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Interdisciplinary Teaching: Solving Real-Life Physics Problems through Mathematical Modelling

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ABSTRACT

Physics and mathematics represent closely intertwined fields, wherein physicists employ mathematical modeling to address intricate problems. A challenge encountered by physicists involves bridging conceptual understanding with mathematical equations, a task that educators can facilitate by supporting students in navigating these two realms of comprehension. Mathematical modeling has exhibited potential in assisting students in recognizing that the domains of physics and mathematics are not insurmountably complex. The present study investigated the capability of science preservice teachers (PSTs) enrolled in an introductory physics course to resolve real-life physics problems by adhering to the stages of mathematical modeling. Data were gathered through the Interdisciplinary Modeling Eliciting Activity, allowing students to collaboratively discuss problems and devise solutions. Analysis was executed utilizing the interdisciplinary mathematical modeling (IMM) framework. The activity provided an inclusive platform for all students, including those who typically remained reticent during classes, to actively participate in group discussions and articulate their ideas. Despite the successful navigation of the problem with the guidance of the IMM framework, groups encountered challenges in certain tasks such as parsing/grouping and generating a context. Overall, the study demonstrated promise in augmenting PSTs' enthusiasm for physics and enhancing their comprehension of mathematical models within the discipline.

Keywords: interdisciplinary mathematical modelling (IMM); science preservice teachers; linear motion

Introduction

Every teacher should possess subject-specific and general competencies for the teaching profession, encompassing knowledge, skills, and attitudes to support their development in their respective fields. Among the subject-specific competencies applicable to science teachers, scientific, technological, and social development competences are essential. The ability to develop students' problem-solving skills is considered a crucial competence expected from teachers, particularly science educators. Therefore, science teachers are expected to impart awareness about potential solutions to their students' daily life problems, which, in turn, necessitates the teachers to possess such skills themselves. Mathematical modeling can provide valuable experience in the development of this competence.

From a mathematical perspective, mathematical modeling plays a significant role in generating solutions to problems faced in daily life. Generally, mathematical modeling includes two subprocesses: developing mathematical solutions and interpreting these mathematical solutions in a real-life context (Borromeo-Ferri, 2006; Lesh & Doerr, 2003). Mathematical models are tools that contain abstractly taught mathematical concepts used to explain real-life situations. For example, the trigonometric functions used in selecting seats to obtain the best screen view in a cinema are mathematical models. Similarly, a watch repairman who notices that the pendulum of a pendulum clock swings slower than it should understands concepts such as gravitational acceleration, period, and function, and uses this information during repair, which is an example of mathematical models.

Mathematical modeling is the process of mathematically expressing a real-life situation that poses a problem and explaining it using mathematical models (Berry & Houston, 1995). However, teaching and learning mathematical modeling can be explained by the formation of different understandings beyond this perspective. Modeling can be classified based on its intended use and the approaches used to handle mathematical modeling in the classroom. Mathematical modeling can be adopted as a tool when used to teach a concept and as an objective when used to foster mathematical proficiency (Julie & Mudaly, 2007; Niss et al., 2007). In this study, mathematical modeling is viewed as a tool for teaching the concept of linear motion while also serving as an objective to assess student competencies in the mathematical component of physics problems. The emphasis is on giving students the ability to create mathematical models and develop mathematical modeling competencies. Through the modeling activity used in this study, students were expected to accomplish the interdisciplinary mathematical modeling cycle. Moreover, as part of this iterative process, they were required to develop mathematical models that can be applied to problems involving physics concepts.

Interdisciplinary Mathematical Modelling (IMM)

Mathematical modelling activities require interdisciplinary connections since the problems represent real-life situations, which often involve a wealth of complications that require the application of various sciences to understand. IMM is a perspective that involves the simultaneous employment of multiple disciplines (Dogan et al., 2018). IMM encompasses the development of solutions to real-life problems through models with the help of both mathematics and science. Within the scope of IMM, a wide range of disciplines can be combined, or merely be a combination of two disciplines. Figure 1 illustrates the IMM process that involves mathematics and science.

Figure 1

Interdisciplinary Mathematical Modelling (IMM) process (Dogan et al., 2018)

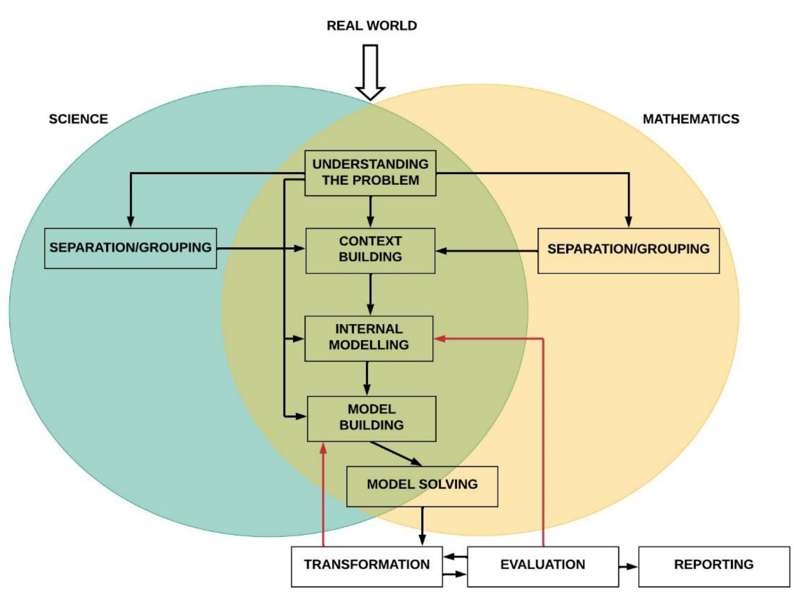


Figure 1 illustrates that the IMM process commences in the context of real-life situations. The following sections and Table 1 provide a description of each stage.

