C-D band. For the Comprehension profile, 51% of the students who obtained Grades I-III, received a grade within the A-B band whereas 49% of these students received a profile grade of C or D.

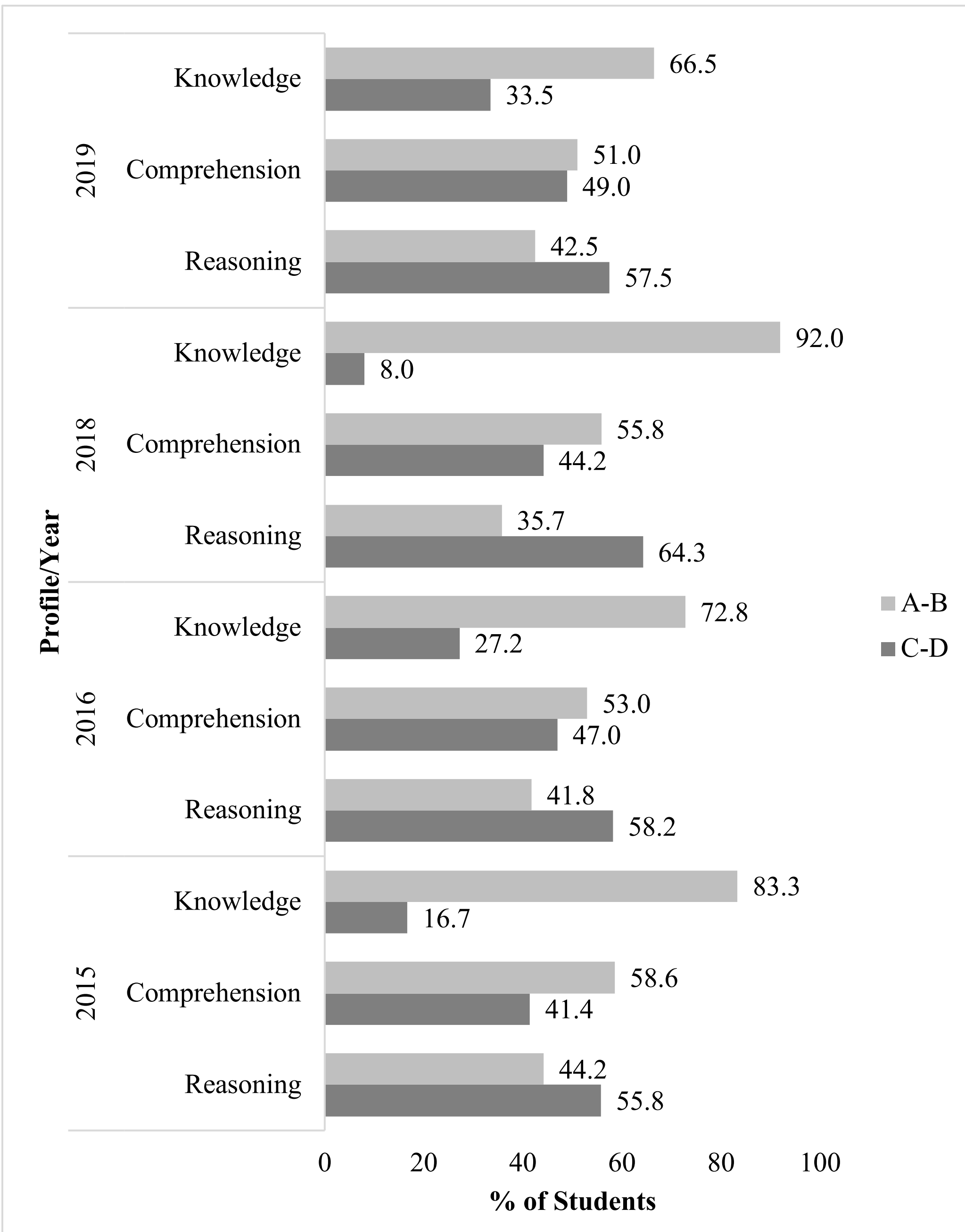
Figure 5 shows that over the years 2015, 2016, 2018 and 2019, there appears to be a regular pattern of grade band attainment as it relates to each of the profiles. For each of the four years under consideration, a close examination of the top band performance for each profile reveals that the knowledge profile has the highest percentage of students obtaining A and B grades, followed by the comprehension profile and then the reasoning profile with the smallest proportion of students exhibiting top band performance. This trend is reversed for students who performed satisfactorily by attaining profile grades C and D. The reasoning profile has the highest proportion of students and that relating to knowledge, the least proportion.

