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Reasoning at the Intersection of Science and Mathematics in Elementary School: A Systematic Literature Review

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ABSTRACT

Despite efforts to integrate science and mathematics learning in elementary school through carefully designed activities, students' cognitive processes remain relatively untapped as a possible place of intersection. We believe *reasoning* is a productive co-curricular concept that could lead to meaningful integration. We conducted a systematic literature review of empirical research published over the past 20 years on students' reasoning in both science education and mathematics education. Articles were summarized and examined for their: (1) methodological approach and experimental design; (2) social dimension in the classroom; (3) definition of reasoning and associated structures; and, (4) evidence of students' engagement in reasoning. For each theme, relationships between scientific and mathematical reasoning research were examined for the purposes of finding intersections and discrepancies between the two subject areas. As a result, we suggest the term *STEM reasoning* that embodies the core skills and thought processes across both subject areas to provide an authentic approach to integrating elementary school science and mathematics learning.

Keywords: mathematical reasoning; scientific reasoning; interactions of reasoning; elementary education

Introduction

Reasoning is a complex skill that is critical to problem solving in students of all ages and across a wide range of subject areas. In the context of solving a problem, reasoning stands as the "power that enables us to move from one step to the next" (Wu, 2009, p. 14). Reasoning is what happens in the time between the end of a question and the resulting answers and discussion that follow. Purzer et al. (2015) identified "reasoning processes such as analogical reasoning as navigational devices to bridge the gap between problem and solution" (p. 2) as mental tools and thought systems in place to solve problems within an interdisciplinary context of science, technology, engineering, and mathematics (STEM) education. We understand STEM education to be the integration of two or more of the disciplinary areas (English, 2016) -- our areas of interest are in science and mathematics. Devoid of reasoning, however, mental tools and thinking processes used in problem solving in science and mathematics are merely a set of tools in a toolbox. The ability to reason serves as our neural tour guide. Reasoning functions as the mechanism that helps students determine which thought pathway to follow - to pick out which pieces of information are critical, how they need to be dealt with, and where to go next.

This reasoning process is an essential skill in the intellectual development of a classroom learner (Alexander, 2017). While forms of reasoning are present in all subject area classrooms, there is a special emphasis on developing reasoning processes in STEM education (Johnson et al., 2021). In fact, reasoning is present to some degree at almost every level of science curricula (Alberta Education, 1996; Next Generation Science Standards, 2013) and mathematics curricula (Alberta Education, 2007/2016; Common Core State Standards Initiative, 2010). Across both subject areas, knowledge-based curricular outcomes are easier to identify and evaluate, while the complex reasoning operations that guide students to their given responses are often implied in curricular outcomes and more difficult to analyze directly. Even for experienced teachers, recognizing and observing the visual signs of reasoning is a process that takes much concentration, awareness, and training (Blanton & Kaput, 2005; Mata-Pereira & da Ponte, 2017; Mercer et al., 1999; Shin, 2020). As science and mathematics educators, we have found that even comparison between science and mathematics curricula can be difficult given the semantic and procedural differences in science and mathematics education. There is also the assumption that since curriculum consumers have a well-formed understanding of reasoning, it is often not explicitly defined or demarcated.

Reasoning in Science Education

Our theoretical perspective on scientific reasoning focuses on the understanding and practice of the relationships between claims and evidence. Encountering a question or puzzling circumstance, children need to examine the context of the question and situation, look for supporting and contrasting evidence, develop, accept, and articulate claims, and justify why the coordination of available claims and evidence has led them to accept certain theories and reject others (Bower, 2009; Varma, 2014). In the problem solving process, scientific reasoning occurs through questions, claims, and evidence to reach conclusions that are critical. Kuhn (1989) explains, "the heart of scientific thinking is the coordination of theories and evidence" (p. 674). Scientific claims or theories stand in relation to actual or potential bodies of evidence, yet, often children make or accept scientific claims not based on evidence but on personal ideas or beliefs, e.g., confirmation bias (Kuhn, 1991; Kuhn & Pearsall, 2000). Further, in "hypothetico-deductive reasoning in which children's working memory accesses and sustains hypotheses from associative memory to be tested and then actively seeks predictions and evidence that follow" (Lawson, 2004, p. 333), it is critical for children to understand how a hypothesis could be tested, evaluated, and sustained (or rejected) based on data sets and evidence that are available. The understanding of correlations between claims and evidence is essential in the scientific reasoning process. Children learn to make, evaluate, and justify claims with evidence as a core value of scientific reasoning. Such scientific reasoning enables students to be critical thinkers as they learn science in school and as they participate in society (Vieira & Tenreiro-Vieira, 2016).

National and international organizations have produced standards emphasizing that critical thinking and induction in problem solving processes are skills that lie at the core of scientific inquiry in science education (National Research Council, 1996; Organisation for Economic Co-operation and Development, 2013). The National Research Council (1996) acknowledges that scientific understanding develops "by combining scientific knowledge with reasoning and thinking skills" (p. 2) and the standards expect students to engage in scientific reasoning as both a process and outcome of learning. Additionally, reasoning in science education has been discussed in complex STEM problem solving contexts by integrating mathematics, causal reasoning, evidence-based evaluation, and argumentation in STEM contexts (Next Generation Science Standards, 2013). As problem solving contexts are complex and integrated with science, mathematics, and technology domains in today's world, reasoning needs to be understood and taught in complex contexts where disciplinary knowledge, skills, and values are intertwined for decision making and justification. Scientific reasoning cannot be independently processed to solve everyday problems.

Reasoning in Mathematics Education

Our theoretical perspective on mathematical reasoning and experiences that promote students' development of mathematical reasoning is informed by seminal publications in mathematics education and established standards within curriculum frameworks which include reasoning as an essential component. Mathematics education research builds on Polya's (1954) demonstrative and plausible reasoning, with ensuing elaborations of the various forms of reasoning such as transformational (Simon, 1996), metaphoric and analogic (English, 1997), imagistic (Thompson, 1996), and indirect (Brown, 2018). Davydov (1990) determined that children reason as they generalize about concepts and relationships which in turn "enables students to think systematically and to apply rules in concrete situations" (Venenciano & Heck, 2016, p. 23). Brodie (2010) defined reasoning as a way to "develop lines of thinking or argument … to convince … to solve … or to integrate a number of ideas" (p. 7). When students reason "about and with the objects of mathematics" (Brodie, 2010, p. 7), they invoke processes like conjecturing, investigating, representing, analyzing, justifying, refuting, generalizing, and convincing (Mason et al., 2010). Reasoning culminates in students' construction of proofs as an essential feature of the discipline of mathematics (Hanna, 1983; Lakatos, 1976). Mathematical reasoning is critical to the growth of mathematically proficient students (Lannin et al., 2011; White et al., 1998) evidenced in heightened achievement (Nunes et al., 2007). Mathematical reasoning is both a way to learn mathematics by actively constructing understanding and a way to be mathematical as a capacity that is developed over time through explicit use in mathematical contexts.