Table 1

IMM Stages and Example of Expected Outcomes

IMM Stages	Example of expected outcomes
In this step, problem solvers are expected to articulate their understanding of the problem, transitioning from the real world to the common ground shared by mathematics and science.	They should be able to express key problem details, such as the velocity of the second car approaching at 80 km/h and the placement of the reflector at a distance of 30 meters behind the broken vehicle.
Separation/grouping: In this step, participants should associate the concepts involved in the problem with the relevant disciplines, mathematics and physics, and grasp them through mental processing.	They should be able to apply equations such as x=vt and represent them through graphs.
Context building: Problem-solvers should observe the interconnections between concepts, form linkages, and make necessary associations. This stage may not occur simultaneously throughout the entire problem-solving process.	At this stage, problem solvers should be able to plan to draw a velocity-time graph by re-associating categorized concepts with individual disciplines such as mathematics and physics.
Internal modelling: Problem solvers should organize available data, produce ideas and assumptions, and engage in required planning to lead them towards a solution.	They should be able to make assumptions, such as that the area under the velocity-time graph can help calculate the distance traveled by the vehicle.
Model building: At this stage, the internal model is translated into a mathematical model by formulating the problem in mathematical terms and constructing a model that leads to the solution.	Problem solvers are expected to produce the solution using the velocity-time graph as a mathematical model.
Model solving: Once the model is established, the problem solver proceeds with the mathematical solution of the model with the help of his previous mathematical knowledge. Even though this stage mostly occurs in the domain of mathematics, given the extensive use of mathematical structures and operations involved, the problem solver takes advantage of scientific knowledge as well.	At this stage, problem solvers are expected to apply their mathematical knowledge to calculate the area of the triangle and rectangle to solve the problem.
Transformation: At this stage, the problem solvers think about the real-life consequences of the solution developed through the application of the model.	At this stage, problem solvers are expected to be able to think about the solution by using the velocity-time graph, which is acceptable for similar situations in real life, etc.
Evaluation: This stage entails testing the real-life applicability and accuracy of the solution.	At this stage, problem solvers are expected to assess the solution using the velocity-time graph to determine its validity for similar situations.
Reporting: When the model is deemed usable in real-life, a report is prepared detailing the mathematical model and its components.	At this stage, problem solvers are expected to be able to decide the solution by discussing the velocity-time graph that is usable in a real-life situation, reporting the details of the mathematical model, etc.

According to Dogan et al. (2018), the theoretical framework of the IMM process allows for a flexible transition between its stages. For example, an individual who cannot find a real-life equivalent to the developed model, or realizes that it cannot be applied to real-life, can still move from the transformation or evaluation stage to the model building stage or to higher-level stages such as understanding the problem. The degree of flexibility provided also means that some stages can be skipped for progress to be achieved. For instance, one can proceed directly to the mental model building stage without separating or grouping the concepts with reference to individual disciplines in the grasping the problem stage.

IMM can be viewed as an effective means of establishing interdisciplinary connections. In fact, a study conducted with mathematics and science teachers found that activities structured around IMM enabled the coverage of different disciplines simultaneously (Gurbuz et al., 2018). Additionally, the same study observed that mathematical modeling was appropriate for associating various disciplines (English, 2015), and the IMM approach that emerged from this feature suggests that the standards belonging to different disciplines can be taught together (Dogan et al., 2018).

Interdisciplinary Component of the IMM: In the Context of Physics and Mathematics

Mathematics is a discipline that has extensive associations with a wide range of fields, making it a useful tool for various sciences. Among these, physics is a field where mathematics is used most extensively (Redish & Gupta, 2010). While some topics in physics are taught with less emphasis on mathematics, acceleration is an example of a topic that is embedded within mathematical formulations (Basson, 2002).

Teaching various disciplines in connection with each other has been shown to help learners develop solutions more easily for the problems they face in daily life (Carrejo & Marshall, 2007; Prins et al., 2009). Additionally, courses emphasizing the connections between various disciplines are thought to pique students' interest and motivation (Dervisoglu & Soran, 2003; Lyublinskaya, 2006; Ogunsola-Bandele, 1996). Several studies have highlighted the importance of teaching mathematics and physics in conjunction, providing a more solid foundation for concepts and promoting effective learning (Erickson, 2006; Munier & Merle, 2009; Redish & Gupta, 2010). To this end, the literature is rich in studies attesting to the effective use of physics models in teaching mathematics and geometry, which can support meaningful learning by rendering abstract concepts more understandable (Bing & Redish, 2009; Munier & Merle, 2009). Conversely, using mathematical models to teach physics concepts has been found to produce positive results, such as developing positive attitudes towards physics classes and facilitating the learning of challenging concepts (Marshall & Carrejo, 2008; Takaoglu, 2015).

However, some topics in physics are related to real-life cases, and students may already have an accurate or inaccurate understanding of these concepts. These concepts may be easier or harder to teach, depending on students' existing knowledge and misconceptions. Motion is an example of a topic that poses such challenges (Aksit & Wiebe, 2020; Bani-Salameh, 2016). Students may experience difficulties understanding concepts such as velocity, position, and acceleration, which can be misleadingly similar but essentially different (Bani-Salameh, 2016). Additionally, interpreting negative and positive acceleration, along with drawing and interpreting velocity-time and position-time graphs, are among the challenges that students may encounter (Goldberg & Anderson, 1989; McDermott et al., 1987; Nemirovsky & Rubin, 1992; Pendrill & Ouattara, 2017). The literature has addressed these issues, providing insights into how to teach such concepts more effectively.

Certain fundamental concepts of classical mechanics serve as the cornerstone of science and physics courses taught in primary and secondary schools (Basson, 2002), with their implications being noticeable in everyday life (Singh & Schunn, 2009). Linear motion and acceleration are among these concepts. As these concepts have implications in everyday life, students often hold normative and/or non-normative ideas about them (Clement, 1982; DiSessa, 1982; Halloun & Hestenes, 1985). Non-normative ideas, based on common sense rather than scientific principles, cause confusion and impede the learning experience for students, particularly when it comes to the concepts of acceleration and velocity. Therefore, during the teaching process, concepts like acceleration should be presented in real-life contexts through clear problem situations and with clear mathematical foundations. Based on this view, the current study proposes a novel approach to teaching physics concepts (particularly linear motion and acceleration) related to real-life cases using mathematical modeling, as opposed to conventional approaches to teaching physics.

Most discussions on the positive effects of interdisciplinary associations reference the significant obstacles teachers face in interdisciplinary teaching processes (Morrison & McDuffie, 2009; Weinberg & Sample McMeeking, 2017). The lack of adequate resources or materials, or the teachers' lack of experience in establishing associations between their own discipline and other disciplines, forces them to focus primarily on their trained discipline (Bybee, 2010). Additionally, students often find the integration of multiple disciplines to be complicated and overwhelming (Dervisoglu & Soran, 2003; Ogunsola-Bandele, 1996). Therefore, teachers require a new, simpler method that can be applied in the context of interdisciplinary integrations. Whereas students would appreciate a new approach to aid in understanding concepts and making the learning process enjoyable through the use of various disciplines in an integrated and interconnected manner. In this regard, the use of mathematical modeling to teach challenging physics concepts stands out as a potentially helpful method.

