

Semiotic Mediation in Mathematics and Physics Classrooms: Artifacts and Signs after a Vygotskian Approach

Maria G. Bartolini Bussi
University of Modena and Reggio Emilia

Federico Corni
University of Modena and Reggio Emilia

Cristina Mariani
University of Modena and Reggio Emilia

Rossana Falcade
SUPSI-DFA

Abstract

The theoretical framework of semiotic mediation after a Vygotskian approach, introduced first for mathematics education is extended to physics education. Beyond obvious epistemological differences between mathematics and physics, dialogue and relationship show fruitful, drawing on the Vygotskian perspective of *obučenie*. The aim of this paper is to show that the theoretical framework of semiotic mediation might offer methodological scaffolding for teachers in order to mediate scientific meanings that have been historically incorporated in artifacts and experimental methods. Cross-fertilization between the two research programs on mathematics and physics education at primary school level is discussed.

A rock from another mountain can be used to chisel your own jade
(Xiao Ya, Shijing Ch. He Ming, XI century BC)

Correspondence concerning this manuscript should be addressed to Federico Corni, federico.corni@unimore.it, University of Modena and Reggio Emilia

Introduction

In the Vygotskian tradition, learning is not considered a direct but a mediated relation between individuals and knowledge: the process of *semiotic mediation* is described by Vygotsky (1978) as the active introduction, from outside the subject, of an intermediate link (a *sign*ⁱ) between the stimulus and the response which constitute the elementary form of behaviour. Inter-psychological functioning, that appears between people and may be used to determine the *zone of proximal development* for individuals, is a precursor of intra-psychological functioning that appears within individuals. As the development of consciousness has its origins in the cultural forms of social interaction, the key to understand all the mental processes of individuals is *internalization* (Vygotsky, 1978).

As a consequence, in school, in the Vygotskian tradition, learning cannot be separated from teaching. This is well evident in the Russian word *obučenie*. Mecacci (1990) in the Vygotskian lexicon attached to the Italian critical translation of *Myšlenie i reč'* quoted the

definition given by Menčiskaya (1966, vol. 3, p. 154): *obučenie* is “the process of transmission and appropriation of knowledge, capacities, skills and methods of human knowing activity. *Obučenie* is a dialectic process, realized by teacher (*prepodavanie* [teaching]) and by learner (*učenie* [learning])”. This circular process of teaching-learning cannot be translated with only one of these words. A similar argument has been proposed years later by Cole (2009). Behind this translation problem, however, there is a hidden deeper contrast. At least in the North American educational research tradition, often, the circular process has been broken and the attention to learning and learners has prevailed over teaching and teachers. For instance, Vianna & Stetsenko (2006) contrasted the socio-interactional version of constructivism (in which Piagetian and Vygotskian insights are often merged) and the one founded by Vygotsky, focusing on the major contrasts concerning history (including cultural artifacts) and the concept of social interaction.

The aim of this paper is to report some basic elements of a wide research program coordinating mathematics and physics education in primary school, after a Vygotskian approach.

Two of the authors (Bartolini Bussi and Falcade) have been conducting research in the field of mathematics education for years and have contributed to the development of a theoretical framework for semiotic mediation in the mathematics classroom (Falcade, 2006; Bartolini Bussi & Mariotti, 2008) with the publication of several research studies (e. g. Bartolini Bussi, 1996, 1998a, 1998b; Bartolini Bussi, Boni, Ferri & Garuti, 1999; Bartolini Bussi, Mariotti & Ferri, 2005, Bartolini Bussi & Boni, 2009)ⁱⁱ. The above studies on mathematics education belong to the continuation of Vygotsky’s approach, as developed by Davydov (1990) and others (see Vianna & Stetsenko, 2006), and try to transform Vygotsky’s seminal theory of semiotic mediation into a tool for designing, realizing and analyzing classroom experiments consistent with the historic-cultural tradition.

The other authors (Corni and Mariani) have carried out research and several teaching experiences in physics education. The disciplinary contents (Herrmann, 2000) are based on the epistemological setting of Continuum Physics Paradigm (Fuchs, 1997), recently further developed by the same author (Fuchs, 2007, 2009). Corni and Mariani propose didactical paths (Altiero, et al., 2010; Corni, et al., 2010; Mariani, et al., 2010; Corni, Giliberti & Mariani, 2010; Mariani, Corni & Giliberti, 2011; Fuchs et al. 2011) to allow children and teachers to develop their understanding of natural phenomena aimed at identifying and differentiating the Force-Dynamics Gestalts (Fuchs, 2009) at the basis of scientific thinking.

