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A Comparative Study of Fifth-grade Mathematics Textbooks Used in Turkey and Singapore

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ABSTRACT

This study reports on comparing the presentation of mathematical content, types of responses required of students, conformity with the curriculum outcomes, and potentially confusing expressions in the subjects of percentages, triangles, and quadrilaterals of fifth-grade mathematics textbooks used in Turkey and Singapore. Document analysis of the two mathematics textbooks indicated that both books remained committed to achieving the curriculum outcomes, that the content of the Singapore textbook was more simply organized, and that the Singapore textbook included more visual representations and multiple solution strategies. The analysis also showed that the questions requiring numerical responses were common in both textbooks, while the Singapore textbook included relatively more questions requiring both response and explanation. The findings also indicated that the Singapore textbook was comparably more error-free; however, the Turkey textbook involved several difficulties concerning syntactic errors, ambiguity, unrealistic task presentation, redundant operation, unclarity in mathematical purposes, and weak mathematical context. We discuss the usefulness and intelligibility of mathematics textbooks regarding established cultural and philosophical traditions and trust and respect for textbooks.

Keywords: comparative study, curriculum outcomes, mathematics textbooks, content presentation

Introduction

Mathematics textbooks are viewed as powerful instructional tools that could act as sources for teaching activities and instructional ideas and hence, provide a particular version of curricular content in a specific sequence (Reys et al., 2004). In so doing, textbooks provide a vision on what and how teachers should teach and how students will learn (Dole & Shield, 2008; Fan & Zu, 2007). Schmidt et al. (1997) define textbooks as micro-organizers of in-class activities, emphasizing their effects on teachers and teaching. This realization led many to focus their attention on understanding the links between teachers and textbooks. Research in this direction produced compelling evidence that mathematics textbooks significantly determine pedagogical approaches adopted by teachers relevant to the delivery of certain contents (Haggarty & Pepin, 2002). For example, Fan and Zhu (2007) state that many mathematics teachers use textbooks as a source while structuring their teaching approach. In addition, the research provides evidence that textbooks affect teaching strategies (Fan & Kaeley, 2000), and points to the similarities between textbook style and teacher's instructional style (Krammer, 1985). Research findings, however, do not necessarily indicate that teachers readily follow and always comply with the textbook prescriptions. It is well established that teachers are selective while choosing textbooks and even adapting certain parts from within the book (Love & Pimm, 1996; Remillard, 2005). Brown (2009) argues that affordances and constraints of textbooks affect teachers' utilization and instructional beliefs and pedagogical understandings shape their reliance on and selection of the content from within the text. The author also emphasizes that this bidirectional relationship had a significant effect on instructional practices in the classroom.

Considerable research attention has also been paid to theorize the connections between the curricular documents and textbooks. For instance, Valverde et al. (2002) classify curriculum into four categories: intended, enacted, potentially implemented, attained and, they define textbooks as the potentially implemented curriculum within this framework. For them, by functioning as mediators between intended and enacted curriculum, mathematics textbooks contribute to the attainment of instructional goals through strategies, activities, and applications. Such a view accentuates textbooks as de facto curriculum (Harwood, 2016), and as materials that bridge formal curriculum goals with in-class teaching (Schmidt et al.,1997). In effect, textbooks could be viewed as envoys that largely influence students' mathematics learning opportunities (Stylianides, 2009) and tools that enable the feasibility of curriculum.

Effects of textbooks on students' learning is another research focus in this area. Remillard (2018), for example, states that textbooks are designed to guide, support, and facilitate learning. On the other hand, Macintyre and Hamilton (2010) provide evidence that selection of content and its presentation have a potential effect on students' participation in mathematical activities and hence their achievement. In addition, Sievert et al. (2019) conclude that mathematics textbooks have a significant impact on students' skills to develop creative strategies, and choose and use appropriate solution methods. Mathematical skills to be acquired by students can be present at different levels in textbooks. For example, one textbook may emphasize multiple solutions while another on problem-solving strategies. Sharing this observation, Valverde et al. (2002) note that the features that make textbooks different from each other can create different mathematical expectations and cause variations in students' mathematical performances and achievement levels.

Research studies have also been conducted to gain insights into the features and components of effective textbooks with regard to student learning and achievement. A typical mathematics textbook includes exercises, problems, tasks, activities, worked examples, strategies, definitions, models, and representations designed to enable learning (Remillard, 2018). The researchers relate each of these components and how they are employed in the creation of the content to the effectiveness of textbooks on student learning and achievement. That is, mathematics textbooks, with their pedagogical potential, and physical and structural features, provide affordances and constraints for students' learning opportunities (Hadar, 2017; Törnroos, 2005; van den Ham & Heinze, 2018). For instance, instructional tasks that encourage students to use multiple solution strategies, use multiple representations, and make explanations are found to be positively correlated with student achievement (Stein & Lane, 1996).

Further, the research establishes links between the textual features and students' attitudes toward and understanding of the content. For example, Zevenbergen's study (2001) offers evidence indicating that students need more help while structuring the information in a text-intensive textbook. With a focus on structural features of textbooks, Fan (2010) puts particular importance on consistency, intelligibility, and error-freeness of the content as discriminating characteristics of effective textbooks. Czeglédy and Kovács (2008) point to the necessity of simple, clear, and concise language and design features that make textbooks easy to follow. O'Keeffe (2013, p.8) relates these features to the notion of readability which is defined as "a number of factors which influence the reader, including interest and motivation, legibility of the print, complexity of the words and sentences in relation to the ability of the reader". The research insights shared hitherto suggest that structural and physical features such as intensity of text, the simplicity of the language, the level of error-freeness, readability, and intelligibility of the texts impacted student learning.