Furthermore, when profile performance bands are analysed across the four years, for the knowledge and comprehension profiles, a larger percentage of students obtained A and B grades than

C and D. For the knowledge profile, the average percentage of students who obtained A-B and C-D profile grades, respectively, was 79% and 21%, while for the comprehension dimension it was 55% and 45%. This difference was therefore, on average, substantially larger for the knowledge dimension (58%) than that of comprehension (10%). This suggests that for the knowledge profile, many more students achieved top band grades than for the comprehension profile.

Figure 5

Student Performance as Per Profiles for 2015, 2016, 2018, 2019

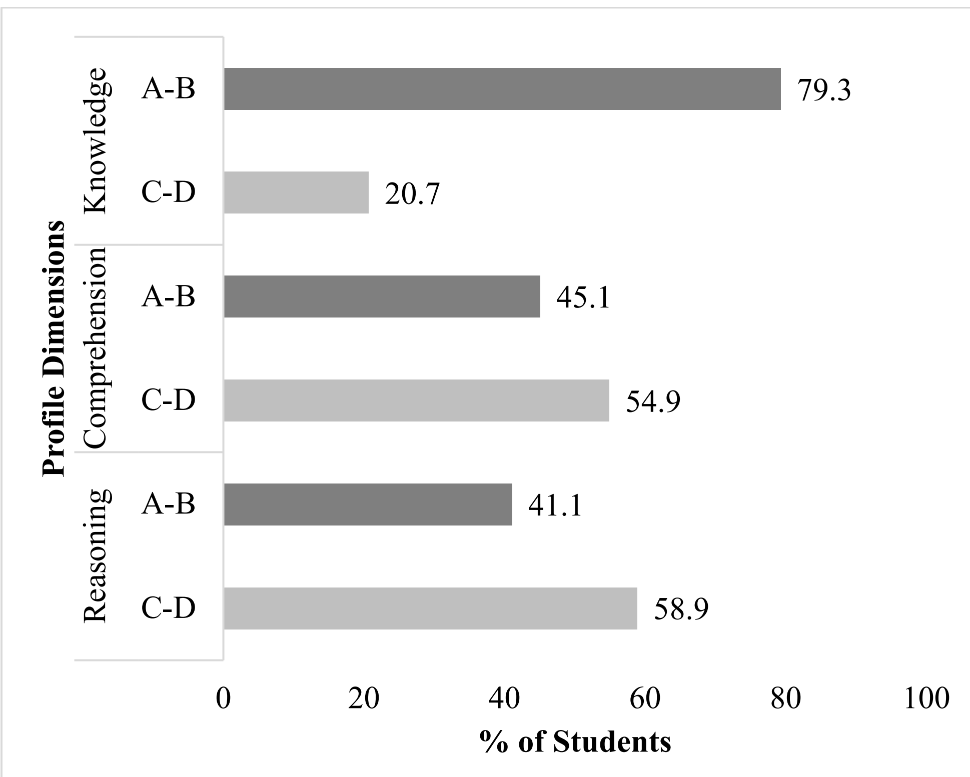


For the reasoning profile, this observed pattern in the knowledge and comprehension profiles was reversed with a larger percentage of students obtaining C and D profiles than A and B, for each of the four years. On average, across the four years, 59% of students obtained profiles within the satisfactory band (C-D) and 41% in the top band. For both the knowledge and comprehension dimensions, the percentage of students who obtained A-B and C-D profiles, respectively fluctuated from year to year. For the A-B band, there was a decrease from 2015-2016 (knowledge: -10; comprehension: -6) and 2018-2019 (knowledge: -26; comprehension: -5), and then an increase from 2016 to 2018 (knowledge: +19; comprehension: +3). For the reasoning profile, there was a small decrease in the percentage of students who obtained A-B profiles, year on year from 2015 to 2018 then a slight increase (+7) from 2018-2019. No student with a passing grade received E and F profile grades.