The shared attention to the dialectic teaching-learning process and some common features of classroom experiments have suggested the need to expand the theoretical framework of semiotic mediation in the mathematics classroom in order to adapt it for physics education.

Artifacts

Artifacts, technical tools and psychological tools

The construction and the use of artifacts - in particular complex artifacts - seems to be characteristic of human activities, but even more characteristic of human beings seems to be the possibility of the contribution of such artifacts to cognitive development. A famous

example concerns the relationship between the invention of mechanical clocks and the development of the sense of time. In ancient ages, time relations were based on the regular cycles of nature (e. g. days and nights; seasons). Later some kinds of measuring tools were introduced, maintaining at the beginning some relationships with the natural environment (e. g. sundials). Kozulin (1999) contrasted this original immediacy with the invention of mechanical clocks and even more of digital clocks, where also the hands of the clock disappeared and hid any reference to a cyclic sense of time. He concluded: “The process of perceiving time has become highly mediated. In order for an individual to read a watch, the whole system of symbols such as digits, language abbreviations, positions on the screen, etc. have to be learned” (Kozulin, 1999 p. 135). With the introduction of mechanical and digital clocks, the perception of time has changed, although it is quite difficult to become aware of this for people who live in a technological culture. Yet anthropologists who study the sense of time in different cultures give evidence of this. Harris (1991, p. 74), for instance, emphasizes “the Western style insistence on synchronization, the modern passion for punctuality, and the tendency to denigrate people who do not conform to rigid patterns of time” and draws inferences about teaching science and mathematics in aboriginal schools in Australia.

The example of mechanical clocks clearly illustrates both the difference and the strict link between technical tools and psychological tools (Vygotsky, 1981). Vygotsky pointed out that in the practical sphere human beings use artifacts (called *technical tools*), reaching achievements that would otherwise have remained out of reach (e.g. drive a nail into a wall by means of a hammer), while mental activities are supported and developed by means of *psychological tools*. The former are directed outward, whilst the latter are oriented inward. In some cases, the same “tool” may be conceived as technical and psychological, as the following quotation shows: “The following can serve as examples of psychological tools and their complex systems: language; various systems for counting; mnemonic techniques; algebraic symbol systems; works of art; writing; schemes, diagrams, maps, and mechanical drawings; all sorts of conventional signs; etc.” (Vygotsky, 1981, p. 137). Another example is mechanical clock, which is directed outward when it determines the cooking time for hard-boiled eggs, but is directed inwards as it changes the sense of time. As we argue below, a distinction which is quite similar (in terms of artifact versus instrument) is made some decades later by Rabardel (1995) in the field of cognitive ergonomics.

Artifacts in cognitive ergonomics

Cognitive ergonomics analyzes cognitive processes in modern industries. It has focused on user-centred design of human-machine interaction and human-computer interaction. In this research field, Rabardel’s instrumental approach (1995) is based on the distinction between *artifact* and *instrument*, between objective and subjective perspectives. According to Rabardel’s terminology, the artifact is the material or symbolic object per se. Conversely, the instrument is defined by Rabardel as “a mixed entity made up of both artifact-type components and schematic components that we call *utilization schemes*. This mixed entity is born of both the subject and the object. It is this entity which constitutes the instrument which has a functional value for the subject” (Rabardel & Samurçay, 2001, p.20). The utilization schemes are progressively elaborated

through the use of the artifact in order to accomplish a particular task; thus the instrument is the construction of an individual, it has a psychological character and it is strictly related to the context within which it originates and in which its development occurs. The elaboration and evolution of instruments is a long and complex process that Rabardel names instrumental genesis (*genèse instrumentale*, Rabardel, 1995, p.135 ff.). Instrumental genesis can be articulated into two processes:

- *Instrumentalisation*, concerning the emergence and the evolution of the different components of the artifact, e.g. the progressive recognition of its potentialities and constraints;
- *Instrumentation*, concerning the emergence and the development of utilization schemes.