Comparative Studies on Textbooks

The textbook can be considered a cultural and historical artifact that mediates and is mediated by teaching and learning traditions by its nature (Rezat, 2006). Cultural and social values affect the development of textbooks, which influences the cultural and social values of future generations (Fan, 2013). Having been produced with the influence of social, cultural, and historical values, textbooks are unique to particular societies in which they are developed. That is, textbooks have features that make them different from those produced by other societies or nations. Such peculiar features of textbooks are described as 'textbook signature' (Charalambous et al., 2010). Comparisons of mathematics textbooks from different countries are essential in coming to know particular textbook signatures and, in turn, shed some light on what societies can learn from each other in teaching and learning mathematics. This line of thinking constitutes one of the motives for researchers to perform comparative analyses of textbooks used in different countries (e.g., Kang, 2014; Takeuchi & Shinno, 2020). The researchers of comparative textbook studies aim to understand and explain the cross-national differences and similarities between learning opportunities presented by different mathematics textbooks (Charalambous et al., 2010), and how content selected by the textbooks vary across countries (Takeuchi & Shinno, 2020). By producing knowledge on instructional practices within particular cultural and educational traditions, comparative studies also allow and encourage the best instructional ideas to travel across borders (Liu, 2019).

A few comparative studies have recently been carried out in Turkey. In this regard, the Turkey textbook has been compared with, for example, that of the USA (Kar & Işık, 2015), Japan (Acar, 2019), and Canada (Kul et al., 2018). One of the main drivers of research on mathematics textbooks in Turkey is Turkish students' consistently low level of achievement in tests such as the Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA). This achievement landscape seems to have prompted research on Singapore's mathematics textbooks, a country with consistently high achievement in these tests. However, the existing comparative research on mathematics textbooks of Turkey and Singapore predominantly focus on the concept of cognitive demand (Engin, 2015; Özgeldi & Esen, 2010; Reçber, 2012; Yılmaz, 2018) and questions (Özer, 2012). Erbaş and Alacacı (2009) compared sixth and seventh-grade mathematics textbooks used in Turkey, the US, and Singapore. These authors found out that the number of activities in mathematics textbooks used in Turkey is higher, textbooks of Turkey and the US included comparably more activities that are connected with real life, Singapore textbooks adopted a more abstract approach, and Turkey textbooks lack significant definitions of concepts. This study also demonstrated that Turkey textbooks have deficiencies in worked examples; which included less modeling compared to Singapore textbooks, while exercises are more enriched through visual aids in both Singapore and the US textbooks. In another study, Erbaş et al. (2012) examined sixth-grade mathematics textbooks used in Turkey, the US, and Singapore in terms of text density, visual elements, organization, weight and the number of topics, and presentation of topics. These authors found that textbooks used in Singapore are distinguished due to their low text density, use of rich visual elements, smooth organization, clarity, and simplicity. A study with a similar focus comparing Turkey 10th grade mathematics textbooks and Singapore mathematics textbooks by Sağlam (2012) showed that less complex problems that require recognition of knowledge/concepts and solution of one-step problems are more common in Singapore textbooks, while Turkey textbooks are richer in terms of student-centered activities.

Although studies comparing Singapore and Turkey mathematics textbooks produced significant results, these studies largely fall short in specifying structural aspects that need to be improved in textbooks used in Turkey. One such inadequacy is related to the approach adopted by textbooks in presenting the content. Hence, comparative studies are needed on textbook approaches adopted to organize and present the mathematical content. Another issue that demands further consideration is textbooks' conformity with curriculum outcomes and potentially confusing expressions in textbooks. One might tempt to think that there is less need for research on textbooks' conformity with curriculum outcomes, and on potentially confusing expressions. This is on account of the assumption that textbooks are normatively developed based on the curriculum outcomes with expectations of rigorous proofreading and a quality assurance process in the course of textbook development. However, in an examination of two secondary school ninth-grade mathematics textbooks, Akkaya (2016) observed that certain curricular standards and acquisitions were ignored and further determined that scientific and syntactic errors were present.

Furthermore, teachers interviewed in Hıdıroğlu's (2016) study mentioned some errors in a different fifth-grade mathematics textbook and stated that these errors formed barriers for learning. Similarly, Çakır (2009) evaluated a fifth-grade mathematics textbook in line with the teachers' views. His findings revealed that most of the teachers in this research thought that the fifth-grade mathematics textbook contained mathematical errors and redundant details. These findings point to the significance of analyzing mathematics textbooks in terms of their conformity with curriculum outcomes and the potential content errors.

Instructional tasks that encourage students to explain how they arrive at an answer or to justify their solutions are found to be important for learning more than simply asking for numerical answers to a question (Stein & Lane, 1996). It is also stated that these tasks in textbooks do not necessarily lead students to engage in mathematical explanations and justifications; yet, such tasks have a genuine potential to serve these purposes (Stein et al., 1996). Examination of the types of responses required is essential in developing our current understanding of the degree to which textbooks encourage students to make explanations; this is partly because solving a mathematical problem also calls for an explanation of the solution process (Li, 2000). Questions that require students to find as well as explain and justify a solution are instrumental for long-lasting and meaningful learning (National Council of Teachers of Mathematics [NCTM], 2000).

On the basis of our considerations shared hitherto, this paper aims to perform a comparative analysis of fifth grade mathematics textbooks employed in Turkey and Singapore. The comparative analyses have been performed regarding four particular dimensions:

1. Content presentations
2. Type of responses required of the students
3. Conformity with the curriculum outcomes
4. Potentially confusing expressions

With these analyses, this study attempts to contribute to textbook research by designating the similarities and differences between the mathematical competencies, learning opportunities, and learning cultures that Turkey and Singapore consider valuable for their students. The findings and results of this study are also expected to yield theoretical and practical insights that textbook developers, textbook evaluators, policy-makers, and teachers may find helpful for reflection purposes while employing textbooks as part of mathematics instruction.

Methods

This study adopted the document analysis technique that requires a systematic examination and interpretation to understand and make sense of data in print and digital materials (Bowen, 2009). Document analysis was used to compare fifth-grade mathematics textbooks of Turkey and Singapore. The reason for the fifth-grade selection was that this grade corresponds to a transitional stage in both countries. In Turkey, the fifth grade is the transitional stage from primary to secondary school levels. In Turkey, a common practice during primary education is that the classroom teachers teach students in all subject fields, including mathematics. However, at the secondary level, mathematics instruction is performed by mathematics teachers who are trained and specialized in this area. In Singapore, fifth grade is the transition stage where the subject-based banding approach begins to be practiced. As part of this practice, either foundational mathematics curriculum (intended for those weaker in their foundations), or the standard mathematics curriculum (designed to further students' mathematical accumulation) is followed. Hence this, in a sense, reflects a transition from a single curriculum followed in the initial four years of education to the implementation of a selective program starting from the fifth grade. In this study, we considered the standard mathematics curriculum for Singapore. Turkey's textbook analyzed was *Ortaokul Matematik Ders Kitabı 5* [Middle School Mathematics Textbook 5] (Cırıtcı et al., 2017), the only textbook developed and distributed for fifth-grade after the introduction of the updated mathematics curriculum. Among the other five available mathematics textbooks in Singapore, we selected Singapore's *My Pals are Here 5* (Kheong et al., 2017) because it was one of the most commonly used textbooks.