National organizations have produced standards emphasizing mathematical reasoning. The National Council of Teachers of Mathematics (NCTM, 2000) identified reasoning as a fundamental process for learning mathematics, where students are thinking analytically and systematically as they investigate mathematical phenomena, highlighting the role of conjecturing and justifying. Students are expected to move from informal reasoning to reasoning inductively toward conventional deductive proofs. Subsequently, adaptive reasoning was described as "the glue that holds everything together" (Kilpatrick et al., 2001) within mathematical proficiency. These two documents influenced the "Standards for Mathematical Practice' (Common Core State Standards Initiative, 2010) where five out of eight standards involve reasoning as noticing and expressing patterns and thinking with "logical progression" (p. 6). NCTM has further elaborated on teaching practices which all incorporate and enhance students' reasoning, along with specifically emphasizing the use of "tasks that promote reasoning" (2014, pp. 17-24) and identified the inclusion of reasoning in early childhood and elementary classrooms as transformational in students' development of "deep mathematical understanding" (2020, p. 9).

Exploring Reasoning as the Intersection

As has been demonstrated above, reasoning is often theorized or reasoning standards for student learning are created within domain-specific sites, either situated singularly within science or mathematics. Even in research about reasoning that transcends the boundaries of science and mathematics, one content area is often emphasized over the other. For example, Wasserman and Rossi (2015) used mathematics problems to investigate reasoning approaches of science and mathematics teachers in the context of STEM education. Limited scholarship exists that addresses the possibility of reasoning impacting the relationship between learning in science and mathematics. Research that investigates reasoning within the context of science and mathematics learning simultaneously often focuses on a type reasoning within a specific domain, like model-based reasoning (Lehrer & Schauble, 2000), proportional reasoning (Dole et al., 2012) or socioscientific reasoning (Owens et al., 2019). More promising, some researchers have recognized reasoning as a cognitive process that could cross the disciplinary boundaries of science and mathematics. Alexander (2017) posited that relational reasoning is "a foundational capacity" (p. 8) that could contribute to STEM learning across associated disciplines. Hwang et al. (2020) analyzed reasoning in Trends in International Mathematics and Science Study data and found that there is evidence for reasoning as "the same cognitive practices were utilized in science and mathematics" (p. 534). These studies open up the conversation for reasoning to exist at the intersection of science and mathematics learning within a STEM approach.

Though subject-specific curricula are written independently, the skills and mental processes we aim to develop in science and mathematics are not dissimilar. Reasoning skills are often distinguished in curriculum with unique labels and slightly variant definitions, but the thought processes they draw on are related and overlapping (Pisesky et al., 2018). Especially in an elementary school environment, the potential for teachers to take advantage of the intersection of science and mathematics through reasoning is immense but often not thoroughly utilized. Even if cross-curricular work is attempted, there is often a focus subject and extraneous outcomes from other subject areas used in superficial ways (Babb et al., 2016). Creation of a simple papier-mache science cell, for example, may be the purpose of an activity, while an estimation of the volume of the object could be layered on top. While this type of interdisciplinary project has definite value, it does not integrate the thought processes of science and mathematics. Rather, it takes a mathematics skill and requires its application on top of an existing science project framework that is not necessarily authentic to mathematics.

The aim of this theoretical study was to better understand the convergence points where scientific reasoning and mathematical reasoning intertwine. A primary literature review was conducted to explore how reasoning is researched as a co-curricular concept in the elementary classroom. We will first evaluate science and mathematics education research on reasoning independently to explore their unique methodologies and conceptualizations of reasoning. Through cross-examination of the findings on reasoning in science and mathematics subject areas, we will note the uniqueness and highlight commonalities between the two disciplines. We focus on the question: *What are the critical intersection points offered by reasoning in science and mathematics?*

Methods

We used a systematic review approach (Cooper, 2015) to create our final subset of literature for this study. This systematic process involved the following procedure: (1) established the required search databases and determined a list of appropriate search terms; (2) ran a search and consolidated the resulting articles; and, (3) used an established screening process to narrow down results to a focused subset of the initial article pool. Our final article pool was determined through several iterative rounds of term selection and literature searching. Our primary inclusion criteria were determined through the following conditions. First, literature must have been published between January 2000 and August 2020. Second, included articles needed to be peer reviewed for an academic research journal. Third, articles had to be focused on elementary education. This definition varies from country to country, but generally ranges from preschool or kindergarten to around grades 5 or 6 (depending on the country's range of elementary grades). Studies that contained large grade ranges (i.e., grades 4-12) were eliminated unless there was a clear way to separate out elementary education data.

Two searches were completed per database, one for science and one for mathematics. This was required because of the synonymous (yet differing) language used around reasoning in science and mathematics education. For science, the search terms used were "science" and either "scientific reasoning" or "logical reasoning." For mathematics, the search terms used were "math" and either "mathematical reasoning" or "logical reasoning." Of note, "reasoning (logic)" was a common keyword that was missed initially and, when employed, produced a large quantity of viable articles for inclusion. Filters were applied to fulfill the peer review requirement and limit dates to the preferred date range. Finally, to specify results to our desired age range, we added search parameters of "elementary," "K-6," "primary" or "early childhood." Of note, we decided to bound our searches with how "reasoning" specifically is used in science and mathematics education because we see it as a broad, integral, cognitive process. We also recognize that terms like "argumentation" are whole domains that are very well-researched in science education (Erduran et al., 2015; Nielsen, 2013), but may be less so in mathematics education (c.f., Staples & Cavanna, 2021; Staples & Newton, 2016), and would have produced unbalanced results between the two disciplines.

These search criteria were applied separately to education databases determined after consultation with an elementary education research librarian. We narrowed our search to two primary engines: ERIC (EBSCOhost) and Proquest. Using these search engines, the following databases were accessed: ERIC, Education Research Complete, Proquest Education Databases, and the Canadian Business & Current Affairs Database. Identical science and mathematics searches were completed using both engines, resulting in 647 total results for science and 542 total results for mathematics. Both lists were exported to Refworks and duplicate articles were removed, leaving 469 unique science education articles and 494 unique mathematics education articles.

A comprehensive screening process was required to eliminate extraneous articles that mention search terms, but are not related to our student focus. Our results were narrowed to short lists for science and mathematics using the following exclusion criteria: (1) Articles needed to be focused on empirical research conducted in a classroom or similar environment. Therefore, literature reviews were excluded. (2) Articles must have been written for academic research journals. Therefore, professional journals or other non-academic publications (even if peer-reviewed) were removed. (3) Articles were required to focus on the reasoning ability of students themselves. Therefore, teacher responses to reasoning, evaluation of existing reasoning skill metrics or examination of reasoning in teachers or preservice teachers themselves were excluded. We did not impose *a priori* definitions of reasoning as there is no single agreed-upon definition in either science education or mathematics education; rather, our aim was to observe how reasoning is conceptualized and operationalized within the empirical literature. Execution of these criteria left our research team with a shortlist of 44 science articles and 43 mathematics articles. A final in-depth screening was completed collaboratively by a team of two faculty members and one PhD student to select the strongest articles that focused most directly on students' reasoning capability. We scanned abstracts and research findings to eliminate any articles that passed through initial filters but did not actually meet our criteria. Many of these new exclusions were eliminated because, though they involved elementary students, they did not focus on student reasoning specifically, such as concentrating on validating a measurement parameter for a diagnostic test or teacher professional development. This final filter left us with 20 science and 21 mathematics articles to proceed with for our analysis.