The two processes are outward and inward oriented, respectively from the subject to the artifact and vice versa, and constitute the two inseparable parts of instrumental genesis. The social dimension is addressed by Rabardel in describing the interplay between individual schemes of utilization and social schemes, as elaborated and shared in communities of practice. Rabardel's theoretical constructs are very useful to start the discussion about artifacts in mathematics and science education. They are not enough, however, because they do not offer tools to cope with the cultural content to be mediated, i.e. the pieces of either mathematics or physics knowledge and they do not consider explicitly the role of the teacher in the classroom interaction.

Artifacts in mathematics and physics experience

Both in mathematics and in physics settings, a number of artifacts have been used for centuries.

For mathematics, consider all the artifacts related to counting, representing numbers or reckoning (e. g. all the kinds of abaci, mechanical calculators and, more recently, electronic calculators) and to drawing figures (e. g. straightedge, compass, squares, templates, perspectographs). They have been used by mathematicians from the ancient times and left traces in the development of mathematics: abaci are related to place value representation of numbers and to the algorithms of arithmetic operations; straightedge and compass are the roots of Euclid's Elements; perspectographs are the roots of modern projective geometry. We come back later to the function of artifacts in the mathematics experience.

For physics, consider all the scientific devices that furnish scientific laboratories, from the simplest to the most complex (e. g. measurement tools, various simple machines and their different applications, up to electric and electronic devices, etc). Several collections of historical interest are stored in science museums all over the world. In experimental activity, the reality that physics deals with is not directly the complex reality of the natural phenomena, but the 'constructed' reality of the laboratory. In the mind of the experimenter, the system under investigation has two distinct representations: the real and concrete image of the system itself and the model consisting of a representation of structure of the concepts provided by the disciplinary framework (Hestenes, 1997, 2006; Gilbert & Boulter, 2000). The idea of scientific laboratory is strictly linked to the idea of professional scientific experience, to the extent that it has

been attracting the attention of historians and sociologists of science (Latour & Woolgar, 1979; Knorr-Cetina, 1999) for decades. In this regard, Knorr-Cetina (1999, p. 6) wrote:

“laboratories provide an ‘enhanced’ environment that ‘improves upon’ natural orders in relation to social orders. [...] Laboratories are based upon the premise that natural objects are not fixed entities that are to be taken ‘as they are’ or left by themselves. In fact, one rarely works in laboratories with objects as they occur in nature. Rather, one works with object images or with their visual, auditory, or electrical traces, and with their components, their extractions, and their ‘purified’ versions”.

Artifacts in Mathematics and Physics Education

The above quotation explains the relevance of laboratories in science education, in order to gain a sort of mirror image of the “real” world, detached from the natural environment. School physics laboratories are full of artifacts that, in a more or less explicit way, hint at natural phenomena. Sometimes the use of these artifacts to answer a scientific inquiry is not simple at all (in Rabardel’s term we could say that *instrumentation* is very burdensome) to the extent that in many cases the real use of these artifacts, both in schools and in science museums, is assigned to experts and technicians.

In mathematics, the idea of laboratory is from the beginning an educational construct. It may be traced back to the late 19th century when invitations to foster a more experimental approach to the teaching of mathematics became popular. The emphasis on the mathematics laboratory was one of the driving forces towards the constitution of the International Commission on Mathematical Instruction (Rome 1908, see Borba & Bartolini Bussi, 2008) and started to be debated in most international conferences on mathematics education since then.

The Theoretical Framework of Semiotic Mediation for Mathematics and Physics Education

Mathematics Education

In this section we outline the theoretical framework of semiotic mediation after a Vygotskian approach (Bartolini Bussi & Mariotti, 2008) in the case of mathematics education. The reference to science (physics) is developed in the next section. There are several elements to be considered. First there is the *piece of knowledge to be mediated* (bottom left vertex of the sketch in Figure 1).