Mathematics Textbooks Used in Turkey and Singapore

Singapore's Standard Mathematics Syllabus covers numbers and algebra, measurement and geometry, and statistics. Turkey's Middle School Mathematics Curriculum similarly covers numbers and operations, geometry and measurement, and data processing for fifth grade. By placing mathematical problem-solving at the center, Singapore's mathematics curriculum adopts a pentagonal instructional framework that includes five interrelated components: metacognition (monitoring one's thinking, self-regulation of learning), process skills (reasoning, communication, connections, thinking skills and heuristics, applications and modeling), concepts (numerical, algebraic, geometrical, statistical, probabilistic, analytical), skills (numerical calculation, algebraic manipulations, spatial visualization, data analysis, measurement, estimation, use of mathematics tools) and attitudes (beliefs, interest, appreciation, confidence, perseverance). On the other hand, Turkey's mathematics curriculum focuses on problem-solving, process skills, mathematical modeling, building connections, emotional skills, psychomotor skills, information, and communication technology skills.

The current curriculum in Singapore was first introduced in 2013 and has been regularly revised every six years. The Singapore textbook analyzed in this study was the third edition of My Pals are Here 5 released in 2017. The mathematics education curriculum was revised in Turkey in 2017 and, different from the previous one, teaching particular manners and certain values have been selected as the primary foci. The values are embedded in every subject matter curriculum, including mathematics, and teachers are supposed to convey eight core values to students during their instructions: justice, friendship, honesty, self-control, patience, respect, love, responsibility, patriotism, and helpfulness (Talim ve Terbiye Kurulu Başkanlığı [TTKB], 2017). Also, various skills and competencies were specified by considering the EU Competency Framework, National Education Quality Framework, and 21st-century skills.

Singapore Ministry of Education (MoE) assigns the task of textbook development to private publishers to increase diversity (Kaur et al., 2015). To allow school principals, department heads, grade level coordinators, and subject heads to make more informed decisions, Singapore MoE publishes an Approved Textbook list for primary and secondary schools every year in August. It demands schools to choose from the list based on their needs (MoE, 2017). Since the mid-1990s, textbooks have been developed by private publishers in accordance with the mathematics curriculum developed by MoE and are approved by an evaluation committee affiliated with MoE. Publishers are asked to revise the textbooks based on the feedback received from the committee and resubmit them for approval (Fan, 2010).

Similarly, in Turkey, the approval of the central education office must be obtained during the textbook development process. Textbooks are first pre-evaluated by the Directorate of Education

Council. Once approval is secured, the process of panel examination and evaluation of the draft textbook begins. Criteria and rubric to be used in examining and evaluating the textbooks are developed and announced by the Directorate of Education Council. Textbooks are graded by the panelists based on these rubrics. Later, a detailed panel report is drafted with the rationales for the given grade. Based on this report, a final decision is made for a textbook for publication by the central government, which is then freely distributed to all students at the appropriate grade level.

Textbook Analysis Framework

In our attempt to carry out a comparative analysis of Turkey and Singapore textbooks, we initially performed a similarity check for the contents of both books. The headings were crosschecked, and corresponding titles with the mathematical substance were examined in detail. Our examination determined two particular topics with considerably higher matches in terms of substance in both textbooks: (1) percentages and (2) triangles and quadrilaterals. Table 1 presents a brief outline for each topic in both textbooks. Our comparative analyses focused on these two topics due to high content correspondences between both textbooks.

Table 1

Two Mathematical Topics Employed to Perform Comparative Analyses of Textbooks

Turkey			Singapore		
Headings	Learning Domains	Content Outline	Headings	Learning Domains	Content Outline
Percentages	Numbers and Operations	Percent, percentages as decimals/fractions, comparison of percentage expressions, finding the percentage of a quantity	Percentages	Numbers and Algebra	Percent, expressing percentages as fractions/decimals, expressing fractions/decimals as percentages, finding the percentage of a quantity
Triangles and Quadrilaterals	Geometry and Measurement	Polygons; types of triangles and quadrilaterals; the measures of the interior angles of triangles and quadrilaterals; finding unknown angles in triangles and quadrilaterals	Triangles and Quadrilaterals	Geometry and Measurement	Classifying triangles; the sum of the angles in a triangle; right-angled, isosceles and equilateral triangles; finding unknown angles; triangle drawings; Parallelogram, rhombus, and trapezoid; finding unknown angles; quadrilateral drawings

We comparatively analyzed the textbook contents in the two topics based on the following dimensions: (1) presentation of the content, (2) types of responses required of students, (3) conformity with curriculum outcomes, and (4) potentially confusing expressions. Table 2 shows the analytical framework used for each dimension.

Table 2

Analytical Framework

Presentation of Mathematical Content	Type of Responses	Conformity with Curriculum Outcomes	Potentially Confusing Expressions
o How is the subject organization scheme? o How is the subject taught? o Are multiple solution strategies provided? o Are visual representations used? o Is technology use encouraged?	o Response only o Response and explanation o Explanation only	o Does the textbook fully cover the relevant curriculum outcomes? o Does the textbook digress from the relevant curriculum outcomes?	o Error resulting from repetition o Syntactic errors o Ambiguity o Unrealistic tasks o Unclarity in mathematical purposes o Redundant procedures o Weak mathematical context

We utilized type of responses required of students and presentation of the content dimensions of the textbook analysis framework developed by Charalambous et al. (2010) in the process of development of our analysis framework shown in Table 2. We added the first two items in the presentation of mathematical content. We also compared the content of subjects in the textbooks and outcomes when examining the level of textbook's conformity with curriculum outcomes. At this point, it should be noted that, unless otherwise stated, the term curriculum refers in this paper to nationally intended curriculum. Potentially confusing expressions emerged as a result of our content analysis of the textbooks. Further details of the data analysis procedure are provided for each dimension below.