The final pool of 41 articles was analyzed for critical elements with respect to reasoning by the whole research team. The articles were initially sorted according to their subject area to support immersion in the orientation to students' reasoning within the specific subject area. Then, the following process was used for both science and mathematics articles. The research question or purpose was highlighted to frame analysis of the findings. Research methods were noted because the authors' findings and research consumers' interests rely on these categorizations, including methodological approach, design of the classroom activities (intervention) or measurement tool (descriptive), and grade level. Additionally, the findings of each study were summarized along with salient quotes that represented both students' reasoning and the researchers' interpretations of reasoning. Making brief notes of the findings enabled preliminary analysis of connections across the articles. The research team met to discuss their collective observations that led to emerging themes and issues addressed by articles across science and mathematics. The emerging themes developed out of both a broad reading of all the articles and a first round of open coding to allow salient ideas to arise in our analysis rather than applying an analytic framework. This emergent process enabled us to discuss elements authors of each article emphasized in their reports.

The synthesis discussion led to a second in-depth round of analysis. In re-reading the articles, we recorded the subject-specific context (e.g., fractions, shape attributes, forces, density), the operational definition of reasoning, the conventional form of reasoning (e.g., inductive, deductive, metaphoric, imagistic), processes of reasoning (e.g., conjecture, hypothesize, generalize, convince), and the social context (e.g., individual students, whole class discussions). The second round of analysis provided finely nuanced insights into the similarities and differences of how reasoning is discussed and researched in science education and in mathematics education. In the second round, we reviewed and compared key ideas that emerged from the first round reading and compared them to develop certain categories to discuss similarities and differences in research on students' science and mathematics reasoning. We focused on what research approach or methods researchers chose to examine students' reasoning, how they perceived and defined reasoning, what types of reasoning researchers focused on, in what context reasoning was researched (e.g., individual cognitive process or social cognitive process, or independent or complex process). We discussed the ideas from the first round of reading and possible categories back and forth to saturate the approach and meanings of students' reasoning in research (Corbin & Strauss, 2008). Four categories were developed to discuss the findings; researchers' approach to examining reasoning, social dimensions, definition of reasoning, and students' engagement in reasoning. We discussed the analysis and developed shared understandings of the categories reported in the results below. Appendices A and B contain the reference lists for science and mathematics education articles, respectively.

Results

In this section, we explore our understanding of how children's scientific reasoning and mathematical reasoning are discussed and researched in the respective subject areas developed through our systematic review of literature published in the past 20 years. Typically, systematic literature reviews in reasoning have been conducted independently in subject areas (for science education, see Engelmann et al. (2016); for mathematics education, see Hjelte et al. (2020) or Jeannotte & Kieran (2017)), but through our exploration we found that critical discussions of interdisciplinarity only occurred as we framed understanding one subject area's conceptualization of reasoning in light of the other area's conceptualization. In this way, ideas and approaches that cursorily appeared to be differences between the disciplines could be better understood as similarities, when viewed through research into children's learning. The differences that remained were generative in enriching both scientific and mathematical reasoning processes rather than inhibiting children's interdisciplinary learning.

We developed tables to provide a structured overview of the results. Because of the substantial list of articles and the two subject areas, each table summarizes the research in one subject area. Table 1 displays research in science education. While maintaining a similar structure for Table 2, which displays the mathematics education research, the subcategories for the "Type of Reasoning" classification differ slightly because of its treatment in the research literature. To succinctly refer to articles in the ensuing results, we have labelled each article with "S" for science or "M" for mathematics, followed by a numeral.

The two overview tables are organized by the categories of general findings discussed below. The four categories include:

1. Researchers' approaches to investigating children's reasoning empirically;
2. Researchers' analysis of a range of social dimensions observed during children's engagement in reasoning;

3. Researchers' working definitions of reasoning, inclusive of domain-general and domain-specific approaches along with conventional forms to structure reasoning statements; and,
4. Researchers' observations and interpretations of children's ways of enacting reasoning in their learning.

In reporting on each of these categories, we first provide an analysis of each of the subject area's findings and then synthesize by interpreting similarities and differences across science education and mathematics education. Our findings provide a foundation to discuss a response to our research question to suggest reasoning as a critical intersection point in integrating science and mathematics learning.

Approaches to Research in Children's Reasoning

Science Education Research

The science articles in the final literature pool (20 total) were diverse in their age range, research methodology, and evidence for reasoning. Target groups varied from pre-kindergarten to grade 7 (using local definitions of elementary education) and were split evenly between early elementary (PK-3, 11 articles) and later elementary learners (grades 4-7, 12 articles) with three articles spanning the full spectrum of elementary learners (S6, S13, S17). Many science articles explored reasoning through the lens of informal play or exploration, often in three- to five-year-old preschool children (S1, S3, S9, S16). Though formal definitions of reasoning are used in the research as they relate to eventual curriculum, these studies access children before they have been formally taught reasoning processes in a classroom setting.

Research in the science group focused on quantitative methodologies, demonstrated through 13 quantitative, five qualitative, and two mixed methods studies. One science article stated that a mixed methods approach was employed in the study; yet, data was presented solely in quantitative manners (S2). For the purposes of this analysis, we have taken the researchers' intentions into consideration and classified it as a mixed methods approach. Most quantitative studies used a pretest-posttest experimental design. The quantitative studies commonly used a range of standardized and/or validated test items over a temporarily extended research period of a few months to a year. Test questions were used to evaluate student reasoning ability (S1, S2, S9 - S14, S17, S19, S20) through analysis of individual test question completion and grading. The majority of the quantitative experimentation focused either on a reasoning-focused lab or class activity, (S1, S3, S6, S9, S16, S19, S20) or a pre-designed individual task or examination (S2, S10, S11, S12, S13, S14, S17) as their treatment. One study appears to be an outlier, as it examined the effect of teacher professional development on student reasoning in the classroom (S4).

Qualitative research in this group used either interviews (S5, S7, S18) or audiovisual recordings (S4, S5, S8, S15) as their means of investigation. Evidence of student reasoning for both data collection designs was qualitatively coded and interpreted. Student statements of reasoning were largely absent in the science reasoning research, though some articles do contain concrete evidence of student voice (S4, S5, S8, S15, S18). Within these articles, the researchers contextualized their investigation of students' reasoning within students' interactions related to actively solving a problem. Data was collected either during the problem solving or in reflective interviews, which resulted in capturing students' expressions of reasoning that were displayed in the articles through extended exchanges.

Mathematics Education Research

With the mathematics articles, age range was similarly diverse, though older elementary learners were emphasized slightly more. There were six articles focused on PK-3 education (M1, M8,

Table 1

Reasoning in Science Education

Article # First Author (Year)	Method			Grade		Social Dimension			Type of Reasoning			Engagement
	Qual	Quan	Mixed	PK-3	4-7	Ind	Small	Class	Process	Concept	Approach	
S1 Bauer (2019)		X		X		X					C	Infer
S2 Chen (2013)			X		X	X			X	X	C E	Conjecture, elaborate, justify, infer, explain, hypothesize, observe, …
S3 Fernbach (2012)		X		X		X					C	Infer, hypothesize, diagnose
S4 Gillies (2013)			X		X			X	X	X	O	Investigate, predict, explain, question, elaborate
S5 Hackling (2015)	X				X			X	X	X	E A	Justify, critique, claim, explain, counterargue, conclude
S6 Hardy (2010)		X		X	X			X	X	X	E	Hypothesize, experiment, justify, generalize, correlate
S7 Hatzinikita (2005)	X				X	X				X	C	Identify, explain
S8 Kim (2019)	X				X		X	X	X	X	E	Analyze, evaluate, justify, explain, claim, argue
S9 Koksal-Tuncer (2018)		X		X		X			X		C	Hypothesize, test, counterargue, refute, revise
S10 Lazonder (2012)		X			X	X				X	C	Hypothesize, experiment, evaluate
S11 Lazonder (2014)		X			X	X				X	C	Predict, justify, hypothesize, infer, question, investigate
S12 Mayer (2014)		X			X	X			X	X	O	Hypothesize, analyze
S13 Osterhaus (2016)		X		X	X	X			X	X	O	Experiment
S14 Osterhaus (2020)		X		X		X			X	X	O	Hypothesize, experiment, analyze
S15 Paparistodemou (2008)	X			X			X		X		E A	Infer, conjecture, generalize, conclude, argue, interpret
S16 Schulz (2007)		X		X		X					C	Explore
S17 Schiefer (2019)		X		X	X	X			X		O	Hypothesize, experiment, theorize, analyze, synthesize
S18 Tytler (2004)	X			X		X			X	X	E	Evaluate, interpret, explore, explain, generalize
S19 Van der Graaf (2015)		X		X		X				X	C	Hypothesize, experiment, evaluate
S20 Van der Graaf (2019)		X			X			X	X	X	C	Hypothesize, experiment, conclude