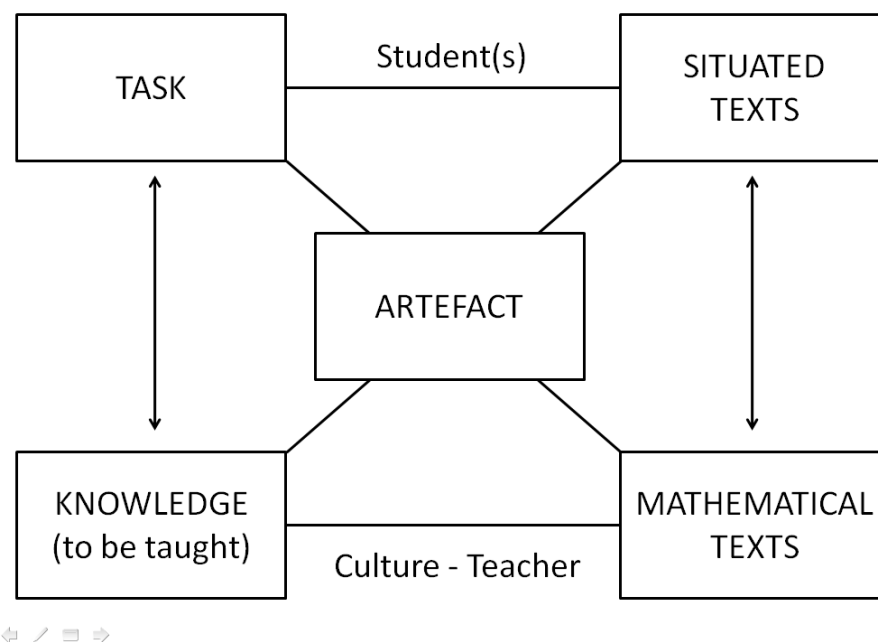


Figure 1. Elements and links between the elements of the theoretical framework of semiotic mediation.

For instance, one may think of the *place value of digits in the representation of natural numbers*. This piece of knowledge, at the beginning is known by the teacher only, who, in order to foster its emergence and its appropriation by students, designs a suitable task to be solved by means of a given artifact (e. g. the spike abacus). The piece of knowledge to be mediated is usually considered in the so-called *epistemological analysis* (for place value see for instance Menninger, 1969; Ifrah, 1981).

The students, either individually or in small group (sometimes even in a large group) produce solutions that might be right (or partially right) or wrong. In this process (top side of the sketch) the students construct personal meanings through utilization schemes of the artifact (see the example below). In the upper part of the sketch the instrumentation process is in the foreground: students are expected to use the artifact to solve a given task, drawing on their previous knowledge, experience, expectations, motivations and so on. The signs produced by students have been called (Falcade, 2006; Bartolini Bussi & Mariotti, 2008) *artifact-signs* as they refer to the context of the use of the artifact, very often referring to one of the parts of the artifact and/or to the action accomplished with it. Hence the texts (i.e. the systems of signs produced by students to solve the task) are *situated*, i.e. related to the concrete situation of using the artifact. The analysis of these situated texts allows the observer to make inference about students' personal meanings (Leont'ev, 1976). In fact, the observer, usually the teacher, may gain access to students' schemes (Rabardel, 1995) by means of the texts produced by students (oral and written texts, drawings, sketches, gestures, gazes, and so on). The aim of the whole process is to nurture the maturation of situated texts (mirroring personal meanings) into mathematical texts, mirroring culturally shared meanings (right bottom vertex of the triangle). The *semiotic potential* of an artifact is the double semiotic link which may occur between an artifact and on the one hand the task to be accomplished, and on the other hand the

mathematical meanings evoked by its use and recognizable as mathematics by an expert. Hence the semiotic potential is represented in the left triangle of the sketch. It is, however, still a *potential* to be constructed and exploited.

The links between the upper part (students) and the lower part (culture) of the sketch are under the teacher’s control and responsibility. The artifact, as a part of the culture, has the potential to foster the maturation, but this process has to be guided by the teacher.

The teacher chooses the tasks (left side) and takes care of the transition (right side). If the transition for some reasons does not work the risk is that mathematics remains in the eye of the observer.

An Example from Mathematics Education: Abacus and Place Value. To illustrate this sketch, consider the following example. It is a short episode concerning a first grade pupil caught up in the task: “Represent the number eleven on a spike abacus” (see Figure 2)

Table 1.

Steps of the shift conducted by the teacher between the situated texts produced by a pupil and some forms of mathematical texts (right side of the sketch of Figure 1).