The pattern observed by year also holds for the entire data set. Figure 6 shows the percentage of students who achieved the profile bands A-B and C-D for each of the three profiles.

Figure 6

Student Performance as Per Profiles

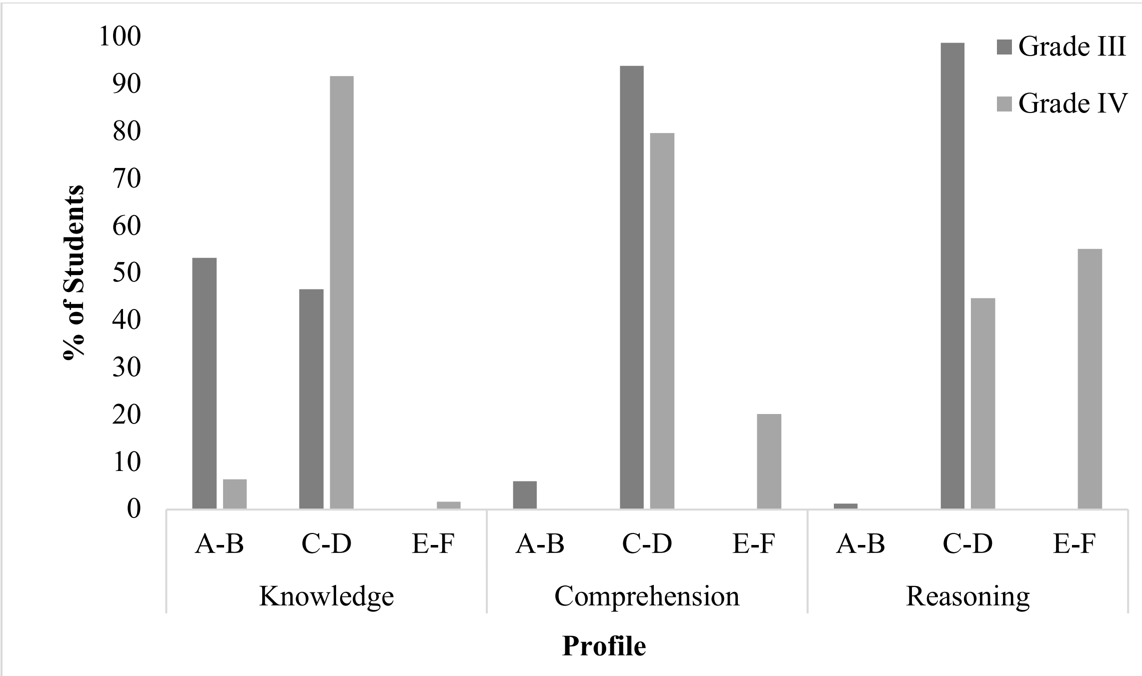


Research Question 3: How do the profile grade bands (knowledge, comprehension, reasoning) of students with a borderline failing grade (Grade IV) and a borderline passing grade (Grade III) compare?

Figure 7 shows the percentage of students with a borderline failing grade (Grade IV) and a borderline passing grade (Grade III) who attained different grade bands (A-B and C-D) for the knowledge, comprehension, and reasoning profiles.

Figure 7

Distribution of Students as per Profile Bands for Knowledge, Comprehension, and Reasoning



For the knowledge profile, the performance of students who obtain a Grade IV is substantially weaker than those who receive a borderline pass of Grade III. While for the former group, only 6.4% of students performed within the top band (A-B). For the latter, A-B were recorded for 53.3% of the students. The modal profile band for students attaining Grades III and IV, was A-B and C-D, respectively. Knowledge is the only profile for which students receiving a Grade IV had a top band (A-B) performance.