Note: Qual = qualitative; Quan = quantitative; PK = pre-kindergarten; Ind = individual; NOS = nature of science; C = causal; E = evidence-based; A = argumentation; O = other

Table 2

Reasoning in Mathematics Education

Article # First Author (Year)	Method			Grades		Social Dimension			Type of Reasoning			Engagement
	Qual	Quan	Mixed	PK-3	4-7	Ind	Small	Class	Logic	Content	Form	
M1 Delay (2016)		X		X			X		X		I	Generalize
M2 Depaepe (2007)			X		X			X				
M3 Flegas (2013)			X		X		X		X		D O	Justify, argue, convince, support, explain, organize
M4 Francisco (2010)	X						X	X	X	X	O	Generalize, justify, convince, explain, describe, explore, …
M5 Gurbuz (2016)		X			X	X				X		Justify, explain
M6 Houssart (2006)	X				X		X	X	X			Conjecture, generalize, modify, analyze, discuss, adapt, …
M7 Hughes (2020)		X			X	X				X		Justify, convince, explain, represent, visualize, …
M8 Hunter (2017)	X			X			X	X		X		Justify, convince, explain, reflect, investigate, …
M9 Jurdak (2013)		X			X	X				X		Generalize, justify, explain
M10 Kumpulainen (2003)	X				X		X			X	O	Hypothesize, argue, explain, organize, represent, test, …
M11 McFeetors (2018)	X				X		X		X		I D O	Conjecture, generalize, convince, explain, refute, …
M12 Mercer (2006)			X		X		X	X	X			Hypothesize, discuss, test
M13 Nunes (2007)		X		X		X				X		
M14 Petrovici (2018)		X		X					X		D	
M15 Reid (2002)	X				X		X		X		I D O	Conjecture, generalize, explain, refute, observe, test, …
M16 Saleh (2018)		X			X	X				X		
M17 Sumpter (2015)	X			X			X		X		D O	Argue, represent, conclude, support, challenge, …
M18 Vale (2016)	X			X	X		X	X	X		O	Conjecture, generalize, justify, convince, verify, extend, …
M19 Vandermaas-Peeler (2015)		X		X			X			X		Predict, explain, conclude, explore, evaluate
M20 Wong (2017)		X			X	X			X		D	
M21 Yankelewitz (2010)	X				X		X		X		D O	Hypothesize, justify, convince, argue, analyze, verify, show, …

Note: Qual = qualitative; Quan = quantitative; PK = pre-kindergarten; Ind = individual; I = inductive; D = deductive; O = other

M13, M14, M17, M19), 14 studies looked at grades 4-7 learning, and only one spanned both age groups (M18). The inclusion of early childhood education also saw less prevalence in the mathematics studies, with only three articles (M14, M17, M19) focused on pre-kindergarten years. In contrast, there were ten studies that focused on the last two years of elementary education (M2, M3, M4, M5, M7, M10, M11, M12, M15, M21).

Mathematics articles were evenly distributed between quantitative and qualitative methods. There were nine articles that adopted qualitative research methods (M4, M6, M8, M10, M11, M15, M17, M18, M21), nine used quantitative methods (M1, M5, M7, M9, M13, M14, M16, M19, M20), and three studies used a mixed methods approach (M2, M3, M12). As with science, methodologies varied throughout the mathematics education articles, although a large subset emerged. These studies shared three major characteristics: they used qualitative methodologies (or mixed methods containing a significant qualitative component), relied on audiovisual recording for data collection, and contained many student statements of reasoning (M2, M4, M6, M11, M12, M15, M17, M19, M21). Explicit evidence of reasoning was much more common in the mathematics studies, with some primarily quantitative research even quoting student statements to support their quantitative coding scheme (M3, M5).

Similarities and Differences

Science and mathematics education research differed somewhat in their approach to exploration and evaluation of reasoning ability. Participants' age focused on the lower grades in the science research studies and higher grades in mathematics. This may have been intentional based on the verbalization of reasoning ability that was often of interest in the mathematics group (M3, M4, M6, M10, M11, M15, M17, M18). Similarly, mathematics did not have as much focus on the pre-kindergarten and kindergarten age groups. This may be due to a common focus in the science subset on reasoning in game-like settings or in a play environment (S1, S3, S9, S16). This existed in only two mathematics articles (M14, M17) and one article that focused on science and mathematics (M19).

Methodologically, there was a clear difference in the science subset towards quantitative research (14 quantitative, five qualitative, one mixed), while mathematics had more balanced research approaches (nine quantitative, nine qualitative, three mixed). Science studies tended to focus on evaluation of individual learner reasoning through standardized testing or as a consequence of the evaluation of novel measurement tools. Even in quantitative mathematics experimentation, reasoning item analysis was less of a focus -- articles seemed more interested in correlational factors like intelligence scores, learning disability status, and classroom relationships (M1, M7, M13, M14).

Within the qualitative research, science and mathematics shared a reliance on audiovisual lesson recording and subsequent coding for evidence of reasoning. Interestingly, this did not translate to the presence of student statements of reasoning in the articles' results sections. Concrete evidence of reasoning was present in 14 mathematics articles and only five science articles. This may partially be a consequence of the increased usage of qualitative and mixed methods in the mathematics subset. This may be further evidenced by the usage of student statements of reasoning that were present on the science side. All five studies containing student quotations were all qualitative or mixed analyses and, as a result, had transcript evidence of these reasoning moments (S4, S5, S8, S15, S18).

Social Dimensions of Engagement in Reasoning

Science Education Research

Science articles mainly focused on students' reasoning ability in an individualized manner. Students worked on activities and researchers examined how they used reasoning to solve the problems. Even if learning occurred in a small group or classroom activity setting (S2, S4, S15, S18), evaluation of student reasoning was still measured through individual testing. In a study by Van der Graaf et al. (2019) (S20), students participated in six inquiry-based lessons and then took pre-, post-, and long-term tests of reasoning skills. Students were engaged in inquiry activities in groups, but their reasoning skills were tested individually. Individual examination occurred for nearly all science articles (exceptions included S5, S6, and S8). Testing was either used as a metric to determine success of an experimental treatment quantitatively (S1-S4, S7, S9-S11, S13, S15, S16, S18, S20) or as a means of item validation for the examination artifact itself (S12, S14, S17, S19). For example, Osterhaus and Koerber (2016) (S14) developed a reasoning testing instrument, the Science Primary School Reasoning Inventory (SPR-I), to measure diverse components of scientific reasoning skills: experimentation, data interpretation, and understanding the nature of science. In this test, students individually worked on problem solving that included inference making based on various conditions.