Presentation of Mathematical Content

To develop an organization scheme pertaining to the presentation of the content, we first examined the introduction sections that included the explanations given by the authors regarding how the textbooks must be used. We then examined the sequence of each construct located in the introduction sections from the first subject to the final subject and schematized them. We changed the names of some constructs based on their intended purpose to facilitate readers' understanding further. For example, we changed the construct 'Learn' in the Singapore textbook and the construct 'Let us Do it Together!' in Turkey's textbook into "worked example'. We presented it as such in the content organization scheme. Also, in this dimension, we carried out a holistic examination of the subject of percentages in each textbook and reported on the presentation of the topic. We later analyzed this report in accordance with how the subject of percentages was taught. We analyzed the use of visual representations, encouragement for technology use, the use of multiple solution strategies by focusing on worked examples, and problems/tasks/drill-and-practice questions assigned to students.

Type of Responses Required

We determined each question as the unit of analysis. Although multiple questions were numbered and itemized under each question, we considered each itemized question as a separate one. We coded the questions based on whether they required students to give a response only, both a response and explanation, and an explanation only. More specifically, a question that required a verbal or numerical response was coded as *response only*, a question that required students to explain how they had found a solution in addition to requiring a verbal/numerical response was coded as *response and explanation*, and a question that required students to give an explanation only without asking them to give any verbal/numerical response was coded as *explanation only*.

Conformity with Curriculum Outcomes

To understand if the textbooks covered the curriculum outcomes, and if they went beyond these intended outcomes, we examined the mathematics curriculum outcomes for subjects common in both textbooks (percentages, triangles, and quadrilaterals), the explanations for these outcomes, and the presence of each outcome, and its explanation in the relevant sections of each textbook. To this end, by following the sequence adopted by the textbooks, we carried out a comparative examination of common subjects and the relevant curriculum outcomes and explanations.

Potentially Confusing Expressions

During the analyses, we also tried to note down any expressions that could confuse students. For this, we selected reading texts, activities, games, research-thinking sections, information boxes, speech-thought bubbles, worked examples, and each question students were expected to solve. Through a line-by-line examination of the texts, we identified and categorized any potential issues such as errors induced by repetition, redundancy, and ambiguity that could confuse students. We also referred to the relevant literature, which helped us make sense of the observed errors, and allowed us to come up with categories for potentially confusing expressions. Among the particular studies that inspired us towards this direction were Adler (2000), Sullivan et al. (2003), and Fan (2010). The categories of confusing expressions employed in this study are listed in Table 3 with explanations and particular illustrative examples.

Reliability of the Findings

We invited an expert not directly involved in our study and asked her to audit our data analysis procedures. We first informed the expert on the purpose, method of our study, and the details of our analytical framework. We then asked the expert to conduct an analysis by using one-third of our data. The expert and one of the researchers carried out an independent analysis of the data, and later met for debriefing. The inter-coder reliability was measured as 100 % for the dimension of conformity with curriculum outcomes, 96% for the type of response required, and 90% for the dimension of confusing expressions. We further discussed the codes on which there was a disagreement and reached a consensus. The remaining sections of the book were also analyzed by a researcher based on the discussed and agreed-upon points. Further, analyses were iteratively revisited, the findings were compared with previous results, and, thus, a final version of the findings was established.

Table 3

Coding Scheme for Potentially Confusing Expressions

Category	Explanation	Example
Error resulting from repetition	An unnecessary repetition of words, numbers or expressions	∠PQR and ∠RSO are right right angles (Kheong et al., 2017, p.135). The word "right" in this statement was written twice.
Syntactic errors	Grammatical errors in the verbalizations	With stadium's shopping center and its lounges becoming the center of our city (Cırıtcı et al., 2017, p.165). The grammatical errors create difficulties in understanding the real intention, which could have been better expressed as "The stadium is becoming the center of our city with its shopping center and lounges."
Ambiguity	Ill-defined situations in which it is hardly possible to corroborate the plausibility of an answer or to make sense of the problem demand	Is there any building around you that is still under construction? If so, how much (i.e., in percentage) of this building is completed? (Cırıtcı et al., 2017, p.165) It is difficult for students to provide a sensible answer to the question as students cannot know the final product of the buildings before they are fully constructed. In addition, in order to check the plausibility of the answer, the teacher must also know the building under construction.
Unrealistic task	Assignment of a duty impossible to realize due to uncertainties involved in a situation and/or the scope	Select a species of endemic plant and animal and then state its percentage in relation to the number of species of that specific animal (Cırıtcı et al., 2017, p.181). In this task, students were asked to state, in percent, the number of one endemic plant and of one animal but there are uncertainties involved in the duty (i.e., percent of endemic species in comparison to what?). The scope is also problematic in that it is hardly possible to determine the number of plants or animals in particular species (unless there are particular records held by authorities).
Unclarity in mathematical purposes	Failure or hardships in making sense of what mathematical purpose a shared text/request serves	Is there any mathematical symbol drawn from the symbol of percentage? Search it! (Cırıtcı et al., 2017, p.166). This question may intend to encourage students to do research. However, it is hard to understand what kinds of answers students were expected to come up with; not clear if such a symbol existed; and if it did, how it was related to the symbol and the concept of percentage.
Redundant procedure	Unnecessary or irrelevant operations, steps, or procedures shared in a solution	There is 25% discount applied to televisions in a store. Let's find out how much discount applied to a TV with a price of 800₺. 800÷100=8 8×25=200 ₺ discount will be made. The discounted price of the television is 800-200=600₺. (Cırıtcı et al., 2017, p.166) Students were asked to find the amount of discount. Although the amount was found, the textbook continued the solution and although not asked, the discounted price was found. Therefore, the worked example included a redundant solution step.
Weak mathematical context	Insufficiencies in creating a context for mathematical discussions or considerations	What should we be careful about so that endemic living species like these butterflies can continue their lives? (Cırıtcı et al., 2017, p.178) This question intends to increase student participation but the mathematical context it tries to develop is weak.