For the Comprehension profile, the C-D band, which is aligned to a satisfactory performance was the modal band of performance for both students obtaining a Grade III and IV. For students obtaining a Grade III, there was a large decrease in the percentage of students obtaining A and B profiles when compared to the Knowledge profile (52.4 versus 6.4). For these students, this trend extended to the reasoning profile, where only 1.2% of students achieved profile grades of A and B. For students who received a Grade IV, the percentage decrease (-6.3%) from the Knowledge profile to that of Comprehension was much smaller than that of the students who obtained Grade IIIs, perhaps because the percentage of students who obtained A and B profile grades for the former group was extremely small (6.4%). For the reasoning profile, no student who obtained a Grade IV had an A or B profile grade and the modal grade band was E-F. For students achieving a Grade III, for the reasoning profile almost all of the students (98.8%) achieved C and D profile grades, and no students obtained E and F grades. It appears that for both grade levels, students performed best and worst in the Knowledge and Reasoning profile, respectively.

Research Question 4: Is there a statistically significant difference in the profile grades of students with passing grades by gender?

Ho: There is no significant difference in the profile grades of males and females. An independent-samples *t*-test was conducted to compare the students' CSEC mathematics performance as per profiles by gender. For the knowledge and comprehension profiles, the mathematics performance of males was statistically significantly lower than females, however, for the reasoning profile the student gendered performance difference was not statistically significant. The gendered student performance for each profile is as follows: Knowledge - males (M = 5.1446, SD = 0.7526) and females (M = 5.1754, SD = 0.7567) $t(50243) = -4.495$, $p = .000$; Comprehension - males (M = 4.6542, SD = .89920) and females (M = 4.7252, SD = .90129) $t(50243) = -8.706$, $p = .000$; Reasoning - males (M = 4.3581, SD = .98962) and females (M = 4.3690, SD = 1.01929) $t(50243) = -1.191$, $p = .234$. The effect size related to the knowledge and comprehension profiles respectively were $d = 0.04$ and 0.08. According to Cohen et al. (2018), this effect size is tiny and a 4% and 8% difference was found between the two groups (males and females) for the knowledge and comprehension profiles respectively. This suggests that the difference between the groups, although statistically significant, was trivial.

Discussion

The data stemming from four years of CSEC mathematics examination administration and approximately 70,000 students saw that the students with the highest overall achievement also had the highest achievement across the profile domains. This is wholly in agreement with Mullis et al. (2012), who reported results related to the TIMMS mathematics component. This research also found a regular pattern of student profile performance for each of the grade levels (I-IV) and across the four years under consideration. Students performed best in the knowledge, then comprehension, and then reasoning profiles. Considering that a different cohort of students was assessed in each of the four years under study, this regularity in performance is interesting and noteworthy. This finding aligns with Mullis et al. (2012) who reported that for the 2011 administration of the TIMMS more countries demonstrated relative strengths in the knowing cognitive domain than the other two cognitive domains. It also agrees with Kaleli-Yılmaz and Hanci (2016) who found that 652 eighth grade Turkish students performed worst in the reasoning domain. Cato (2020) who investigated the CSEC mathematics results for a Caribbean sample also reported similar findings. However, results contrast with Kaleli-Yılmaz and Hanci (2016) who reported that their sample performed best in the applying domain. This finding from the present research also deviates from that of the 2019 offering of the TIMMS assessment for mathematics which indicated that students in more countries performed better in applying and reasoning than in knowing (Mullis et al., 2020). For an Australian cohort in the mathematics component of the TIMMS 2003, Thomson (2006) also found divergent results to the present study in that for both Years 4 and 8, comparatively, achievement in reasoning exceeded that of the other domains. For Year 4 and 8, respectively, achievement in applying and knowledge were lower compared to the other domains.

This study also found that the cognitive domains such as knowledge, comprehension, and reasoning have large, significant, positive relationships to mathematics achievement. This finding coincides with Pogoy et al. (2015), whose research related to the TIMMS 2011 data across countries. The author of this paper recognises that comparing CSEC student performance across multiple consecutive years with that of the TIMMS mathematics assessment that takes place every four years has its limitations. However, it is noted that in contrast to the consistent trend in performance across and within each of the four years that the analysis for this study revealed, for the TIMMS data, there appears to be a fluctuation in the cognitive domain that students in most countries perform best and worst in. For example, in 2011, candidate performance was superior in the knowing domain. This is in alignment with the present research whereas in 2019 it was the applying and reasoning domains. It may be that countries use the assessment results to improve student profile performance for the next TIMMS, hence the observed variation.