There were studies (S4, S5, S6, S7, S8, S18) which examined students' reasoning by coding classroom dialogues or conducting interviews. In the Hackling and Sherriff (2015) article (S5), researchers analyzed classroom discourse based on argumentation structure to examine how students provided their reasoning for claims and critiquing and justifying of ideas. In this approach, students' reasoning was understood as a social cognitive process, and was thus examined while students were engaged in social dialogical interactions in classrooms. This approach, however, was uncommon amongst the science subset with only five articles (S4, S5, S7, S8, S18) using a qualitative methodology to analyze group discussions. Hardy et al. (2010)'s article (S6) used collaborative coding of classroom discussions but then translated results into categorical numerical data and analyzed it quantitatively. In these classroom settings, students were engaged in inquiry activities and classroom talk as a whole or small group to discuss their ideas during and for problem solving. Researchers observed and analyzed classroom talk between the teacher and students or students themselves to examine students' reasoning skills.

Mathematics Education Research

Substantial research in mathematics education has resulted in an emphasis on mathematical discourse as an important contributing element to meaningful mathematics learning (Cazden, 2001; Herbel-Eisenmann & Cirillo, 2009; Sfard, 2008). Mathematics articles most commonly used small groups as the unit of analysis for the study. Social engagement was used as a driver for classroom discussion that was then recorded and analyzed for reasoning themes and keywords. The usage of this discussion element was not agreed upon -- most articles used small group or classroom discussion as a tool to elicit reasoning talk (M2, M3, M4, M6, M8, M15), but did not intend to explore the effect of that social dynamic itself. Houssart and Sams (2006) (M6), for instance, detail the importance of group discussion on exposing reasoning thought, but focus their analysis on successful reasoning on game strategies. Depaepe et al. (2007) (M2) even noted the likely effect of social interaction, but concluded that their data was not sufficient to explore the social effect on reasoning. Other studies focused on the importance of the collaborative impact of social interaction on reasoning itself (M1, M3, M10, M11, M12, M17, M18, M21). These were all qualitative studies that analyzed small group or classroom discussion transcripts except for the notable Delay et al. (2016) study (M1). This experiment was unique in its examination of the effect of peer dyads on mathematical reasoning ability. Though there was an implied criticality to the relationship under investigation, this study was quantitative and interactions between dyad pairs were not observed directly.

There was a small subset of mathematics articles that focused on individual reasoning ability (M5, M7, M9, M13, M16, M20). Most of these studies were quantitative experiments (exception M7), with a strong emphasis on exploring correlations to reasoning ability using standardized measurement instruments (M5, M9, M13, M20). Generally, this research demonstrated that students' ability to reason mathematically impacted their overall achievement in content-specific mathematics topics. The exception article by Hughes et al. (2020) (M7) focused on the importance of critical mathematical vocabulary in communicating reasoning through students' written responses. The researchers noticed that students' written reasoning statements were limited, and researchers reported results through summary of data rather than display of students' responses.

Similarities and Differences

Scientific and mathematical reasoning take dramatically different approaches to understanding the social dimensions of reasoning in the articles we reviewed in this study. The science articles focus mainly on evaluating students' individual reasoning ability. Even in those articles that explore classroom discussion and/or small group environments, reasoning was still often tested at the individual level. This was mostly done in pen and paper examinations or via one-on-one interviews with researchers. Mathematics had a small subset of studies that took this approach, but the majority of mathematics articles focused more on evidence of the reasoning process as it presented itself in group conversation. It was much more common in mathematics to use qualitative methods to record, code, and analyze verbal discussion than it was in science. This may have been a result of the focus on higher elementary grade levels and thus stronger communication abilities. Or it may have tied to outcomes or learning goals in mathematics that focus on the explanation of thought processes.

Defining Reasoning and the Types of Reasoning Invoked in the Research

Science Education Research

Scientific reasoning is defined or discussed with various terms and processes in the articles. Some articles explained scientific reasoning with a specific definition (S2, S3, S6, S7, S9) and some articles explained the elements of scientific reasoning (S4, S5, S8, S10, S11, S12, S13, S14, S17, S19, S20). Some articles did not explain what scientific reasoning was but suggested key ideas such as inference (S15), causal relationship (S16) and coordination of theory and evidence (S18). For instance, Chen and She (2013) (S2) paraphrased definitions of scientific reasoning from various scholars such as "science reasoning skills are the ability to define a scientific question, plan a way to answer the question, analyze data, and interpret results" (National Research Council, 1999, p. 3). Lazonder and Kamp (2012) (S10) explained that "children can start to develop proficiency in the scientific reasoning skills of hypothesis generation, experimentation, and evidence evaluation" (p. 69). Lazonder and Kamp (2012) did not state the definition of scientific reasoning, yet their statements on the components of scientific skills explain what they emphasize as scientific reasoning. Based on the review of the articles, the key ideas to explain scientific reasoning are categorized as follows; causal reasoning (S1, S2, S3, S7, S16), the coordination of theory and evidence (S2, S5, S16, S18), hypothesis, test, and evidence evaluation or analysis (S2, S4, S9, S10, S11, S12, S13, S14, S17, S18, S19, S20), control of variables or cause-effect relationship (S10, S11, S19, S20) and evidence-based, including data-based, information-based, and/or rule-based processes (S2, S5, S6, S8, S15). In the category of causal reasoning, we included articles that emphasized cause and effect relationships, causality, and causal inference. The category of the coordination of theory and evidence also includes claim-evidence relationships. When researchers emphasized the process of hypothesis, test, data analysis, and conclusion, they were categorized in hypothesis, test, and evidence evaluation or analysis. There were also studies that particularly focused on students' understanding of the control of variables as scientific reasoning during inquiry processes. Also, some researchers employed the term evidence-based to emphasize the importance of data, information, theories, rules, and so on as evidence to justify one's claims.

Even though scientific reasoning was explained with different terminologies in different contexts, there were core values of students' scientific reasoning amongst the articles, that is, understanding of the relationship between claim and evidence. In science classrooms, students are engaged in diverse science inquiry activities to develop reasoning and problem-solving skills. In a problem solving process, students attempt to design methods, gather data (or information), and suggest conclusions and solutions to the problems. In the process, students need to understand how the problem, test design, data, and conclusion are related and to justify their conclusion in relation to data and the problem. Thus, it is critical that students make, evaluate, and justify claims based on evidence in scientific problem solving and communication. In the relationships between claim and evidence in scientific contexts, claims include hypothesis, prediction, inference, theory, and conclusion. Evidence includes data, information, knowledge, theory, etc. For instance, students make a claim that the air moves up and expands when it is heated (claim) on the phenomenon of hot air balloons floating (evidence). In scientific explanations, students need to explain how evidence supports, refutes, or revises claims, and that is evidence-based reasoning. As a classroom example, the articles (S2, S4, S6, S8, S9, S10, S11, S12, S15, S18, S20) explain that students are engaged in scientific investigations which include questions, hypothesis-making, testing, and conclusion and in this process, students need to understand the relationship among questions, hypothesis, data from testing, and conclusions. Additionally, the importance of students' causal reasoning is emphasized in controlling variables to test the hypothesis and in justification for the conclusion based on data (evidence). Overall, understanding and explaining how claim and evidence are related, that is, evidence-based reasoning, is presented as the key element of scientific reasoning in the articles.