Utilization schemes	Description	Words	Mathematical meanings
Counting and stringing beads on the unit stick	1. The child strings beads one by one on the right stick telling off the number sequence.	One ... two ... three ...	One to one correspondences words-gestures and words-beads word — gesture \ / bead
	2. A bead falls out of his hands; he turns his eye towards the bead, while the hand picks up another bead from the heap.	... four ..	
	3. He goes back to counting.	... four ... five ... six .. seven ... eight ... nine	
Replacing ten beads on the unit stick with one bead on the ten stick	4. He tries to string the tenth bead on the stick. It does not fit. He stops and thinks, He rests it on the table, slowly unstrings the nine beads and gathers them into a heap. He moves the tenth one closer to the others, grips them in the hands and looks up.	... ten.. it does not fit onto the stick It is a ten.	Ten units make a ten
	5. He strings a bead on the ten stick.	.. I string a bead here ... it is a <i>ten-bead</i> , ten...	Place value
	6. He strings a bead on the first stick. He smiles.	.. and eleven ... done!.	Eleven is one ten and one unit

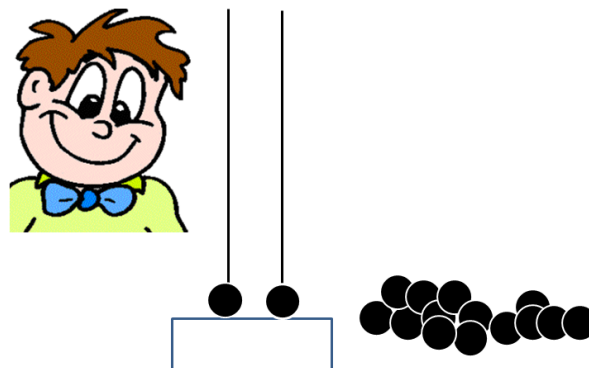


Figure 2. The abacus and the number eleven”.

The teacher’s role (right side of the sketch of Figure 1) is now to construct the shift between the situated texts produced by the pupil and some forms of mathematical texts (suitable to the pupil’s age) that are de contextualized from the specific situation and, at the same time, able to evoke the concrete experience. An interesting example is given, in step 5, by the composed word *bead-ten* (literally “*pallina-decina*”). The presence in the same protocol of different signs: *bead* (*pallina*), *ten* (*decina*), *bead-ten* (*pallina-decina*) witnesses the fact that the pupil’s utterances are hybrid, as they contain different signs: “bead” is an *artifact-sign* belonging to the concrete world (situated), “ten” is a *mathematical sign*, belonging to the mathematical world, “bead-ten” is what has been called (Bartolini Bussi & Mariotti, 2008; Falcade, 2006) a *pivot-sign*, that may be used by the teacher to create a link between the plane of concrete experience and the plane of mathematical experience: it is a bead, but, because of the position on the second stick, it is worth a ten. Further examples concerning this role are discussed by Bartolini Bussi & Mariotti (2008), Bartolini Bussi & Boni (2009).

Physics Education

The Figure 3 shows the same scheme of the Figure 1, extended to the case of Physics education. We elaborate more on this point below in order to show the usefulness of applying this theoretical framework, which is already known and used in the literature on mathematics education.

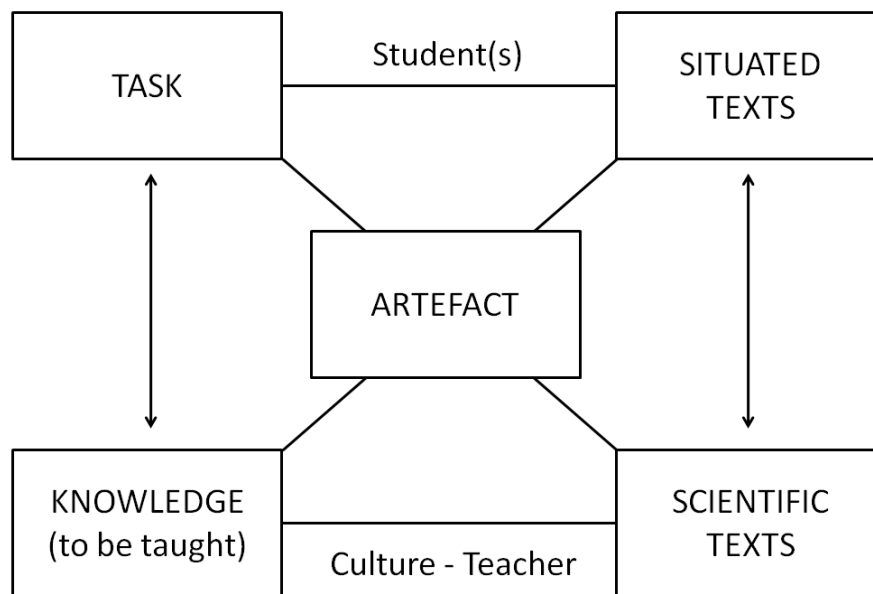


Figure 3. Elements and links between the elements of the theoretical framework of semiotic mediation, addressed to science (physics) education.