Research Purpose and Questions

In light of current research, it can be concluded that the use of IMM clearly aids in increasing competence and knowledge levels across all disciplines by fostering comprehensive interdisciplinary connections. Previous studies conducted with science preservice teachers (PSTs) have indicated that physics courses are often taught in a discipline-based manner with relatively low levels of success (Michaluk et al., 2018; Pollock, 2006). Thus, the present study aims to investigate problem-solving processes through an IMM activity requiring the combined use of mathematics and physics by PSTs enrolled in the Science Teaching Program.

The evaluation of PSTs' modelling skills applicable to problem-solving processes is critical in terms of establishing their subject-specific competencies and problem-solving skills necessary for teaching in real classrooms. Developing mathematical modelling abilities in PSTs is essential since they will eventually teach science from an interdisciplinary perspective to middle school students. Moreover, instilling in-service and PSTs with sufficient mathematical modelling abilities, as well as executing modelling assignments in the classroom, is essential for the efficient integration of mathematical modelling into science education programs at all levels.

Therefore, this study focuses on PSTs' completion of an IMM task. The research questions guiding this study are:

1. To what extent do science PSTs solve a real-life physics problem by following the stages of mathematical modelling?
2. Which stages of IMM pose the most significant challenges for PSTs to complete during the mathematical modelling process?

Methodology

Research Method

The present study utilized a collective case study research approach, which is a qualitative research methodology that allows for an in-depth review of a pre-defined system. According to

Creswell (2007), case studies are typically conducted on a single person or a group of people, an event, or other entity that is less well-defined than a single person. In this study, we examined the real-life physics problem-solving processes of two groups of science PSTs and identified how they progressed through the stages of interdisciplinary mathematical modelling. To achieve this goal, we conducted an articulated analysis of each case's discussions and decisions. Therefore, we designed our data collection procedure based on multiple case studies.

Participants

The study group for this research comprises six PSTs, consisting of four females and two males. These PSTs were enrolled in an introductory physics course, which forms part of a science teaching program offered at a university in Türkiye. As the study was extracurricular, participants were entirely voluntary. This study particularly focused on science PSTs, as they constitute a fundamental element of the learning environment. The research aimed to examine the problem-solving process of PSTs, whereby their interpretation of the problem setting was influenced by real-world information. PSTs are required to acquire mathematical modelling skills, since they will eventually teach science from an interdisciplinary perspective to middle school students.

Context of The Study

The PSTs in the study were divided into two groups based on their performance levels on the midterm exam scores. This was done to ensure that the students were evenly distributed among the groups with respect to their high, medium, and low scores. Since most of the questions on the midterm exam were related to linear motion and acceleration, the scores served as a reliable indicator of their preparedness. Previous research studies have emphasized the importance of group work in the effective implementation of mathematical modelling activities (Antonius et al., 2007; Erbas et al., 2016). Therefore, the present study was designed to incorporate a group activity centered around modelling, which required the participants to work collaboratively.

The Practice Exercise

Before commencing the modeling activity, the participants received an introduction to mathematical modeling. To enhance their understanding of the process, the *Water Tank* activity designed by Erbas et al. (2016) was implemented, encompassing various physics concepts. In this activity, the pre-service teachers (PSTs) collaborated in groups to formulate a mathematical model addressing a problem related to creating an altitude-volume graph. This graph aimed to assist in developing an animation for three differently shaped water tanks. The participants were tasked with utilizing mathematical knowledge, including functions, derivatives, and graphical representation.

Engaging in this mathematical modeling practice facilitated communication within the groups and provided the participants with an experiential understanding of how to navigate through the stages of the Instructional Model for Mathematics (IMM). Following the allocation of adequate time to the PSTs, we conducted discussions with all groups, outlining the tasks required for each step of the Water Tank exercise and demonstrating approaches to the various phases.

The IMM Activity

After ensuring the participants' comprehension of mathematical modeling strategies, we introduced the *Braking Distance of a Car* activity developed by Erbas et al. (2016) to assess their group discussions and problem-solving strategies for addressing a real-life problem. This activity encapsulates a real-world context of linear motion as a physics concept and linear functions as a mathematical context. To derive a solution for the activity, pre-service teachers (PSTs) were required to possess a comprehensive understanding of various skills, including the analysis of linear motion, the interpretation of the velocity-time graph, the determination of the quantity of motion and acceleration, and the application of mathematically correct operations throughout the process.

Data Collection Instruments and Process

To gather data, we encouraged the participants to explain their ideas and express their opinions aloud while working on a small whiteboard and worksheet. The worksheets were designed based on the steps of IMM related to the activity, and the PSTs were required to explicitly describe and justify the answers they generated at each step of their worksheets. The group conversations were recorded using both sound recorders and video cameras. Our data consisted of their work on the worksheet and whiteboard solutions, as well as their open discussions.

The activity lasted approximately 45 minutes, and the researchers visited each group every five minutes to observe their group work and discussions. If groups had questions about the stages or the problem, the researchers guided them to find the answer on their own. After completing all the stages, the groups were given five minutes to present their solutions and answer questions from other groups. Finally, the discussions between the groups were followed by the presentation of the formal solution to the problem given by the researchers. The process flow for the modeling activity is presented in Figure 2.

Figure 2

Process Flow Chart for IMM Activity

Data Analysis

We conducted a content analysis of the data collected from worksheets, audio recordings, and video recordings in this study. Content analysis is a scientific method for examining communication content by analyzing the meaning, circumstances, and intentions expressed in messages. To effectively conduct content analysis, it is necessary to narrow down the data to concepts that define the problem under study (Elo & Kyngäs, 2008). The first step in our study was to transcribe the data obtained from video and audio recordings (Berelson, 1952).

Subsequently, two researchers independently read all the data and attempted to comprehend the process as a whole. Following this, the researchers analyzed the students' worksheets using the IMM stages presented in Figure 1. The data were analyzed and categorized according to the individual stages of the IMM framework. Our analysis focused on revealing how each group's discussion and the mathematical model development process evolved in each stage.

After the initial individual analysis, the two researchers discussed the coding of the content until 80% of their codes were in agreement. Quotations from the solutions developed by the groups and the remarks they made are presented below to support the results of the analysis. Additionally, images are included to reinforce the presentation of the findings.

Findings

This section furnishes an analysis of the process undergone by participants during the *Braking Distance* activity. It delineates the process of each group individually, taking into account the stages inherent in the IMM (Interactive Multimedia Module) process. To streamline this section, each participating Preservice Teacher (PST) has been identified with a code number signifying both the group number and the order of the students within the group. For example, the second member of group 1 is denoted by the code number G1S2.

The IMM Process of Group 1 Through the "Braking Distance of a Car" Activity

This section presents a detailed analysis of the problem-solving process of Group 1, which is broken down into stages based on the IMM process. The participating PSTs were identified by a code number that denotes the group number and the order of the students in the group. For instance, the 2nd member of Group 1 is referred to as G1S2.