To examine and discuss students' scientific reasoning skills, some researchers developed students' activities in certain science concepts or topics and some developed activities in general contexts of causal reasoning or relationships. Articles that discussed specific science concepts can be categorized in the content area of physics (S2, S4, S6, S8, S10, S11, S 18, S19, S20), chemistry (S2, S5,S7, S18), biology (S15, S18), and earth and space science (S2). Further, S13 and S14 focused on the Nature of Science. Despite the variety of science topics and content areas, only a half of the articles (S2, S5, S6, S13, S14, S18) mentioned the relationship between conceptual development and scientific reasoning, and the rest used the topics to contextualize students' inquiry and reasoning process in science content areas.

Mathematics Education Research

In mathematics education research, reasoning is of great interest to researchers within a broad range of perspectives on what constitutes reasoning demonstrated through varied definitions, contexts of mathematical topics, and conventional structures. Close to half of the articles do not include a definition of mathematical reasoning (M2, M3, M4, M8, M9, M12, M13, M14, M19), which is not a surprising finding considering Lithner and Palm's (2010) acknowledgement that mathematics educators tend to rely on an "implicit assumption that there is a universal agreement on its meaning" (p. 285). Analyzing the remaining articles that did state the researchers' definition of reasoning does not reveal a universal agreement. Articles that provided an explicit definition of reasoning described it as a process that is a way of thinking that is logical and systematic (M1, M5, M6, M11, M15, M17, M18) that is seen as inherently mathematical. Others described the product of reasoning as a focus in their definitions, rather than a process, as arriving at explanation (M7, M10), conclusion (M16, M21) or solution (M20). Reid (2002) (M15) contains the most deliberate development and discussion of reasoning among this corpus, contributing "*ways* of reasoning…, *needs* to reason…, *formulation* or awareness of reasoning" (p. 6, emphasis in original) to the field's investigation of students' mathematical reasoning. Many of the articles point to a detailed list of specific actions of reasoning to animate their explicit definitions (M5, M11, M15, M17, M18) or as a way to indicate what reasoning looks like and focus their investigations (M3, M4, M6, M7, M9, M10, M16, M21). These actions will be explored further in the fourth category below.

Situating these conceptualizations of reasoning, researchers explored students' mathematical reasoning within varied contexts and structures. In many of the articles, researchers referred to mathematical reasoning, or logical reasoning, as the focus of their investigation (M1, M3, M4, M6, M11, M12, M14, M15, M17, M18, M20, M21). These authors foregrounded students' actions of logical thinking as they engaged in problem solving tasks that happened to be related to mathematical topics or in domain-general tasks. Examples of domain-general tasks include abstract strategy games (M6, M11, M14) and outdoor play (M17). Focusing on logical thinking, researchers investigated the nature of students' use and/or development of mathematical reasoning. All of the qualitative studies contain thick descriptions of students' statements of reasoning that illustrate both students' actions and the way students structured their mathematical reasoning through conventional forms. These forms included mainly deductive reasoning (M3, M11, M14, M15, M17, M20, M21) and inductive reasoning (M1, M11, M15), but were also supplemented by other forms, such as indirect reasoning (M4, M11), reasoning by cases (M4, M21), or metaphoric and analogic reasoning (M10, M11, M12, M15). A smaller number of articles foregrounded mathematical topics and studied students' achievement within these domain-specific areas given that reasoning occurred. The mathematical topics ranged from reasoning about quantities (M4) to fractions (M4, M7, M16) to computations (M5, M13) to algebra (M9). Overall, these studies did not identify the conventional forms of reasoning, as the focus varied, and most of the studies were quantitative, where students' statements of reasoning were not collected. Additionally, a few articles foregrounded qualities of communicative interaction in social situations discussed above as a context for reasoning (M2, M8, M10, M19), where results focused on the nature of communication rather than qualities of reasoning.

Similarities and Differences

In all the articles in both science and mathematics, researchers established that the investigation and understanding of students' reasoning is critical to pursue to improve learning in elementary school classrooms. Consensus is apparent; and yet, the complexity of how reasoning can be characterized often leads to implicit use of the term "reasoning" or absence of a clear definition by researchers. This commonality between science and mathematics research represents a shared-as-given understanding of reasoning that is held by researchers, even if it is not coordinated or goes so far as to be contested. More often, researchers chose to include a list of ways that students could engage in reasoning to illustrate their stances on what constitutes reasoning, where science emphasized different approaches to reasoning (e.g., causal, theory-evidence coordination, inquiry cycle), while mathematics referred to both the processes (e.g., conjecturing, exploring, analyzing) and product (e.g., explanation, conclusion, solution) of students' reasoning. The illustrations of reasoning point to a difference in science education researchers consistently situating specific moments of reasoning within the broader frame of the scientific inquiry process, while mathematics education researchers employed notions of logical thinking as a broader frame within which they identified processes that lead to products.

One of the ways that researchers in both science and mathematics emphasized the importance of investigating reasoning is highlighted in the contexts of their studies. In both subject areas, almost two-thirds of the research was conducted within domain-general areas related to either science or mathematics. In this way, science researchers were emphasizing students' scientific inquiry process or causal relationships and mathematics researchers were emphasizing students' logical thinking. The remaining articles explore students' reasoning but foregrounded domain-specific content, both in science (e.g., sound, electricity, changes to materials) and in mathematics (e.g., quantities, computations, algebra). While differences in content occur because of the different subject areas, the similarity in primarily focusing on students' thinking while problem solving and how they reached a solution indicates that the two fields are united in their purpose for understanding students' cognitive processes during moments of reasoning.

Investigating the structures within which students experience and express reasoning was evident in both the science and mathematics reports on research. An intersection between scientific and mathematical reasoning appears in common conventional forms of reasoning such as inductive (mathematics) or evidence-based (science), deductive (mathematics) or causal (science), and several other forms that emphasize the relational aspect drawing on several ideas (science) or representations (mathematics). In science, the coexistence of inductive and deductive reasoning in students' thinking within an inquiry activity occurred more frequently as researchers indicated that experimentation begins deductively as students hypothesize based on known facts or prior experiences, then incorporate inductive reasoning, as they create a claim based on evidence from analyzing data. Mathematics research differed, in that researchers observed and interpreted distinct moments of inductive, deductive, or many other well-defined forms of reasoning in excerpts of students' logical thinking rather than looking at how the different conventional forms of reasoning are coordinated. Despite distinctions in focus related to reasoning structures, there are possible opportunities for reciprocity between subject areas to enhance students' interdisciplinary learning through reasoning: science research could begin to identify more varied forms of reasoning, while mathematics research could begin to coordinate the forms of reasoning.

Students' Engagement in Reasoning Processes

Science Education Research

Teaching science as a process has been emphasized for decades to improve students' scientific reasoning and problem-solving skills (Harlen, 1999). Science process skills include basic process skills (observing, inferring, measuring, using tools and equipment, communicating, classifying, predicting, etc.) and integrated process skills (formulating hypotheses, controlling variables, interpreting data, experimenting, formulating models, etc.). These terms have been widely used to describe students' engagement and actions during scientific inquiry and problem-solving tasks, in addition to the learning objectives that students are expected to develop in science classrooms. Researchers in the articles demonstrated these process skills to explain what process students were engaged in to practice scientific reasoning. These researchers provided students with tasks that engaged basic process skills such as observing and inferring (e.g., S1) and also integrated skills such as experimenting, controlling variables and justification (e.g., S10, 12, 17). When students are engaged in problem solving inquiry tasks, basic process skills are integrated into the complex process skills. For instance, students' observing and using tools and apparatus are all part of the experimenting process. In the hypothesis-verification process, students' engagement in reasoning is more complex, with the process of questioning, hypothesizing, testing, analyzing, and justifying conclusions. In these processes, scientific reasoning, such as the coordination of claim and evidence and justification with evidence, is practiced and developed.