There are several explanations for the finding that students performed best in the knowledge, then comprehension, and reasoning profiles. The present author proposes several such hypotheses next. This finding may suggest, consistent with Mullis et al. (2003), that the performance in the knowledge profile dimension serves as the gatekeeper for students' ability to comprehend and reason. In the comprehension dimension, students apply procedures, concepts, and facts to routine problems. This dimension is akin to the applying dimension in the TIMMS. Mullis and Martin (2017) state that for students to engage optimally within this dimension, they need to possess knowledge of facts, concepts, and procedures, which they can then use in solving familiar mathematical problems involving different contexts. This indicates that knowledge may act as a prerequisite for application and reasoning (Pogoy et al., 2015; Thomson, 2006). This view is also supported by the National Council of Teachers of Mathematics (2014) who states that learners should "acquire conceptual knowledge as well as procedural knowledge, so that they can meaningfully organize their knowledge …and transfer and apply knowledge to new situations" (p. 9). The current finding and these explanations could lend support to the argument that was briefly introduced earlier regarding whether Bloom's taxonomy was hierarchical or not. They appear to support the notion that the taxonomy may be hierarchical, but future research is needed to further delineate this. This contrasts with Kilpatrick et al. (2001) who appear to prioritise reasoning over knowledge and comprehension although acknowledging that all cognitive dimensions are important and mutually influential. This research provides empirical evidence to support the previously mentioned assertion of Kilpatrick et al. (2001) and is significant in that it adds to the existing literature, new ways in which the profiles or cognitive domains are related to each other. Reasoning has also been found to help students see connections between their present and prior knowledge thereby augmenting the prospect of retaining and recalling this knowledge (Ball & Bass, 2003; National Council of Teachers of Mathematics, 2009). Consequently, it is plausible that the group of students in this research who are competently able to reason are using this to boost their knowledge and comprehension scores.

Other explanations related to the research context may explain this observed pattern that the students who participated in this research performed best in the knowledge, then comprehension, and reasoning profiles. In Jamaica, like international jurisdictions, both policy and curricular documents emphasise that mathematics teaching should aim to develop analytical, reasoning, and critical thinking skills (Jeannotte & Kieran, 2017; Ministry of Education, 2013, 2017). However, the yearly examination reports, based on the performance of test-takers in the CSEC mathematics examinations, consistently note that in general, candidates' reasoning and problem-solving skills were wanting and made recommendations relating to instruction to teachers in that regard (Caribbean Examination Council, 2020). Further to this, in Jamaica, the current curriculum in use in Grades 7-9 was recently developed based on the finding in early reform reports that the curriculum and related teaching were too focused on the retention of factual knowledge and not on the development of transferable skills and competences (Ministry of Education, 2016). Consequently, it is not wholly surprising that the reasoning cognitive domain would be the weakest since it has been given the least emphasis in practice. Stigler and Hiebert (2009) describe teaching as a cultural activity which "evolve(s) over long periods of time in ways that are consistent with the stable web of beliefs and assumptions that are part of the culture" (p. 87). Furthermore, they point out that the instructional approaches that most teachers apply to their practice, closely resemble those that were used by their own teachers when they were students and in earlier times as well. Based on the aforementioned, it is reasonable to conclude that the teaching methods that present teachers tasked with developing students' reasoning and problem-solving skills were exposed to would have seldom focused on the development of reasoning or critical thinking skills. Instead, they would very likely have mainly centred on the acquisition of knowledge and skills. This would suggest that it is plausible that many present-day Jamaican teachers would have weak reasoning skills in mathematics and so would likely find it difficult to facilitate their development in mathematics lessons. Previous research (e.g., Mata-Pereira & Da Ponte, 2017; Richland et al., 2012) has linked the development of students' reasoning capabilities directly to the mathematics instruction that they receive. In this regard, Brodie (2009) states that "mathematics reasoning is challenging to learn and teach. For teachers who learned mathematics and learned to teach mathematics in traditional ways, the challenges are enormous" (p. 3). Based on this contextual consideration, the finding that the reasoning cognitive domain would be the weakest is reasonable.