The first stage of the IMM process is Understanding the Problem, and Group 1 commenced this stage by reading and interpreting the problem. During this process,

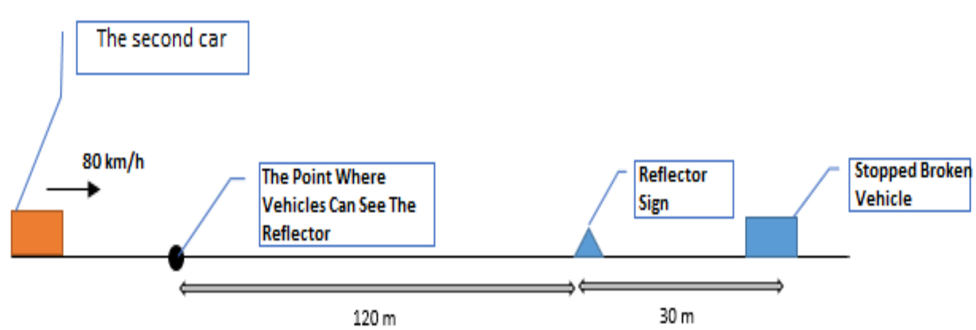
> G1S1: … *I call it nonsense* [refers to the second drivers' claim]*! There are no visible tire marks on the road. And the surveillance does not allow us to see the point where the second car hit the brakes. But we are still expected to shed some light on how the accident happened.*

The statement "I call it nonsense!" suggests that the student does not agree with the second driver's statement. Later on, in the Transformation stage for Group 1, the same student made another statement that indicated a generalization based on an incorrect piece of knowledge from their daily life. However, the other members of the group did not raise any objections regarding this point.

Following the initial statement, the group proceeded to draw a visual representation of the problem's provided input, as depicted in Figure 3. This visual representation helped the group members to understand the problem more clearly and aided them in formulating a solution.

Figure 3

Group 1 Created a Model of the Drawing to Assist Grasping the Challenge.



During the conversation that ensued while creating the drawing, the group members asserted that the second car was approaching at a velocity of 80 km/h and that the reflector was positioned 30 meters behind the broken-down car. They further claimed that the second car noticed the reflector at a distance of 120 meters from it, but there were no tire marks on the road. It is apparent that the group misinterpreted the visibility distance, mistaking it for the distance from the broken car instead of the reflector, which resulted in an incorrect estimation of the braking distance (120 m). Despite the idea originating from G1S1, none of the other group members voiced any objections and accepted the notion. Apart from this misinterpretation, Group 1 did not have any further issues in the "Understanding the Problem" stage. The remainder of the conversation held by Group 1 is presented below.

> G1S1: *The second driver claims to have hit the brakes as soon as he saw the scene. But that is only his statement. That statement can be wrong as well.*
> G1S2: *If he hit the brakes, the car would have stopped anyway.*
> G1S1: *But he couldn't stop. And also, there are no tire (brake) marks on the pavement.*
> G1S3: *We can calculate the change in velocity at 2-second intervals.*
> G1S2: *The change in velocity is already apparent on the table.*

The presented case highlights the importance of accurately understanding the problem to come up with an accurate solution. However, the group's prejudices towards the second driver's statements as inaccurate still persisted.

Separation/Grouping

This stage required the group members to associate the concepts involved in the problem with the disciplines of mathematics and physics. An excerpt of the conversation that took place during this stage is presented below.

> G1S1: *We can use the formula x=vt. We can calculate it at 2-second intervals. In other words, we will increase the time in 2-second increments.*
> G1S3: *Wouldn't we have different results then? Shouldn't we be drawing a graph?*
> G1S3: *I guess so. What would the slope of the graph represent? Area under the line?*
> G1S1: *This velocity-time graph... But the velocity is not increasing in a uniform linear manner.*
> G1S3: *It did not increase because the driver hit the brakes.*
> G1S1: *Multiplying v by t would yield x, which is the distance. Multiplying the base by the height gives us the area of the triangle... So, what does it mean by stating 2-second intervals?*

At this stage, Group 1 was able to spot the method of multiplying velocity and time to calculate the distance the vehicle covered before it stopped and to calculate the distance with reference to the area under the velocity-time graph. However, they were still unable to accurately interpret the data provided on the velocity-time table with 2-second intervals and realize that it actually represented linear motion. As they found out, by multiplying the velocity by time during the IMM activity, they were able to spot the complementary aspects of the formula associated with both mathematics and physics.

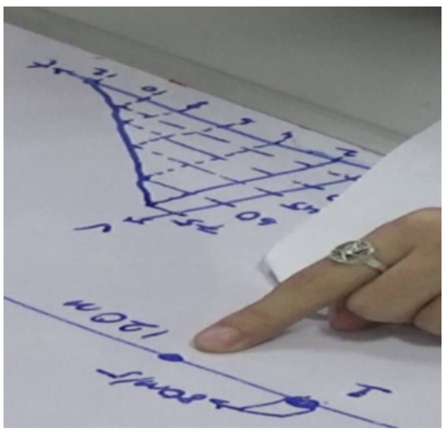
Moreover, while interpreting the graph, they were able to step into the domain of mathematics with reference to concepts such as the area of a triangle. However, in the separation/grouping stage, Group 1 failed to assign meaning to the uniform-linear motion concept in the field of physics. This observation may attest to their inability to establish adequate associations between the concepts involved in the problem and the relevant disciplines, as well as their failure to complete the separation/grouping stage.

Context Building

Group 1 was observed to associate the relevant concepts with the applicable discipline but failed to group the linear motion in the previous stage. In the subsequent stages of the process, Group 1 embraced the idea of drawing a velocity-time graph through the re-association of the concepts they categorized with respect to individual disciplines (mathematics vs. physics). See Figure 4 for this information.

Figure 4

Velocity-time graph by Group 1



This is a testament to moving to the internal model-building stage. For example, there were no explicit references to the phase of identifying the context in Group 1's interactions. Although Group 1 evaluated the data on the problem in the context of two separate disciplines in the separation/grouping stage, they moved on to the next stage without interpreting the data they obtained at this stage in the context of the common field of both disciplines.