In the articles, many researchers emphasized students' knowledge and skills of evidence-based reasoning and causal relationships, and thus provided a hypothesis-verification process to examine and develop students' scientific reasoning. Starting from questions, students develop, test, and justify hypotheses through controlling variables, analyzing, and justifying with evidence. Students' inquiry process was explained with the verbs of process skills. The verbs can be categorized in the processes of making claims (question, infer, hypothesize, predict, conjecture), testing claims (test, experiment, explore, control variables), analyzing (analyze, interpret, examine, evaluate, synthesize, conclude), and justify (argue and counter argue, critique, refute, generalize, explain, theorize). Some verbs such as investigate and research pointed to the entire inquiry process, and thus, were not included in this categorization. Some verbs such as explain, theorize, or conclude could be included in various categories such as making claims and justifying.

Mathematics Education Research

A critical focus of reasoning in mathematics education is investigating the processes students invoke that can be categorized as reasoning, and here we refer to as the ways in which students engage in mathematical reasoning. Jeannotte and Kieran (2017) identify these processes as one of two aspects of mathematical reasoning. The processes of reasoning are commonly represented by a broad range of verbs, where the verbs are signifiers of actions taken by students and demonstrate how they go about engaging in mathematical reasoning. Inclusion of statements of students' reasoning or examples of the ways students engaged in processes of reasoning within articles' results followed methodological distinctions: quantitative studies contained no examples (M1, M7, M13, M14, M16, M20) or minimal examples (M5, M9, M19); mixed methods contained varied amounts of student data (M2 had no examples, M12 had limited examples, and M3 had many examples); qualitative studies contained many illustrations of students' engagement in reasoning (M4, M6, M8, M10, M11, M15, M17, M18, M21). Articles that contained many examples tended to focus their investigation on logical thinking (all except M8 and M10) and portrayed the complexity of students' reasoning through many different actions; whereas, articles with limited examples emphasized domain-specific reasoning (except M12) and the product of student reasoning with little variation in actions.

Researchers incorporated students' data and then interpreted it by labeling the action with a particular verb that highlighted the specific reasoning process. Our analysis revealed six clusters of verbs linked thematically; however, it must be noted that these clusters are not mutually exclusive nor applied in a linear fashion in that students' actions are complex and contribute to different purposes at different moments in their problem solving. Conjecturing is giving "a 'conscious guess'" (Lakatos (1976) as quoted in Houssart & Sams (M6), 2006, p. 60), which are "put forth without being considered to be true or false--as something subject to testing" (Reid (M15), 2002, p. 14) and sometimes labeled as a hypothesis or prediction. While an initial conjecture was often made after some mathematical activity, like building and observing a pattern (M10, M15, M18, M21), it can also be revised throughout problem solving to respond to counterexamples (M6, M11, M15).

Investigating (M4, M8, M10, M11, M12, M15, M18, M19) may occur near the beginning of the activity when students explore and play, when observing and acting within the problem with various approaches, and even near the end as they test emerging generalizations. Rather than the reasoning imbedded in investigating remaining ephemeral, researchers highlight the importance of students representing (M3, M4, M6, M10, M11, M17, M21), or tangibly demonstrating with communicative "tools to assist the students' sharing" (Kumpulainen & Kaartinen (M10), 2003, p. 366), to make external and visible their reasoning so that it can be consolidated and interrogated by others. Articles report a wide variation of representations, including show, use manipulatives, gesture, draw diagrams, make images/pictures, organize, describe, discuss, and label. Students used their representations to systematize their investigating, which moves their actions toward analysis (M6, M11, M15, M17, M21), or "consider more completely" (McFeetors & Palfy (M11), 2018, p. 115) strategies and patterns expressed earlier to begin moving to generalizing. Researchers pointed to analysis as a process of mathematical reasoning using verbs like visualize, modify, adapt, specialize, compare, contradict, give counterexamples, and refute.

Students' engagement in the range of activities eliciting mathematical reasoning result, in all the studies, in expressions of generalizing and justifying. Generalizing occurs through students "finding a similarity" and establishing a "a general formula or fact or a meaning of an object or idea" (Vale et al. (M18), 2016, p. 876) and can also be described with the verbs like concluding, relating, and extending. Examples include creating winning game strategies through repeated success of discrete moves across games (M6, M11), creating a rule from a pattern (M3, M4, M9) or a statement accepted by the group (M15, M17, M18, M19). While generalization is illustrated in just over half the articles, a greater emphasis is placed on students' justification (all articles expect M6, M12) as a process to "construct generic arguments and persuade" and "develop mathematical arguments" (Flegas & Charalampos (M3), 2013, p. 71). The articles seem to advocate for students supporting a solution is more important in the reasoning process than arriving at a solution (generalization) itself. Justifying is represented by other verbs such as convince, argue, explain, support, reflect, and evaluate where the commonality is that the students are using various representations to defend, in a convincing manner, their processes of mathematical thinking and solution to the task as an invocation of mathematical reasoning.

Similarities and Differences

In both science and mathematics articles, students' reasoning is emphasized through processes rather than outcomes of thinking and knowledge. Researchers designed their studies to engage students in the overall process of investigating, researching, and problem solving to understand how students develop their ideas scientifically or mathematically. These researchers characterized the processes of students' thinking and reasoning with various verbs in the categories of testing ideas (predict, hypothesize, experiment, collect data, manipulate materials), reaching solutions (analyze, synthesize, conclude, generalize), and justifying (explain, argue, evaluate, justify), especially if there was a clear emphasis on justification in both areas. For instance, in science education, students need to explain why certain information can be evidence to certain claims (evidence-based reasoning) or how conclusions are justifiable and valid through controlling variables (causal reasoning). In mathematical problem solving, students need to explain how they reached certain solutions (patterning, use manipulatives, gesture, represent with diagrams or images/pictures, organize, explain, discuss, and label) and how their solutions need to be supported deductively as an early process leading toward constructing proofs (conjecture, analyze, synthesize, generalize, justify).

Students' engagement in scientific reasoning is often discussed in the whole process of scientific inquiry. Students practiced scientific inquiry processes, which developed scientific reasoning. Thus, scientific inquiry skills often denoted students' reasoning skills and vice versa. In mathematics articles, the whole process of and the phases of problem solving are not emphasized in the discussion of mathematical reasoning. Certain processes such as generalization or justification are incorporated as one specific skill of mathematical reasoning. As the hypothesis-based inquiry has been emphasized in science education, hypothesizing (conjecturing in mathematics) and related processes such as controlling variables are frequently discussed, which is rarely mentioned in the mathematics articles.

There are the discipline-specific terms and differences between science and mathematics education (see Table 3). Yet, it is also evident that students' reasoning is explained as a process rather than a product and that students understand their meaning-making process of how questions, phenomena, and knowledge are related, interact, and are justified for problem solving. Encountering questions and problems, students are engaged in reasoning processes to develop reasonable and logical explanations and justification to their questions that are represented in a variety of ways. We acknowledge that similarities are evident in the different terms and actions.