Findings

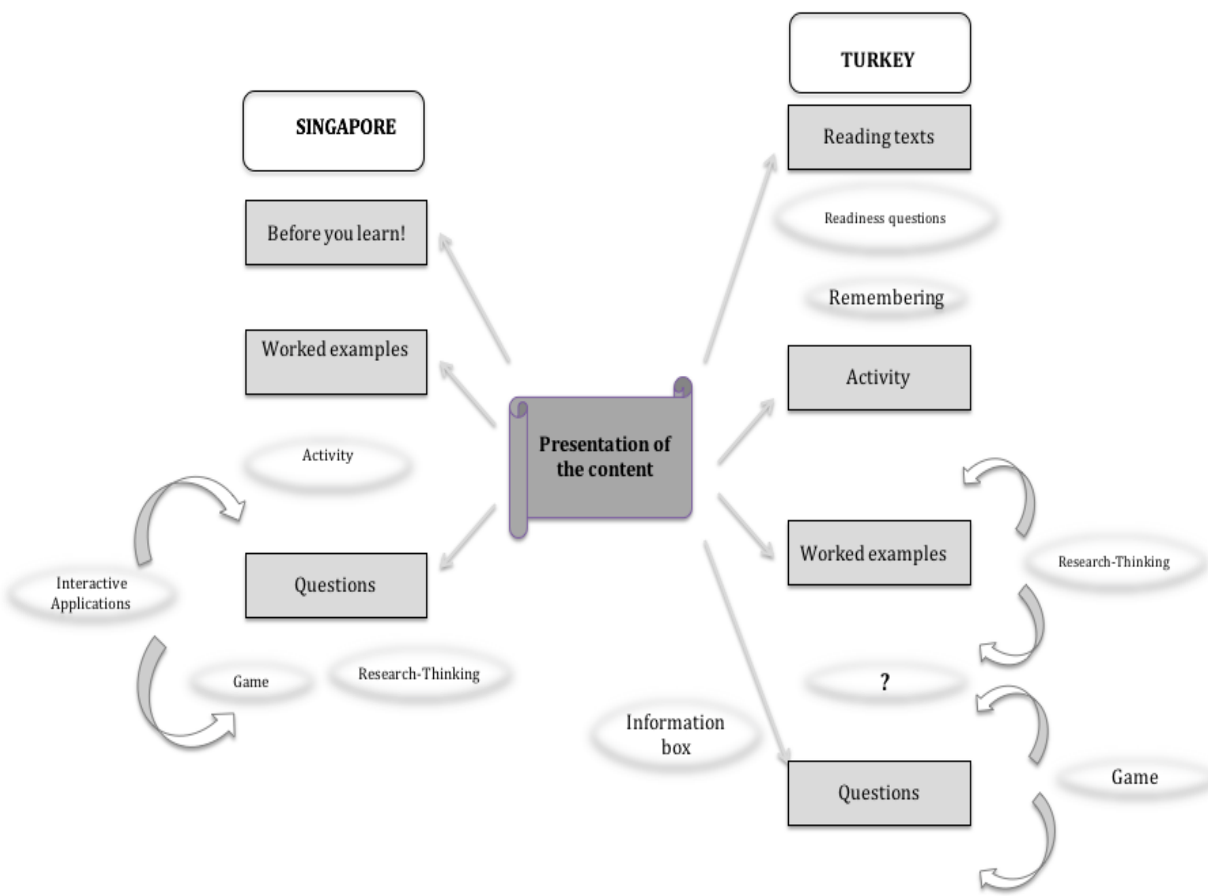
We present the findings under four separate headings: presentation of mathematical content, type of responses, conformity with curriculum outcomes, and potentially confusing expressions.

Presentation of Mathematical Content

In this part, we present the schemes regarding the organization by wholly studying both textbooks as a result of our analysis of the sequence of constructs, such as worked examples, questions, activities, and games and developed schemes indicating the organization of the subjects (see Figure 1). We then explain the presentation of the content by analyzing the way the subject is taught, multiple solution strategies, visual representations, and the use of technology in the subject of percentages.

Figure 1

Textbooks' Organization Scheme of Subjects



Constructs in bold in the figure above were those that had always been used, and constructs in light color were those used at intervals. Both textbooks included activities, games, worked examples, questions, and research-thinking parts. Different from Turkey's textbook, previous learning was presented under the worked examples in the Singapore textbook. In comparison with the Singapore textbook, the Turkey textbook included reading text and questions aimed at increasing readiness, while the Singapore textbook included 'Before You Learn' and interactive application parts. The Singapore textbook concluded the unit with unit evaluations, unit summary, and non-routine problems after presenting the subject. The Singapore textbook also adopted a much simpler organizational structure and used more subheadings.

Each unit in the Singapore textbook began with a colorful cover that was relevant to mathematical content in the unit and in which cartoon characters spoke and invited students to think. The covers have taken mathematical context as its focus and had designs that students could find appealing. They included numbers that must have enabled students to carry out procedures easily. The mathematical content to be taught was introduced through speech bubbles. Also, the classes to be taught were given, and the main mathematical idea of the unit was presented under the 'Big Idea' heading. The Turkey textbook did not make efforts to create a mathematical context, except in the headings of the subjects to be taught in the colored unit covers.

The introduction of subjects through revisiting students' prior knowledge was periodically evident in both textbooks. Each subject started with a reading text in the Turkey textbook. These reading texts were expected to increase students' interest and invite students to think about mathematical concepts. Some questions related to these texts were also posed. Each class started with a question under 'the Before You Learn' heading in the Singapore textbook. Students were expected to think on the question, develop connections between concepts and visual representations, and occasionally make explanations. Lower text density and the use of plain language were also distinguishing features of the Singapore textbook.