The Setting. To stimulate the students' emotional and cognitive involvement, Corni and Mariani propose problematic situations posed by an illustrated story (Corni, Giliberti & Mariani, 2010). The direct action on the concrete artifact is mediated and introduced by the story. The story, built according to specific criteriaⁱⁱⁱ, is an instrument of semiotic mediation not only for the task, but also for the concrete artifact, the situated texts and the scientific texts. In a more general meaning, the story is part of the artifact. It can be considered as an environmental context of the concrete artifact, one of the various contingent conditions determining the concrete artifact children have to overcome.

The story-artifact is a context that involves children that, under the guidance of the teacher, take active role in the development of the utilization schemes of the concrete artifact. At the same time children, who are helped to enter and to exit the story and then to change the planes of observation and reasoning several times, are facilitated in the process of meaning construction (Corni, Giliberti & Mariani, 2010; Pagliaro & Mariani, 2011). Our choice to build this environment has pedagogical and methodological advantages (Corni, Giliberti & Mariani, 2010), but it requires that the teacher has to ensure that the children are able to enter and exit the affective scene of the story to focus on the concrete artifact.

Artifacts. Concrete artifacts are reproductions in the laboratory of a piece of reality, which in principle is complex, with the aim of simplifying it and focusing particular aspects (according to the analysis of Knorr-Cetina quoted above). Such reproduction/simplification and its use follow from the task to be solved and children, if adequately involved, could acquire the ability to question and to examine phenomena. Because of the setting in the story-artifact, the concrete artifact is enriched by the story, that gives it a human sense^{iv} and allows describing it in a narrative way.

Tasks. The tasks relate to the experience of a natural phenomena in a “purified laboratory version” (see the quotation of Knorr-Cetina (1999, p. 9). The task, in some cases, reflecting a fundamental step of scientific research, could be expressed in terms of inquiry, stimulating children to become able to formulate good questions by themselves. The formulation could imply at least three main kinds of answers: the interpretation of an observed phenomenon, the prediction of the behaviour of a phenomenon under certain conditions, or the resolution of a problematic situation. In addition, other tasks can be proposed to children thanks to the use of the story-artifact. For example the children are asked “to continue the story” and/or “to send a letter to a story character anticipating or explaining which errors could be made and could be avoided”.

The piece of knowledge to be taught. In our approach, the epistemological framework is based on some physical concepts which are basic in term of their role in the discipline and elementary in terms of their affinity with the primitive images derived from early experience elaborated by child mind. The images at the basis of scientific thought have been identified within the theory of Force Dynamic Gestalts having the aspects of quantity or substance, quality or intensity, and force/power or energy (Fuchs, 2009). The tools for this identification can be found mainly in cognitive linguistics, particularly in Talmy's (1988) theory of embodied schemas of causation (called the theory of Force Dynamics by Talmy). The first two FDG aspects refer to the extensive and intensive quantities, respectively, that characterize the different areas of physics. For example, in the case of fluids, the fluid volume is the extensive quantity, while pressure is the intensive one. Table 2 summarizes the extensive and intensive quantity values associated with the main physical areas commonly studied in primary school.

Table 2.

Commonly studied physical areas and corresponding extensive and intensive quantities.

Physical area	Extensive quantity	Intensive quantity
Fluids	Volume	Pressure
Thermodynamics	Entropy	Temperature
Mechanics (translations)	Momentum	Velocity
Mechanics (rotations)	Angular momentum	Angular velocity
Electricity	Electric charge	Electric potential
Chemistry	Amount of substance	Chemical potential

When the substance is submitted to a difference of intensity, which corresponds to the concept of difference of potential in the generalized sense, it is subjected to a driving force that induces flow.

The force/power gestalt refers to the idea of being able to do something, to cause a process, to hinder, to resist, and are at the basis of the scientific concept of energy. This fundamental physical concept acts as controlling principle of natural phenomena that can be schematized in the increase of potential of a quantity at the expense of the decrease of potential of another quantity. For example, the water wheel rotary motion is produced by the pressure drop of water breaking on its blades.