Internal Model Building

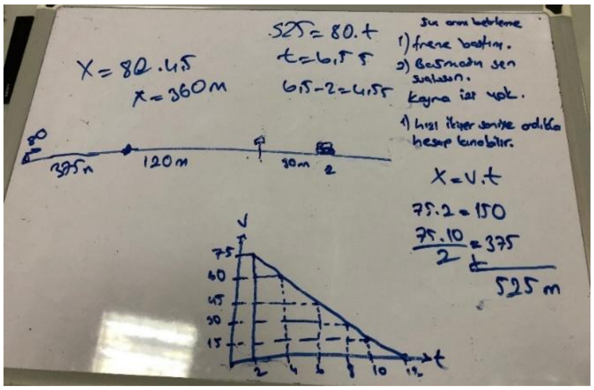
Regarding this stage, Group 1 made mistaken assumptions early in the process. As they interpreted the data available, they were observed to embrace a mistaken perspective built around the maxim "*you are to blame as you did not hit the brakes*", after reading the statement by the second driver, who said he was unable to stop even though he hit the brakes as soon as he saw the reflector. However, later on, through a better analysis of the data presented in the problem, the group arrived at the idea to draw a velocity-time graph to calculate the braking distance of the vehicle. In doing so, they were able to come up with an accurate representation of the straight linear movement as well as calculate the distance covered by the vehicle by calculating the area under the velocity-time graph.

Model Building

The graph is the mathematical form corresponding to the model that Group 1 constructed in their minds with respect to the movement of the vehicle. In other words, at this stage, the solution will be produced by using the velocity-time graph as a mathematical model. As shown in Figure 5, the group has apparently developed the x=vt formula to present the area below the graph with a view to calculating the distance as another mathematical model application.

Figure 5

Solution by Group 1



Both models are accurate and could be used in the solution of the present problem. At this stage, Group 1 has achieved success by using suitable models for the problem. However, even though Group 1 came up with the correct models, they included data items with different units in the same calculation. For instance, the y-axis on the graph shown in the image shows the velocity, while the x-axis represents time. Yet, the data shown on the x-axis is in seconds, while the data shown on the y-axis is presented in km/h. And doing so led to an inaccurate solution. As the deceleration of the second vehicle is presented on a time scale of seconds, they should have used m/s as the unit of velocity, rather than km/h as presented in the problem and applied the necessary conversion.

Model Solving

A glance at the solution presented in Figure 5 and the dialogue recited above reveals that Group 1 came up with the correct model (a velocity-time graph) and used their mathematical knowledge about the calculation of the area of the triangle and the rectangle to solve the problem. The transcript of the dialogue that Group 1 had on the way towards a solution is presented below.

G1S3: [Once the graph was drawn, let's calculate the area below the graph.] *And then we calculate the distance covered.*
G1S1: *Then we should multiply 75 by 12 and divide the result by 2* [trying to come up with a solution based on the area of the triangle].
G1S3: *No, see, this is a rectangle.*
G1S1: *Actually, it's a trapezoid. Take a look here! Bottom and top... Bottom plus the top...*
G1S3: *I don't see why you are trying so hard. Divide the graph into two. Make this section a rectangle. And calculate the area of the triangle here.*
G1S1: *That is one way to go. The trapezoid provides an even more direct route. 75 times 2 equals 150 here. And this part is 75 times 10, divided by 2.*
G1S2: *So it's 375?*

During the discussions within the group, G1S1 stated that the graph looked like a trapezoid. Yet, it is noteworthy that G1S3 insisted on going with the calculation of the area of the rectangle and the triangle since she said that calculating the area of the trapezoid was difficult. Based on this statement, we can reach the conclusion that the students had adequate knowledge of rather conventional forms, such as triangles and rectangles, but did not know how to calculate the area of a trapezoid and they were able to associate the graphical model with a mathematical model to interpret linear motion. Additionally, when we look at the graph produced by Group 1, it reveals that the velocity was expressed in km/h and the required conversion was not applied. Therefore, the solution they came up with was incorrect, even though the models they developed were correct.

Transformation

At this stage, Group 1 started to relate the real-life consequences of the solution developed via the model. Group 1 had the following dialogue regarding this stage:

> G1S1: *So, if the distance is 525 meters, what this guy says is not accurate. 525 minus 150 equals 375 meters. So, the driver noticed the other car and hit the brakes 375 meters before the vehicle. Had he really done so, why do we have this gap of 150 meters? So, looking at this, we understand that he did not hit the brakes as soon as he saw the car. The distance is 525 meters.*
> G1S3: *Maybe it is the driver of the first car who is not providing the correct information. How do you know that?*
> G1S1: *That man did not engage in any action. All he did was stop on the road. And he placed that thing as a safety measure.*
> G1S2: *In any case, the car that hits the other one, rather than the one that had stopped, is always deemed the faulty party.*

According to the results Group 1 reached through the interpretation of the incorrect solution they came up with, they commented that the driver of the second car is to blame. So, when they began to think about the real-life consequences of the solution they developed, G1S2 came up with the comment, "*In any case, the car that hits the other one, rather than the one that has stopped, is deemed the faulty party*". This can be considered an incomplete assessment of the real-life picture. According to the traffic rules, that comment can be applied only in cases where certain other requirements are met. Furthermore, the same student voiced the view that a car that hits another one from behind would always be considered the guilty party. G1S3, in turn, came up with the comment that the driver of the second car did not know how to drive the car as the actual reason for the accident in the real-life context. Such comments suggest that Group 1 failed at this stage.

Evaluation

Group 1 had the most extensive discussions among other groups during the evaluation stage. They began with a quick return to the "understanding the problem" stage. They had the following conversation regarding that stage:

> G1S1: *Are we being asked how many seconds have passed? Are we supposed to find the second in which he applied the brakes?*
> G1S3: *We are going into a loop now.*
> G1S3: *The question is, at what point did he hit the brakes at this 150-meter distance? We just calculated the distance...*
> G1S2: *We calculated the distance he saw the reflector at.*

G1S1: *I've got it here. The distance between the two vehicles when this one applied the brakes. It is shown in the camera footage. The distance from the time of hitting the brakes. Got it?*
G1S3: *Nope. I still don't get it.*

Based on the preceding conversation, it is apparent that G1S3 encountered difficulties in comprehending the problem. Conversely, G1S1 made concerted efforts to decode the input provided in the problem to aid the other student's understanding. Eventually, G1S3 expressed comprehension by stating, "I see." Subsequently, the group members deliberated on the solutions they formulated during the problem-solving phase. However, the group failed to reach a consensus and resumed working on the solution. Furthermore, the group members employed mathematical elements based on disparate units (km/h vs m/s) in the same calculation. Although they arrived at a mathematically correct solution due to a fortuitous occurrence, they were able to make the correct determination. At this stage, they abandoned their initial belief that the second vehicle was at fault and concluded that the car that came to a stop on the road was responsible.