Both textbooks chose to teach the content through reliance on worked examples. Reading texts were followed by an activity that was followed by worked examples in the Turkey textbook. Worked examples were presented in a row, and then several questions were posed altogether in a row. On the other hand, the Singapore textbook first presented a worked example and then a relevant question to consolidate learning. Turkey's textbook included two child cartoon characters (one girl, one boy) with speech bubbles. These speech bubbles contained procedural knowledge, additional knowledge, and a repetition of the expression included in the solution. The Singapore textbook, in contrast, had six real characters (three girls, three boys) depicted as Singaporean kids. They were presented in thought/speech bubbles, which included steps/models for the problem-solving process, a question that can facilitate a specific solution step, and a question to elicit students' learning, information, or alternative solution processes. While Singapore's textbook included an interactive application for each subject accessible under the heading 'app-tivity', the Turkey textbook lacked such a practice. To delve more into the differences between the two textbooks, Table 4 presents the instructional structure of the percentage topic in both textbooks.

Regarding the presentation of mathematical content, we focused on worked examples and problems/tasks/drill-and-practice questions assigned to students to designate the use of visual representations, encouragement for technology use, and the use of multiple solution strategies. To this direction, we initially determined worked examples and questions/problems about percentages, which are presented in Table 5. In the Turkey textbook, there were 12 worked examples and 58 questions/problems. In the Singapore textbook, 13 worked examples along with 89 questions/problems were presented.

Table 4

Instructional Structure of Percentage Topic

Turkey	Singapore
The meaning of the concept of percentage was given after presentation of the reading text, game, and worked example.	The meaning of the concept of percentage was given as soon as the unit started under the heading 'Big Idea' in the unit cover that explained the unit and the lessons to be taught.
Solution strategies in worked examples were not clearly separated from one another.	Solution strategies in worked examples were presented in separate sections under the headings Method 1, Method 2, or in speech bubbles. Sometimes 3 different strategies were presented.
100-square grids were used for the connections between percentage and fractions.	100-square grids and percentage scale were used for the connections between percentage and fractions. A linear scale was used for the connections between percentage and decimals.
A scarce use of models was present.	A frequent use of models is present. In the student directed questions, ready-given models were included, the solution path was divided into steps and gaps were left, that is, guided tasks were presented.
Worked examples on conversion of fractions to percentages were carried out through fractions with a denominator lower and higher than 100.	Worked examples were presented through selection of fractions with a denominator lower and higher than 100 and selection of fractions, the denominators of which, cannot result in 100 through reduction and expansion.
Trio conversions, the conversion of fractions to decimals, and conversion to percentages was given in addition to dual conversions.	Students were asked to convert percentage either to fractions or decimals, and to convert decimals or fractions to percentages, which meant there were only dual conversions.
Discount and wage raise related problems were presented, while there were no interest rates and tax questions.	Discount, wage raise, and tax related problems were presented. The meanings of interest rates, discount, and tax were given in speech bubbles.
Strategy of estimation of the solution before starting the solution process was used only in one problem, and after the solution was completed and estimated, the real solution was not compared.	Polya's four-step problem solving strategy that intends to answer the questions such as: What is given? What is expected? How can I solve it? Is my solution logical? was used only in one problem. However, the justification of the solution was not sought.

Table 5

Examples and Questions Presented in the Percentage Section of the Books

Types	Turkey	Singapore
Worked examples	12	13
Questions/problems	58	89
Total	70	102

Multiple solutions, visual representation, and use of technology in tasks on percentages are shown in Table 6. Since the intensive use of models in the Singapore textbook draws attention, our analyses also included this. As seen in Table 6, the Singapore textbook gives more space for multiple solutions, visual representation, modeling, and technology than the Turkish textbook. The high use of visual representations in the Singapore textbook (32%) and modelings are two distinct characteristics. The Turkey textbook gave much less space to multiple solutions (4%) and technology (1%).

Table 6

Multiple Solutions, Visual Representation, and Use of Technology in Percentage Section

	Turkey (*n*=70)	**Singapore** (*n*=102)
Use of visual representations	18 (26%)	33 (32%)
Use of modellings	9 (13%)	33 (32%)
Use of multiple solutions	3 (4%)	7 (6%)
Use of technology	1 (1%)	12 (12%)

We can briefly summarize the main differences in the presentation of percentages as follows. While the Singapore textbook chose to introduce the subject with a mathematical question that required students to think and to explain their thinking, the Turkey textbook chose to use reading texts with social contexts to introduce the subject. Worked examples were given by wholesale in a row, and questions were posed altogether in the Turkey textbook. On the other hand, the Singapore textbook prioritized the immediate consolidation of learning by introducing a similar problem right after giving a worked example. The Singapore textbook is different from the Turkey textbook in terms of including a higher number of and more diverse visual representations, more intense use of modelings, utilization of multiple solution strategies in the worked examples, and promotion of technology use. Provision of modelings of problems posed to students, the guidance provided to students by leaving blanks in the problem-solution process, presentation of the subjects in a more simplified organization, less and more cogent explanations, and low text density were among the other salient features of the Singapore textbook.

Type of Responses Required

This part presents findings related to the type of responses for problems in the percentages, triangles, and quadrilaterals. The type of responses both textbooks required of students were examined based on three different categories: response only (RO), response and explanation (RE), explanation only (EO) (see Table 7). It could be observed that the majority of the questions in both textbooks only ask the students for answers and less for explanations. In addition, it has been determined that the questions in the Turkey textbook require only answers at a relatively higher rate. In contrast, the questions in the Singapore textbook encourage students to make more explanations.

Conformity with Curriculum Outcomes

We have observed that both textbooks remained committed to the relevant curriculum outcomes; they covered every target outcome and did not go beyond these outcomes.

Table 7

Type of Responses Required of Students in the Textbooks

	Turkey			Singapore		
	Percentages (n= 58)	Triangles (n=47)	Quadrilaterals (n=33)	Percentages (n=89)	Triangles (n=75)	Quadrilaterals (n=77)
RO	57 (98%)	44 (94%)	32 (97%)	80 (90%)	44 (59%)	56 (73%)
RE	1 (2%)	2 (4%)	1 (3%)	8 (9%)	21 (28%)	20 (26%)
EO	-	1 (2%)	-	1 (1%)	10 (13%)	1 (1%)

Note. RO: response only; RE: response and explanation; EO: explanation only

Potentially Confusing Expressions

The findings of the confusing expressions observed in both textbooks within the scope of two analyzed sections are presented in Table 8, where we shared the frequencies of the determined incidents. Compared with Singapore's, the volume and diversity of confusing expressions were strikingly higher in Turkey's textbook. While we observed one minor error resulting from the repetition in the Singapore textbook, seven instances could mislead or confuse the users of the Turkey textbook. Unclarity in mathematical purposes was the most common difficulty observed in the Turkey textbook.