The activity that we give as example is addressed to the identification and differentiation of the quantity of water (substance) and the pressure difference (potential difference) determined by the difference between two free surfaces as relevant variables for water transfer in a system consisting of two containers connected by a pipe (communicating vessels).

An example from physics education: communicating vessels

The setting: the illustrated story. The story is organized in several acts, each structured in different scenes. We refer here to the first act of a more articulated didactical path about fluids in which the specific aim is the identification of the difference of water level as the driving force for water flow. The story “Rupert and the dream of the swimming pool” tells the adventures of a frog named Rupert, who has to fill his swimming pool (represented by a small container in the artifact) connected to an aqueduct (represented by a large high container) on which attachment points are provided at three different altitudes. The story act is composed of three scenes: the first refers to the pool connected to the highest point of the aqueduct that does not allow an adequate filling, the second one refers to the connection to the intermediate point that allows a full filling of the pool, and the third one to connection to the lowest point causing water to overflow.



Figure 4. Pico and Rupert, the characters of the story.

The character Pico (Figure 4) enters the scene in correspondence of problematic situations. He is a child, the fil-rouge of all the stories, who supplies guidance to pupils. He escorts the children in various situations, stimulating them with requests, proposing experiments with artifacts, suggesting tasks and questions to be answered.

The artifact. The concrete artifact consists of two containers of different sizes connected by a pipe. The larger one is full of water and maintained at constant level by a pump. The artifact needs to be explored because it is composed of several distinct elements that can be connected in different ways (Figure 5) thus conveying different relational information.

This step is related to the instrumentalization process, according to Rabardel (1995), where the objective features of the artifact are explored.



Figure 5. The artifact used for the experimental activities.

In the proposed activity, the artifact provides three possibilities of pipe connection at different heights, leaving all other parameters fixed. The opportunity given to pupils, through the artifact, to focus on one variable at a time gives strength to the meaning of difference of level as a cause of water flow (later, the concept of level difference will be made to evolve into difference of pressure). At this stage it is possible to find some relatively stable and structured elements in activity and action of the children: a conscious control on each component of the concrete artifact (container with a specific capacity, pipe with characteristic resistance, tap) and a partial control on their mutual relationship emerges. The utilization schemas (e.g. connecting the pool to the reservoir) can be described at different levels, according to the age of the students and to their previous knowledge. Actually an expert could operate with an “automatic” action synthesizing various mental operations.

Tasks. The questions and the tasks suggested by Pico to the pupils in this segment of the story are listed in Table 3, which indicates the time when they are posed

Table 3

Questions and tasks related to storyboard steps

Story board	Task after storytelling	Task before the experiment	Task of the experiment	Task after the experiment
... Rupert began to enjoy the idea of swimming in the pool...	1) Draw story's Rupert until now	2) How will Rupert fill the pool?		
To his surprise he noticed that only a fraction of the pool was filled. What was happening? What		3) According to you, why is Rupert's pool not filled?	4) Observation and drawing of the artifact	5) Why is the pool not filled? At what level will the water stop?

had not worked as it should?				
The grasshopper gave indications to lower the position of the pool.	6) Draw the current situation	7) According to you, once the tap has been opened, will the pool be filled? Why? How much will the pool fill?	8) Observation and drawing of the artifact	9) What happens to the water? How do you explain it?
...Rupert indicates to the beetles to move the connection further down, to the foot of the hill....The Blackbird said: Rupert! Are you sure? you did not just use your head: I have a suspicion ...	10) Draw the current situation	11) In your opinion what does the blackbird suspect? After opening the tap, will the pool fill? Are there any differences compared to the other experiments?	12) Observation and drawing of the artifact	13) What happens to the water? How do you explain it?

The students' texts. Examples of children's drawings and texts are presented. In Figure 6 we show two examples of children's drawings on story situation about Rupert's problem to fill the swimming pool (task 1).