These new findings are wholly plausible and in line with Camilli and Dossey (2019) who note that "educational systems of different participants have distinct profiles of mathematics achievement" (p. 1). Consequently, the present author acknowledges that the findings related to student profile performance, and the relationship between achievement grades and that related to profile/cognitive domains, presented here do not constitute a typical picture of high school mathematics achievement in general. For a different sample of high school students, other patterns of performance could emerge. The findings are noteworthy since this study reveals a regular pattern of profile performance across achievement grade levels and multiple years of the administration of the high stakes, large-scale mathematics examination. This has not been reported in previous research or large-scale assessments in mathematics (e.g., Kaleli-Yılmaz & Hanci, 2016; Mullis et al., 2020). In this regard, the findings from this study add to the mixed findings related to the student mathematics performance as per cognitive domains. The present findings also provide new evidence relating to the potential impact of the knowledge domain on the other two profiles/cognitive dimensions and the relationship between the profiles and mathematics achievement.

The disaggregation by years of the profile grade band performance of students who obtained passing grades revealed that for each of the years being considered in this research (2015, 2016, 2018, 2019), for the knowledge and comprehension profiles, a larger percentage of students obtained A and B grades than C and D. This pattern is reversed for the reasoning profile. This is a cause for concern for stakeholders in education in Jamaica since the island, for the first time, will participate in the next administration of the Programme for International Student Assessment (PISA) in 2022 (Hunter, 2019). This large-scale, international assessment measures 15-year-old students' ability to apply their knowledge and skills related to reading, mathematics, and science in real world problem solving situations (*Programme for International Student Assessment*, 2021). A major focus of the mathematics component of this assessment is mathematical reasoning due to the central role it plays in mathematical literacy in the 21st century (OECD, 2019). While the CSEC mathematics examination data involves 16-year-old students and the PISA 15-year-olds, the CSEC examination data suggests that Jamaican students may struggle with the mathematics component of the PISA. The aforementioned presents an example as to how a country's national data can be used to provide insights that could improve educational programs, establishments, or systems, enhance practices in education, and support the learning processes of individuals (Koeppen et al., 2008) and ultimately help prepare students for an international assessment. This approach if not yet utilized can be replicated in other jurisdictions.

In the 2016-17 academic year, a new national curriculum that emphasises the development of higher order thinking skills such as reasoning in mathematics, was implemented in Jamaican primary and lower secondary schools. The previously mentioned findings could form the basis on which to monitor changes in achievement in the cognitive domains, post implementation of the new curriculum. Changes in achievement may reflect the changes made in the national curriculum in Jamaica. This confirms Mullis et al. (2012) and serves as yet another example of the key role that assessment can play within educational reform. Future research could explore achievement in the cognitive domains post this implementation.

This research found that there is a significant gendered performance gap in favour of the females as it relates to the three profile dimensions of knowledge, comprehension, and reasoning. This finding coincides with previous work related to CSEC mathematics achievement in Jamaica (George, 2022) and international research relating to mathematics assessments (e.g., Cimpian et al., 2016;

Lindberg et al., 2010; Lubienski & Pinheiro, 2020). Gender disparity in education is unfavourable from the perspectives of educational or social justice and equity. For this reason, among others, the Sustainable Development Goals, an international plan of action proposed by The United Nations and adopted by many countries including Jamaica, as a global agenda aims to "eliminate gender disparities in education" (UNESCO, 2021). As a result, there is a research imperative to uncover the presence of these disparities and to decrease them, with the aim of ultimately eradicating their occurrence (Evans, 1997; OECD, 2015; UNESCO, 2017). In this regard, the findings of this study related to gender differences favouring females are important. Mathematics plays a crucial role in an individual's scholastic and vocational success. Also, it is an essential component in fields related to Science, Technology, Engineering and Mathematics (STEM) which are instrumental in advancing innovation and competition in the 21st century (Marginson et al., 2013; Waite & McDonald, 2019). It also promotes financial productivity and development for a country's citizens, industries, and businesses (Ukpata & Nancy, 2012). Evans (1997) notes that gender differences in education highlight human capital issues since developing countries need educated persons with a wide range of knowledge, skills, and competencies.