Table 8

Potentially Confusing Expressions in the Textbooks

Potentially Confusing Expressions	Turkey (*f*)	Singapore (*f*)
Error resulting from repetition	1	1
Syntactic error	1	-
Ambiguity	1	-
Unrealistic tasks	1	-
Redundant operation	1	-
Weak mathematical context	1	-
Unclarity in mathematical purposes	3	-

Discussion

The first focus of our comparative examination was on the presentation of mathematical content. Findings that are relevant to this aspect highlight that the Singapore textbook utilizes more subheadings and adopts a simpler presentation style. The Singapore textbook presents a more comfortably capacious layout with lower text density and more space on the pages. This observation corroborates the findings by Erbaş et al. (2012), who pinpointed low text density and a simpler form as two prominent features of the Singapore textbook. Given that low text density can serve to limit the time needed for completion of instruction and tasks (Morrison et al., 1988) and considering that students need more assistance when faced with higher text density (Zevenbergen, 2001), it could be argued that the Singapore textbook can enable more effective instruction.

Another significant difference between the two textbooks regarding the presentation of the content is related to definitions and explanations of the concepts commonly used in daily life. For instance, the Turkey textbook does not explain the concept of "discount" when presenting discount problems in the percentage topic. On the other hand, if students do not fully comprehend the meaning of these concepts, the definitions of concepts such as tax, interest (the meaning of income and loans), and discount were provided in speech bubbles in the Singapore textbook. The complexity of the words and sentences is one of the most important factors affecting the readability of textbooks (O'Keeffe, 2013). In this respect, we might argue that the Singapore textbook appears to be more concerned with readability. Furthermore, the Singapore textbook also aims at intelligibility (Czeglédy & Kovács, 2008; Fan, 2010) with its simple organizational structure, low writing density, spacious page structure, frequently used visual representations, models, and plain language. One reason for using plain English might be because this textbook is prepared for non-native Singaporean student speakers of English (Wang-Iverson et al., 2009). That Turkey's textbook fails to present explanations on technical terms and the use of a more intense language might generate some issues for students whose mother tongue is not Turkish and who have not yet developed a good command of the Turkish language.

An essential feature of the Singapore textbook was related to the importance attached to the use of multiple solution strategies (even occasional presentation of three different strategies), a feature lacking in the Turkey textbook. Employment of multiple solution strategies is an important feature that characterizes Singapore's mathematics education approach (Soh, 2008) in supporting the development and diversification of learning opportunities (Yoong & Hoe, 2009). The Turkey textbook's failure to include multiple solutions might be limiting students from developing multiple approaches to problems, and from developing skills that could otherwise enable them to handle problems from diverse perspectives.

Singapore textbooks use modelings and visual representations more often. This approach might serve various purposes. Modelings in mathematics education in Singapore are used as tools in analyzing and processing the information and developing logical steps needed for problem-solving (Kaur et al., 2015), in visualizing the problem (Ng & Lee, 2009), and in concretization (Fong & Lee, 2009). This is a well-established practice in Singapore mathematics education, which follows through a three-stage developmental trajectory known as concrete-pictorial-abstract (Kaur et al., 2015). In this approach, ideas are first introduced with concrete materials or practical activities followed by visual illustrations, which are eventually translated into abstract mathematical conceptualizations (see Ginsburg et al., 2005; Jaciw et al., 2016). The effect of this approach on textbook production could easily be observed in many Singapore mathematics textbooks (Soh, 2008). Yoong and Hoe (2009) argue that, with this approach, real-life and abstract ideas are better connected, and concepts and skills are further developed.

Singapore textbook developers frequently use visual representations and modelings due to the positive effects on the development of mathematical understanding (Arcavi, 2003; Naroth & Luneta, 2015). Bora and Ahmed (2019) argue that mathematical modeling supports students' creative and original problem-solving skills, improving their self-perception and self-esteem. Bahmaei (2011) similarly discusses the impact of modeling on enhancing students' knowledge and skills, and helping them grow into more self-confident mathematical thinkers. In the tasks/problems presented in the Singapore textbook, pre-developed modelings are presented to students right after worked examples. Also, spaces related to problem-solving steps and some hints are provided to guide students more effectively. Such an approach comes with pros and cons. For example, the guided tasks adopted by the Singapore textbook may limit students' creativity and cognitive efforts during the problem-solving process.

On the other hand, guided tasks can serve many significant purposes. They could prevent students from developing incorrect models, minimize frustration by allowing them to discover the solution paths, build their self-confidence and self-efficacy, and provide further scaffolding for struggling students. Turkey textbook's avoidance of guided tasks might avert certain limitations, yet this avoidance could also result in certain disadvantages for students to develop self-confidence and self-efficacy. Given that the trade-offs involved in guided tasks have not been fully resolved, future research on the strategic integration of guided tasks into mathematics textbooks by considering the cultural fabric of different communities could help develop a more informed view.

Singapore has been formally using modelings at the elementary school level since 1983 (Ng & Lee, 2009). Other basic approaches adopted by Singapore textbooks include the employ of modelings in visualizing a problem, using pictorial representations in displaying the connections, and developing strategies for problem-solving (Soh, 2008). Textbooks also include modelings and diagrams to show basic concepts, non-routine tasks, use additional diagrams and pictures to display different thinking methods, and include extra exercises, guided practices, and problems that become more difficult (Ginsburg et al., 2005). These approaches that have been identified with Singapore textbooks are also defined as "textbook signature" for these books (Charalambous, et al., 2010). The Singapore textbook also includes modelings that conform to this signature through visual representations, multiple solution strategies, guided practices, and non-routine tasks. It then becomes evident that Singapore textbooks, including the one analyzed in our study, are developed based on an established instructional philosophy of mathematics education, and that deep historical traditions shape textbooks' content and structural organizations. These traditions seem to evolve into certain educational visions and produce pedagogic approaches that have been tried over the years and have been effective (see also Oates, 2014).