The Mathematical Modelling Process of Group 2 Through the "Braking Distance of a Car" Activity

In this section, we closely looked through the problem-solving process of Group 2 by breaking down the whole group work with reference to the IMM process.

Understanding the Problem

In this stage, Group 2 tried to interpret the problem through the following statements:

G2S1: *The broken vehicle was hit by another vehicle, even though the latter applied the brakes. Furthermore, the velocity was falling at 15 km/h every two seconds.*
G2S2: *We need to find out the distance at which the second car hit the brakes.*

In its bid to understand the problem, Group 2 did not make sufficient references to any other data provided in the problem. This observation suggests that the requirements applicable to the solution of the problem were expressed in an incomplete form by Group 2. The PSTs were, arguably, unable to meet the required level of competence for this first stage of the problem-solving process.

Internal Model Building

Group 2 did not engage in any acts applicable to the separation/grouping and context building stages. They skipped these stages and moved on directly to the internal model-building stage. At this stage, the PSTs were expected to organize the data they had with respect to the problem, come up with assumptions regarding the solution, and clearly express what was provided and required for the problem. The dialogue regarding this stage is transcribed below.

G2S2: *Let's write the input we are provided. We have the table here, and we are told that the car hit the brakes at some distance, and we are also provided with the velocity of the vehicle. I mean, we have the velocity-time data for the vehicle that hits the other one. We know that its initial velocity was 80 km/h.*
G2S1: *We should also note that its velocity fell by 15 km/h every 2 seconds after hitting the brakes.*

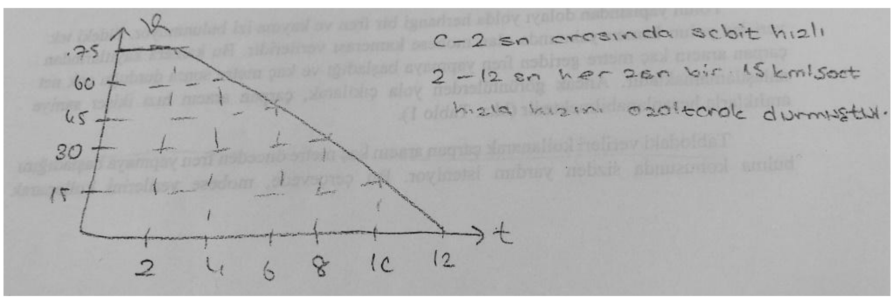
Group 2 was successful in naming one of the required variables with a specific reference to the velocity and time data. Nevertheless, they still fell short of the requirements by failing to come up with any assumptions. Group 2 accurately expressed the requirements but still failed to establish a connection between the data that was provided and the data that was required. Therefore, Group 2 cannot be considered the most effective at this stage.

Model Building

At this stage, Group 2 opted for a velocity-time graph to help with the solution of the problem. See Figure 6 for the graph that Group 2 produced.

Figure 6

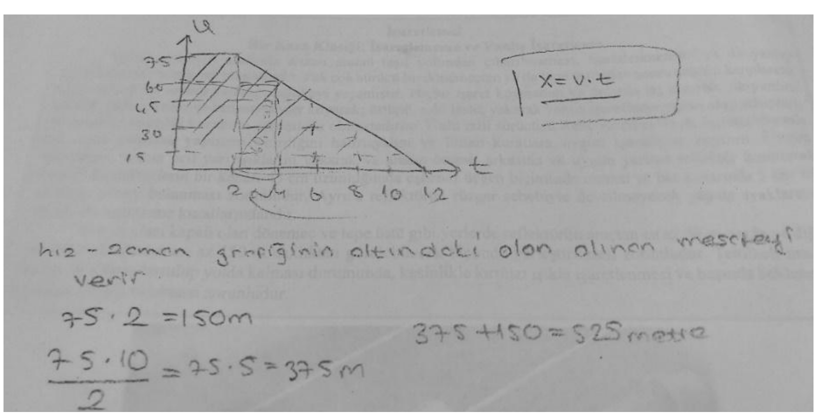
The Mathematical Model that Group 2 Produced to Help With the Solution of the Problem



Upon examination of Figure 6, it is evident that Group 2 opted to employ a graph as a mathematical model, specifically a velocity-time graph for the second car. Although the selected mathematical models were deemed appropriate for the task at hand, they were constructed using an erroneous approach to the data. Notably, the data on the x-axis was presented in seconds, while the data on the y-axis was expressed in km/h. Additionally, this phase marked the first time that the PSTs referred to linear motion after constructing the graph. Following this, the group proceeded to solve the model and derived the subsequent solution, which was based on the x=vt formula. See Figure 7 for this information.

Figure 7

Solution by Group 2



The solution by Group 2 contains the accurate statement that "*the area below the velocity-time graph always represents the distance covered*." As they solved the problem, Group 2 employed the formula to calculate the area of triangles and rectangles and determined the area under the velocity-time graph. However, because the PSTs did not convert the km/h and m/s for uniformity in units while calculating the distance, they obtained the incorrect response.

Transformation

At this stage, G2S1 voiced her opinion, ascribing the fault in the accident to the vehicle that hit the broken one. Her group mates also affirmed that view, and embraced the idea that the vehicle that hit the other from behind was at fault. However, this view was incorrect. As they later interpreted their mathematical solutions in a real-life context, they said, "*We saw that the vehicle decelerated smoothly after hitting the brakes. In daily life, we move with a given velocity, even if all we do is walk. And the distance we cover is a function of our velocity and the time we spend. So, in order to calculate the distance, we covered, we need to multiply the velocity with the time.*" Here, it is evident that the PSTs have been interpreting the models they used rather than the mathematical solution they produced. One can argue that if they really focused on interpreting the mathematical solution and the party was at fault, they could have noticed the shortcomings of their solution.

Evaluation

At this stage, Group 2 said, "*We don't think we have been mistaken. Calculations about actual real-life cases also follow this route*" claiming that the solution they developed would lead to accurate results in real life as well. However, they did not engage in a discussion of the real-life applications of this case.

Conclusions and Discussion

Linear motion, a foundational topic in physics, is typically instructed using traditional methods emphasizing formulas (Ropii et al., 2019). Unfortunately, this approach often results in the rote memorization of concepts and formulas, hindering meaningful learning and comprehension of the underlying principles (Reif, 1995). In this study, our objective was to address the challenges faced by students in a teacher education program taking an introductory physics course in grasping motion concepts and developing a mathematical understanding of physics. To achieve this goal, we employed an Interactive Multimedia (IMM) activity to analyze students' problem-solving processes and establish connections to real-life contexts.