Although this research found significant gender differences in mathematics achievement the related effect sizes were small and so from a practical perspective this difference is negligible. This difference is also not as remarkable as has been found in other Caribbean territories, such as Trinidad & Tobago (OECD, 2016) who reported the largest gender gap in favour of females in the 72 countries who participated in the 2015 PISA mathematics component. Notwithstanding the small effect size, Stoet et al. (2016) asserted that the gender disparity is important and "one of the main psychological and educational research aims is to determine which factors can explain the sex difference in mathematics performance" (p. 2). Future research could seek to explore this issue in greater depth. The present finding also contrasts with Stoet and Geary (2013) who analysed data from four administrations of PISA and found that in most countries, boys scored higher than girls in mathematics assessments.

Conclusion

Mullis et al. (2000) asserted that "it is important that educators, curriculum developers, and policy makers understand what students know and can do in mathematics and what areas, … need more focus and effort" (pg. 57). In this regard, this study explored different facets of students' mathematics achievement as per three cognitive domains, *knowledge, comprehension, and reasoning,* to provide educational stakeholders with a more fulsome and deeper understanding of this achievement. In particular, it investigated student profile performance in the CSEC mathematics examinations as per overall achievement grades (I-IV), across four years (2015, 2016, 2018, 2019), and by gender. It also explored the relationship between the profiles and mathematics achievement.

This study is significant in several ways. First, it found a new and regular pattern of cognitive domain performance across multiple years for different cohorts of students. Students performed best in the knowledge, then comprehension, and reasoning profiles. The Jamaican mathematics policy and curricular, in alignment with current research and international practices within mathematics education, emphasise that the development of reasoning skills is a central focus of instruction. This research shows that this goal is not currently being attained. Students appear to have a relative weakness in reasoning which signifies to teachers and other educational stakeholders, such as teacher educators, an area that requires more effort and focus as per Mullis et al. (2000).

Also, students with a borderline failing grade had notable deficiencies in the knowledge domain and that the homogeneity of student profile performance decreased with each grade level from I-IV. This indicates that perhaps a wider variety of resources and/or pedagogical approaches including the use of differentiated instruction may be needed for engaging with lower performing students than for high performing students. Teachers could aim to improve students' performance in the comprehension/applying domains by investing instructional time in improving students' knowledge of mathematical facts, concepts, and procedures. This is particularly important for low performing students whose low performance in the knowledge domain appears to impact their performance in the other two domains.

This research also found that there is a strong, positive relationship between achievement as per the three cognitive domains and general mathematics achievement. Therefore, a focus on strengthening student mathematics achievement in the three cognitive dimensions is likely to result in improved student achievement. For Jamaica, this would be a key educational imperative considering the longstanding poor achievement of students in mathematics. This could be prioritised especially in the earlier grades of high school so that adequate time can be spent on facilitating the development in all the cognitive domains. In order to implement the aforementioned recommendations relating to teaching, there is the need for training of teachers in developing students' higher order thinking skills. This could be undertaken by teachers' training institutions or the central Ministry of Education.

This research focused on the cognitive domains related to students' mathematics achievement. Since performance outcomes can be classified with regard to content and cognitive domains, future research could centre on student achievement, as per the content domains assessed in the CSEC mathematics examination. That research could be undertaken using the raw scores of students to overcome a limitation of the current research, that is, to analyse overall grades. Since teachers are expected to facilitate the development of students' competencies regarding the cognitive domains, future research could also explore mathematics teachers' existing knowledge and skills relating to the three cognitive domains to ascertain whether and where gaps exist that need to be addressed.