On the other hand, the Turkey textbook appears to suffer from the lack of similar established approaches on mathematics education and the lack of a fundamental philosophy that could have, otherwise, guided the content and structural organization of the textbook. The presence of certain guiding instructional approaches on the content and presentation of a textbook may contribute to the development of an established culture within textbooks. Such established approaches may also potently serve as prevention and mitigation of wasted labor and sources by eliminating the need to initiate new instructional pursuits in every book development process and allowing new textbook and material development under the guidance of certain corroborated principles. That the first edition of the Singapore textbook we examined in this study was dated 2005, the second edition was dated 2008, and the third edition was dated 2017, which underscores the protection of labor needed for textbook development and the refocusing of efforts on improving the existing textbook. Rather than using a textbook for a while and then pulling it out from the market, we argue for the instrumentality of an approach for sustaining, updating, and improving the quality of an existing textbook following emerging instructional innovations, an approach evident in the Singapore textbook.

In the percentage topic, the Turkey textbook showed the estimation of percentages through the use of a calculator only in one worked example, while the Singapore textbook asked students to use a calculator in 12 questions. The Singapore textbook encouraged calculator use in tasks that include big numbers (e.g., 20% of 7.500), and procedures that are difficult to carry out without a calculator (e.g., 7% of 699). Research evidence on the use of calculators in mathematics instruction indicates benefits, such as the potential to assist students to formulate calculations, to interpret these calculations (Ruthven, 2009), in developing concepts, carrying out exercises, improving problem-solving, saving time (Van de Walle, 2007), and in developing positive attitudes (Och & Indoshi, 2011). Calculators are also supported in mathematics education due to their potential in increasing the mathematical expertise of students by making the mathematical tasks more accessible and, thus, enabling students to better cope with complex tasks (Ruthven, 1996). Also, the use of calculators in mathematics textbooks is argued to be an effective instructional practice when mathematical tasks do not aim not to find a calculation-based solution, but to solve a complex problem (Thompson & Sproule, 2000). Therefore, the use of calculators can be further encouraged in the problems that include larger numbers when the aim is to discover the potential links in the given data.

Regarding the second focus of our study, which is the type of responses required of students, both textbooks predominantly presented questions/problems expecting students to give only a numerical response. However, the Singapore textbook tended to elicit more explanations from the students than that of Turkey. Mathematical explanations encourage students to think on mathematical ideas, open these ideas for further discussions, review, and update. All these features contribute to students' mathematical development and support students' communication in mathematics (NCTM, 2000). Providing students with the opportunities to explain their solutions in mathematics textbooks is significant in the social construction of mathematics inside the classroom (see Yackel, 2001). Given the possible effects of explanations on students' mathematical growth and understanding, we support the further inclusion of more tasks that require students to make mathematical explanations.

The use of tasks that require only numerical responses seems to be a global practice. As a matter of fact, Li (2000) similarly showed that most problems included in both the US' and China's mathematics textbooks required only numerical responses from students. The author reported that only 19 % of the problems in the US textbook expected students to make an explanation for solutions and that no problem in the Chinese textbook required explanation for any solution. Törnqvist's (2019) study that examined Sweden's and China's textbooks and the study on Kosova's and Albania's conducted by Vula et al. (2015) also confirmed the common practice of questions that require numerical response only. In examining eighth grade Singapore and Turkey mathematics textbooks, Özer (2012) also showed the predominant use of tasks requiring numerical responses in both textbooks. However, Özer (2012) found that Turkey textbooks at that grade required comparably more explanations from students.

Regarding the third focus of our study (conformity with the curriculum outcomes), it has been determined that the textbooks remain loyal to the country's mathematics curriculum, include each target acquisition, and do not go beyond the acquisitions. This could be because compliance with the officially issued curriculum documents is considered an important criterion for the textbook production process in both countries and that care was given to realize the expected compliance. In fact, compliance to the national set of standards devised by the official curriculum is an important condition (Fan, 2010; TTKB, 2015) employed to evaluate the adequacy of textbooks in both Turkey and Singapore.

As to this study's fourth and final focus, our observations of potentially confusing expressions in percentages, triangles, and quadrilaterals subjects (only one in the Singapore textbook and nine in the Turkey textbook with seven different categories) are also interesting. The presence of only one error caused by the repetition of the same word twice in the Singapore textbook is a testament to higher standards for textbook development and a more rigorous evaluation process. This is supported by Kaur et al.'s (2015) argument on meticulous mathematics textbook preparation and the approval process in Singapore. Fan's (2010) contention that publishers seriously consider the input given by the approval committee and carefully revise the book accordingly also helps explain our result. Our study has categorized and defined seven difficulties that potentially reduced the textbook's instructional and practical value, intelligibility, and readability: repetition, syntactic error, ambiguity, unrealistic tasks, unclarity in mathematical purposes, redundant operation, and weak mathematical context. The Turkey textbook we examined in this study included all these types of difficulties, and others (e.g., Çakır, 2009; Hıdıroğlu, 2016) also reported similar observations in different fifth-grade mathematics textbooks. Questions that have unclarity in mathematical purposes, procedures that are redundantly lengthened by the continuation of further solutions, tasks that aim at increasing interest but have weak mathematical context, unrealistic tasks, and ambiguous expressions might increase students' cognitive load and diminish their motivation for learning. Such potentially confusing expressions might also decrease users' trust in textbooks and compromise respect for the textbooks.

Turkey's textbook examined in this study is distributed to millions of students and thousands of teachers free of charge. It is evident that expressional issues present in this book influence students' learning opportunities and teachers' interpretation of the content. This might negatively affect the productive application of the content presented through the book. Research focusing on the use of mathematics textbooks by teachers in Turkey (e.g., Özmantar et al., 2017) showed that although textbooks are distributed at no cost, teachers commonly refrain from using these textbooks. While explaining their reluctance to use Turkish textbooks, teachers often referred to the difficulties related to the ones observed and reported in our study. Based on analysis of potentially confusing expressions that emerged from our study and the prior research, we can arguably suggest that expressional issues form a barrier for the achievement of outcomes set by the textbooks and have the potential to influence readability negatively. We also wish to point out the need for further studies to develop organizational and systematic structures to detect such issues in the textbooks before they are published.

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