Our focus on Pre-Service Teachers (PSTs) was twofold. Firstly, as future educators, they need to cultivate mathematical modeling skills to teach science from an interdisciplinary perspective. Secondly, many students perform poorly on exams due to a lack of understanding of fundamental concepts in physics and mathematics (Teodoro & Neves, 2011). Therefore, teaching physics concepts alongside their mathematical components has become increasingly important, particularly at the secondary level.

Upon analyzing the data based on the IMM stages, we observed a variety of common and unique issues at each stage. During the *understanding problem* stage, students frequently relied on their real-life experiences and common-sense knowledge to attribute meaning to the problem, resulting in misconceptions. This phenomenon is not uncommon, as physics concepts encountered in daily life can lead to non-normative ideas or models (Clement, 1982; DiSessa, 1982; Halloun & Hestenes, 1985). However, students should develop a scientific understanding of the problem and establish a connection with physics, rather than relying on preconceived notions.

Furthermore, existing misconceptions of linear motion among students impact their interpretation of the problem and, consequently, their ability to solve it. Motion, being a topic closely associated with students' preconceptions based on real-life experiences, is an easier topic to teach but also poses challenges in avoiding misconceptions (Bani-Salameh, 2016). Overall, our findings suggest that students require sufficient experience to interpret data realistically, highlighting the importance of teaching physics concepts in conjunction with their mathematical components.

In the *separation/grouping* stage, difficulties were observed among pre-service teachers (PSTs) in effectively linking mathematical and physical concepts within the context of the problem with their respective disciplines. Those who faced challenges in comprehending linear motion during the initial stages of problem understanding also encountered difficulties in connecting this idea with physics and categorization. However, some students did not explicitly reference this stage during the process. The accentuation of certain stages in the Integrating Mathematics and Physics (IMM) process may be attributed to insufficient experience within the study group. The third stage of the IMM process, context building, did not overtly occur in all groups, possibly due to the ineffective implementation of the preceding separation/grouping stage.

Gurbuz et al. (2018) reported that the separation/grouping and context-building stages were not explicitly performed by their participants, as they were experts in the physics topic and collaborated effectively. However, it cannot be conclusively asserted that the same reason led to the present study's participants bypassing these two stages. According to Chi et al. (1981), novices tend to categorize physics problems based on surface aspects, while professionals categorize them based on the physics ideas needed to solve them. Therefore, novice students may overlook the separation/grouping and context-building stages, which involve establishing connections between the concepts related to the relevant physics ideas. Moreover, despite the sequential listing of all stages of the IMM process on the provided worksheet, a flexible transition occurred between them, as affirmed by previous studies (Borromeo-Ferri, 2018; Doerr, 1997; Gurbuz et al., 2018).

The primary issue in the *internal model* stage concerned the limitations of the PSTs in developing assumptions. One of the main causes of these shortcomings was the students' misconceptions based on their daily life experiences. According to Aydin-Guc (2015), the students were only able to develop a limited number of new assumptions about the actual state of affairs through the mathematical modeling process. This lack of competence in assumption-making was linked to the students' insufficient knowledge and experience regarding the context of the activity. Similarly, when information about real-life cases was not explicitly provided in the problem context, individuals encountered difficulties in generating assumptions for mathematical modeling practices (Blum, 2011). Other studies have yielded similar findings regarding the development of this competency based on experience (Blum & Borromeo-Ferri, 2009; Bukova-Guzel, 2011). As the students regarded the input provided as assumptions, they experienced challenges in developing new ones. Consequently, one could argue that the perceived shortcomings were mainly due to the students' inability to comprehend the input as part of the problem. For instance, the students read and discussed the problem statement multiple times but still faced difficulties in developing a model. According to Maaß (2006), students often encounter difficulties in developing a model when they fail to comprehend the presented case through written statements.

Another issue observed in this stage was the students' inability to provide a scientific description of the motion of the vehicle that hit the other vehicle from behind. Consequently, they attempted to calculate the distance covered by the decelerating vehicle through the equation expressing the connection between velocity and time. This finding was consistent with Marshall and Carrejo's (2008) observations, as the students predominantly overlooked the change in the speed of the moving vehicle during the specified time frame, opting instead to solve the problem by applying the instantaneous velocity formula. At this stage, two groups failed to recall the linear motion formulas they had learned in class and were unable to apply them to the problem. Tuminaro and Redish (2004) assert that students' inability to apply their mathematical knowledge and skills in physics classes is a significant challenge in understanding physics. From this perspective, it is plausible to claim that the participants' mathematics performance levels were correlated with their physics performance levels in this study.

In the fifth stage of the study, it was observed that all participating groups chose applicable models, namely the velocity-time graph and the distance formula. However, it was found that simply drawing the velocity-time graph based on provided values did not necessarily lead the students to use the linear motion formula, rather than the instantaneous velocity formula. This finding is consistent with previous studies (McDermott et al., 1987; Nemirovsky & Rubin, 1992; Phage et al., 2017), which also reported difficulties in interpreting velocity-time graphs and building mathematical models based on them. Intra-group conversations suggested that students' limited mathematical skills contributed to their inability to interpret the graphics, as noted by Potgieter et al. (2008) and Scott (2012).

Although the students struggled with interpreting the graph on an individual basis, they were able to reach the correct conclusion that the area below the velocity-time graph represents displacement through group discussions and shared knowledge. However, the use of km/h instead of m/s as the unit of data during graph drawing led to an incorrect calculation of displacement and an inaccurate interpretation of the solution. Aydin-Guc (2015) observed that heuristic strategies, such as applying unit conversions, were used by students only in response to the researcher's suggestions. Moreover, the study found that guiding students to use a graph to represent one-dimensional motion through the stages of IMM led to a clearer understanding of the problem and facilitated problem-solving, as noted by Phage et al. (2017).

During the model-solving stage, it was observed that the students avoided using the formula to compute the trapezoid's area, as expected, instead dividing the area into smaller triangle and rectangle-shaped regions, which they were more comfortable with. Ozer-Keskin (2008) attributed this to students' inclination to utilize formulae they were familiar with. However, the utilization of incorrect units in the velocity-time graph, as highlighted in the model-building phase, resulted in incorrect outcomes at the end of the process. In the transformation stage, it was evident that students interpreted the models they utilized rather than the mathematical solution they derived. Group 1, for instance, connected their solution to a real-life context despite their incorrect assumptions. No group was observed to be successful in this stage. Other studies in literature have found that this stage is frequently disregarded or involves a superficial and inadequate consideration of the real-world implications of the solution (Hidiroglu et al., 2014).