Table 3

Reasoning Verbs and Emphasis in Science and Mathematics Education

	Science reasoning & problem solving	Mathematics reasoning & problem solving
Verbs	• question • predict, hypothesize, infer • experiment, manipulate materials, control variables, collect data • analyze, interpret, synthesize • conclude, generalize, • explain, evaluate, examine, argue, justify, theorize	• question • conjecture, predict • investigate, explore, inquire, examine, use manipulatives, systematize • analyze, synthesize, pattern, integrate, modify/adapt, evaluate • generalize, classify, pattern • convince, explain, gesture, represent, justify, refute, argue, discuss, label
Emphasis	• evidence-based reasoning (claim-evidence-justification) • causal and relational reasoning	• logical, systematic thinking using varied conventional structures to present justifications such as inductive, deductive, analogic, imagistic, indirect, metaphoric, by contradiction, etc.

Discussion

Based on the systematic review, similarities and differences in educational approaches to scientific and mathematical reasoning were evident. The terms and pedagogical contexts for students' problem solving were different and diverse according to the disciplinary foci of science and mathematics. Yet, students' reasoning processes were similar as a logical and holistic meaning-making process both in science and mathematics. Once problems are framed and suggested in classrooms, students explore the problems and ways to find answers, develop their answers, and evaluate why their answers are reasonable and justifiable. This process of reasoning in problem solving processes needs to be understood as interdisciplinary, rather than a disciplinary-specific approach belonging exclusively to science or mathematics.

An interdisciplinary approach for science and mathematics learning in elementary education requires students to develop knowledge integration and skills in interdisciplinary problem-solving processes (Honey et al., 2014). Students approach complex STEM questions by seeking, testing, justifying, and negotiating knowledge claims. Reasoning skills are critical throughout STEM problem solving in science and mathematics education. Scientific and mathematical reasoning skills are emphasized within their individual disciplinary domains, and yet, with the recent interest in a STEM-centered approach to teaching (English & King, 2015; Estapa & Tank, 2017; Stohlman et al., 2012), students' reasoning process is increasingly interdisciplinary (Mayes & Gallant, 2018; Tan et al., 2022). In our findings, domain-specific terms and emphases were well defined and researched in science and mathematics education. As examples, skills could include hypothesis testing or scientific method in science, and conjecture and methods of proof in mathematics. Though similar in purpose and implementation, terminology differences make direct comparisons and cross-curricular connections more difficult. Our findings show that both subject areas emphasize students' logical thinking (inductive, deductive, abductive, etc.) to seek out solutions and justify the process and products of their problem solving. When students' reasoning skills are integrated across science and mathematics, however, direct translation of meaning is not inherently obvious and thus pedagogical discussion needs to follow. A key takeaway from these findings is that the development of students' understanding of the relationships between problems, observations, data, and knowledge claims, rather than focusing on the discipline-specific terms and characteristics, is critical to the advancement of reasoning in science and mathematics education.

We suggest the term 'STEM reasoning' to describe the interdisciplinary nature of reasoning that could be shared by students in science and mathematics classrooms. To occupy that theoretical space, STEM reasoning would have to take on a broad meaning and incorporate both the processes of student engagement and subsequent ways of thinking through problem solving processes. It would aim to identify the finely nuanced facets of reasoning that science and mathematics both offer independently and incorporate them into the process of STEM problem solving. This approach is not, however, intended to focus on the historic integration of science and mathematics -- typically using mathematics formulae as a simple, procedural tool to solve the various substeps of complex science or technological problems (Bosse et al., 2010; Bursal & Paznokas, 2006). Rather, from this literature review we are suggesting that the field can aim for much higher cognitive activities, integrating mathematics-style logic, systems thinking, and methods of proof into approaches of nature-of-science style scientific inquiry.

There is some research to suggest that authentic integration of science and mathematics can be effective (Treacy & O'Donoghue, 2014). Existing definitions for subject-specific reasoning that overlap may converge or disappear through the process of students' problem-solving inquiry. There may also be room for new terminologies that describe complex co-curricular thinking that did not previously occur. Overall, we suggest that STEM reasoning could exist as an umbrella term for the thinking and reasoning that science and mathematics share in their approach to solving problems because of the similarities we identified in students' engagement in reasoning across both disciplines. Students' reasoning and decision-making process in problem solving contexts is integrated with diverse knowledge, skills, and values in problem contexts. Thus, we believe a key takeaway is that a focus and use of STEM reasoning could lead to more authentic and meaningful integration of science and mathematics learning experiences for students.

STEM reasoning in this study focused on the intersection of scientific and mathematical reasoning. There remains an unexplored area in how STEM reasoning may extend to the other area disciplines of technology and engineering. Finding the intersection points between scientific and mathematical reasoning and their engineering (Tan et al., 2022) and technology (Kennedy & Kraemer, 2018) parallels would be critical to formalizing the idea of STEM reasoning. In acknowledging this limitation, we hope to see expansion and clarification of STEM reasoning emerge in future research, consolidating reasoning integration in all aspects of the STEM disciplines. Slavit et al. (2021) recently posited that there is precedent for the consolidation of reasoning skills and abilities in science and mathematics and seem to agree that this happens frequently in practice, but lacks epistemological foundation in the literature. This may include an exploration of the intersection in reasoning with critical thinking and design engineering (Silk et al., 2009; Siverling et al., 2017) or with computational thinking and computer science (Kennedy & Kraemer, 2018; Olabe et al, 2014; Weintrop et al., 2016). Inclusion of these subjects is difficult in this study, however, as relevant literature in both engineering and computational thinking is lacking at the elementary level. Students' reasoning to solve every day-related problems is not a sole discipline-based reasoning but an interdisciplinary logical thinking process to reach out solutions. The demarcation among disciplinary thinking and reasoning is neither present nor meaningful. To understand and develop interdisciplinary STEM reasoning, cross-curricular and problem-based approaches could be useful. This could be approached through cross-pollinated activities involving both science and mathematics curricula. Expanding science and mathematics into STEM integration could occur through curriculum-ambiguous activities that apply science and mathematics more subtly -- perhaps through broad, project-based learning activities or digital or analog gameplay.

Regardless of the curricular grounding for future research, there is much in the way of methodology and experimental design that can be shifted among science and mathematics. Our findings show that science could gain valuable insight into student reasoning processes from shifting more frequently to a qualitative methodology. Despite the importance of integration of diverse knowledge, skills, and values in collective STEM knowledge building and problem solving, students' reasoning abilities were examined in individual contexts in most of the studies in our review. As STEM problem solving requires integrated, collective, and social domains of learning (Crippen & Antonenko, 2018), students' reasoning needs to be examined and evaluated in interactive and social dimensions. Evidence derived from students' conversations and interactions in the classroom could help bridge the gap from individual student moments of reasoning to the overarching nature of science problem solving. Conversely, our findings show that mathematics researchers may benefit from the application of some of the reasoning inventories developed in science education. Taken together, STEM reasoning research would be invited to include science and mathematics education standards -- both to ground it in previous literature and to fully realize the benefits of their subject counterparts. However, this would need to be expanded to technology and engineering reasoning and problem solving. STEM reasoning could provide a unique approach to STEM education that combines individual disciplinary thinking into a common critical approach to problem solving.

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Appendix A: Science Education Articles

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