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Appendix A

Figure 1

Specimen Caribbean Secondary Education Certificate Mathematics Question 2 (Caribbean Examinations Council, 2015, p. 87)

(a) Simplify: $p^3q^2 \times pq^5$ **(1 mark)**

(b) If $a * b = 2a - 5b$, calculate the value of

(i) $3 * 4$ **(1 mark)**

(ii) $(3 * 4) * 1$ **(1 mark)**

(c) Factorize completely: $3x + 6y - x^2 - 2xy$ **(2 marks)**

(d) A string of length 14 cm is cut into two pieces. The length of the first piece is x cm. The second piece is 5 cm **longer** than half the length of the first piece.

(i) State in terms of x, the length of the second piece of string. **(1 mark)**

(ii) Write an expression, in terms of x, to represent the TOTAL length of the two pieces of string. **(1 mark)**

(iii) Hence, calculate the length of the first piece of string. **(2 marks)**

Total 9 marks

Appendix B

Figure 2

Profiles Assigned to the Solutions to Question 2 (Caribbean Examinations Council, 2015, p. 99)

	PROFILES			Total
	K	C	R	
(a) $p^3q^2 \times pq^5 = p^4q^7$	1			
	1	**–**	**–**	**1**
(b) $a * b = 2a - 5b$				
(i) $3 * 4 = 2(3) - 5(4) = 6 - 20 = -14$		1		
(ii) $(3 * 4) * 1 = (-14) * 1 = 2(-14) - 5(1) = -28 - 5 = -33$			1	
	–	**1**	**1**	**2**
(c) $3x + 6y - x^2 - 2xy$				
$= 3(x + 2y) - x(x + 2y)$		1		
$= (3 - x)(x + 2y)$	1			
	1	**1**	**–**	**2**
(d) (i) Length of 2nd piece $= \frac{1}{2}x + 5$		**1**		
(ii) Sum of two lengths $= x + \frac{1}{2}x + 5 = \frac{3}{2}x + 5$	**1**			
(iii) $= \frac{3}{2}x + 5 = 14$; $= \frac{3}{2}x = 9$; $x = 6$			**1**	
Length of 1st piece is 6 cm		**1**		
	1	**2**	**1**	**4**
TOTAL	**3**	**4**	**2**	**9**

Appendix C

Figure 3

Specimen Caribbean Secondary Education Certificate Mathematics Question 4 (Caribbean Examinations Council, 2015, p. 89)

(a) The line BC passes through the point A(−5, 3) and has a gradient of $\frac{2}{5}$.

(i) Write the equation of the line BC in the form $y = mx + c$. **(2 marks)**

(ii) Determine the equation of the line which passes through the origin and is perpendicular to the line BC. **(2 marks)**

(b) The functions f and g are defined as:

$$f(x) = \frac{2x - 1}{x + 3} \qquad g(x) = 4x - 5$$

(i) Determine f g(3). **(2 marks)**

(ii) Derive an expression for $f^{-1}(x)$. **(3 marks)**

Total 9 marks

Appendix D

Figure 4

Profiles Assigned to the Solutions to Question 4 (Caribbean Examinations Council, 2015, p. 103)

	PROFILES			Total
	K	C	R	
(a) $A(-5, 3);\ m = \frac{2}{5}$				
(i) Equation of line BC is $y - 3 = \frac{2}{5}(x + 5)$		1		
i.e. $y = \frac{2}{5}x + 5$		1		
(ii) Gradient of line perpendicular to BC is $-\frac{5}{2}$	1			
Equation of line through (0, 0) is $y = -\frac{5}{2}x$			1	
	1	2	1	4
(b) $f(x) = \frac{2x-1}{x+3};\ g(x) = 4x - 5$				
(i) $g(3) = 12 - 5 = 7$	1			
$\therefore\ fg(3) = \frac{14 - 1}{7 + 3} = \frac{13}{10}$			1	
(ii) $\frac{2(f^{-1})-1}{(f^{-1})+3} = x$			1	
$2(f^{-1}) - 1 = xf^{-1} + 3x$				
$f^{-1}(2 - x) = 3x + 1$		1		
$f^{-1} = \frac{3x + 1}{2 - x}$	1			
	2	1	2	5
TOTAL	3	3	3	9