During the evaluation stage, only one group (Group 1) engaged in a process to test the real-life applicability and accuracy of the solution. The literature is replete with studies indicating that students, confident in the accuracy of their solutions, do not feel the need to verify them, leading them to skip checking for the correctness of their solutions and calculation errors (Blum & Borromeo-Ferri, 2009; Maaß, 2006; Sen-Zeytun, 2013). Blum and Borromeo-Ferri (2009) and Sen-Zeytun (2013) note that this is due to students' conviction that the instructor's role is to verify the solutions. In conclusion, the utilization of the IMM stages to solve real-life physics problems proved to be an effective approach to engaging pre-service teachers' problem-solving skills. Although pre-service teachers typically find physics classes dull, they expressed enjoyment in applying their skills to tackle the problem at hand. Although the groups followed the recommended stages of IMM diligently, certain tasks such as parsing/grouping and generating context posed challenges. Nevertheless, the outcomes of this study present a promising avenue to stimulate and uphold pre-service teachers' interest in physics classes and their comprehension of the mathematical models used in physics.

Limitations of the Study and Recommendations for Researchers and Instructors

Despite the overall positive effects of IMM activities on all groups, individual student performance plays a significant role in determining group performance levels. In this study, groups were intentionally formed to include high-, medium-, and low-performing students, but groups consisting of PSTs with higher levels of personal interest and motivation demonstrated greater diversity of ideas and more extensive discussions. A member of Group 1, who displayed a more active stance and higher level of performance, was particularly effective in leading group discussions and implementing IMM stages as required. Consequently, group composition is essential for group dynamics and the success of IMM activities. Familiarity with IMM and internalizing the meaning and requirements of each stage also contributes to more effective implementation of these activities. PSTs' negative preconceptions about physics, resulting from prior experiences in physics courses, challenged them to complete all stages of IMM as required. Participants reported that they had never before experienced such a direct relation between physics and mathematics with real life. They appreciated working on a problem in a sequence that helped them to think, discuss, and revise their models. Additionally, they were more engaged in the activities and voiced their opinions more actively than in other sections of the same course. Instructors who intend to apply IMM activities in their classes could expect to demonstrate applied execution of each stage of this model and reinforce the insights gained through debates involving the entire class. Teaching physics content to science PSTs via modeling activities such as IMM, where students actively work on a real-life problem and apply their knowledge to more tangible models, can motivate and alleviate PSTs' prejudices against learning physics concepts. Our recommendations for future research are presented below:

- We strongly advise instructors who plan to use IMM activities to practice implementing such activities a few times before achieving satisfactory results. Novice students may need time to become familiar with some stages of IMM.
- Researchers should create an environment that allows for the implementation of multiple IMM activities to ensure that study groups are familiar with one another and the stages of IMM. This should focus on developing mental model-building competencies that will help students understand and solve the problem.
- The parsing/grouping stage requires students to associate the concepts of mathematics and physics involved in the problem with the relevant discipline. However, the groups in our study had difficulty making discipline-related separations or groupings. Therefore, we recommend that future researchers exercise this stage in different interdisciplinary contexts.
- Context building was another challenging stage because students did not complete the separating/grouping stage. Therefore, we recommend that each group establish a control mechanism before proceeding to the next stage of future research. Asking the right questions or establishing checkpoints may help these groups successfully complete each stage.
- PSTs in our study failed to test the real-world applicability and accuracy of their solutions. They were generally self-assured in their solutions. For future studies, another similar real-life problem may be provided to help students test their solutions. This way, they can confront their errors or incorrect approaches.

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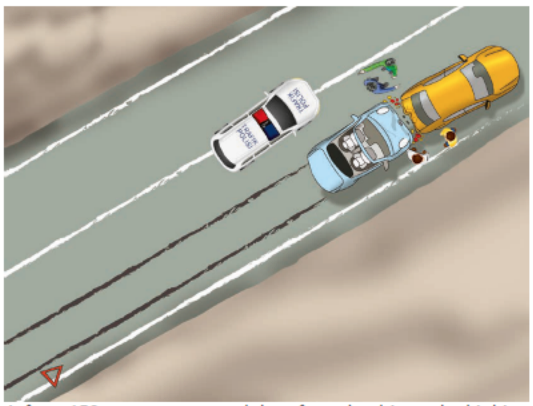
Appendix

Braking Distance of a Car

Marking!

An accident classic: wrong marking

Among the causes of the accident, the failure to remove the broken vehicle from the road, not marking or incorrectly marking, has an important place. Many drivers have experienced the danger of a broken vehicle that suddenly appears after a bend or hilltop. Unfortunately, vehicle owners who make wrong markings by burning old tires, putting no signs or laying stones on the road, putting first aid kits, drums, jacks, and similar things on the road are responsible for fatal accidents. Let us remember that the famous rally driver Renç Koçibey also lost his life by crashing into an unmarked vehicle, and please make the markings according to the rules. Proper marking is done by turning on the vehicle's emergency warning lights and by placing a reflector in front of, behind, and in appropriate places. The reflector must be in the form of an equilateral triangle with a length of 45 cm and a reflective surface of 5 cm on each side. It is also among the marking rules that the reflector has legs that will not topple over due to the wind.



It is mandatory to place the reflector at least 30 m away from the vehicle and visible from a distance of at least 150 m by other drivers in places such as bends and hills with limited visibility. In the event that the vehicles carrying dangerous goods break down and stop on the road, they must be marked with a red light and kept under surveillance.

An auto traveling at a speed of 80 km per hour slightly hits another vehicle from behind due to a breakdown on the road. The owner of the stationary vehicle claims that he placed a beacon reflector within 30 meters of his vehicle in a place where drivers could see it from 150 meters away, and therefore, the driver who hit his vehicle was one hundred percent guilty.

On the other hand, the owner of the vehicle that caused an accident blames the owner of the defective vehicle, saying that he could not stop despite pressing the brake when he saw the reflector. Knowing how many meters before the impacting car slams the brake will be a crucial parameter in calculating crime rates.

Due to the nature of the road, there are no signs of braking or slipping on the road. The only data available is the surveillance camera data, which is just near the road. From the surveillance camera recordings, it is not clearly understood how many meters behind the crashing vehicle started to brake and how many meters it stopped. However, the speed of the impacting car may be computed in two-second intervals using the photos (see Table 1).

Using the data in the table, you are asked to find out how many meters before the crashing vehicle started braking. In this context, using surveillance data, find out how many meters before the driver stepped on the brake and explain in detail in a way that the authorities will understand.

Table 1 Speed-time data of the crashing vehicle

Time (h)	Speed (km/h)
4:00:00 pm	75
4:00:02 pm	75
4:00:04 pm	60
4:00:06 pm	45
4:00:08 pm	30
4:00:10 pm	15
4:00:12 pm	0