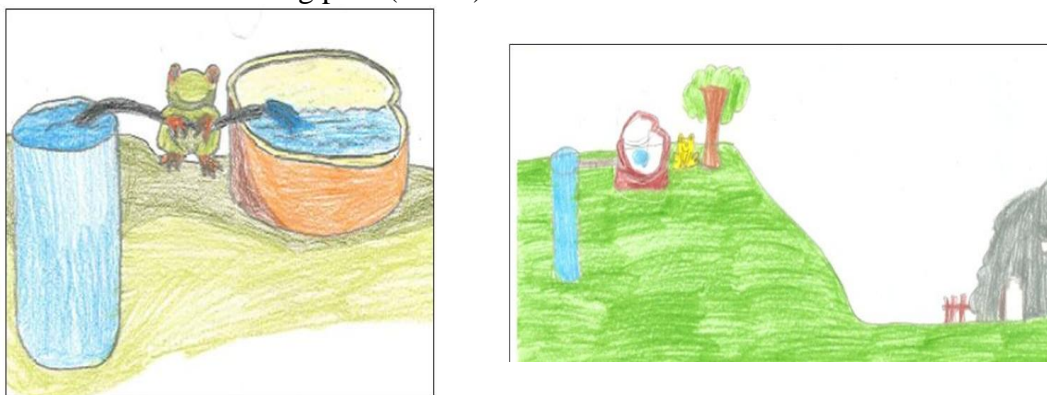


Figure 6. Examples of situated drawings of two pupils

Two children's answers to task 2 are the following: *"The bath fills with water by opening the tap which sends down the water"*; *"Rupert has to take a bucket, look for one pond, thereby continuing to fill the pool"*. In Figure 7 according to task 8, a child's drawing of the artifact is shown. In addition, the following sentences give an example of child 'answer to task 11: *"(1) I think the blackbird suspects that it was not a good idea"*; *"(2) when Rupert opens (the tap), the water will come out from the pool"*; *"(3) In my opinion the water come out from the pool because the (water in the) aqueduct is too high. Instead*

in the first situation the water filled (the pool) very little. In the second case the water filled up to the level of the aqueduct”.

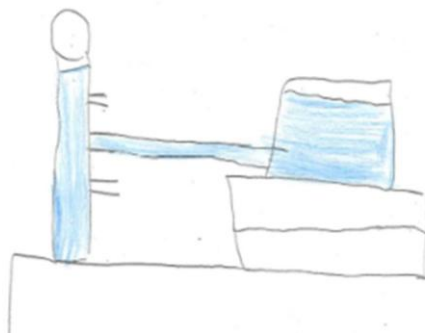


Figure 7. An example of pupil's situated drawing

These productions are in response to artifact activity to describe, assemble pieces, identify problem variables, formulate hypothesis and give explanations and interpretations.

By means of all these activities children produce some situated texts (where artifact-signs are present and may be used as pivot-signs) that the teacher may use for the evolution towards scientific texts. In Table 4 there are some expressions (fourth column) which frequently emerged in classroom experimentation specifically connected to artifact activities (second column) and to the scientific meaning (fourth column).

Table 4

Steps of the shift conducted by the teacher between the situated texts produced by pupils and some forms of physical texts (right side of the sketch of Figure 2).

Utilization schemes	Task	Description	Children' words	Scientific meaning
Connecting at the highest point	2	The child speculates on the basis of its experience as someone could fill the pool	Pipe Pump Bucket To fill To take	Substance can be transferred
	3-5	S/he connects the pool to the highest aqueduct tap	Thrust to climb Greater level Glasses of water Aqueduct Pipe	Substance flows from higher to lower potential
Connecting at midpoint	7-9	S/he connects the pool to middle aqueduct tap; s/he turns her/his eye to follow water	To lower Lower To reach To exceed Faster To pass the pipe	The potential difference produces the substance flux.

		stream until it stops.	Up to a certain level Right place Right level To flow Equal	
Connecting at the lower point	1 1-13	S/he connects the pool to the lower aqueduct tap; s/he turns her/his eye to follow water s which continues to flow and to escape from the container	Height Differences Get out Overflow Higher than	If the potential difference is maintained, the substance continues to flow

The teacher's role. Using the children's expressions (pivot signs) the teacher builds the relationship between situated texts and scientific texts located in Figure 2. We give two examples to show the correspondence between them. The teacher could use "right level" or "right place" (pivot signs in Table 4) to move towards the physics meaning of balance in condition of equal levels. As a second example the teacher starting from "difference", and "higher than", "lower than", "greater than" could induce children to think about "quantity" and "height" which are usually not well distinguished, and to identify the height difference (later to be better acknowledged as difference of pressure) and not the quantity difference as the cause of water flow.

Here language plays an important role and the teacher has to master the various languages available to children and their grades of formalization. According to the pupils' age and competencies, the scientific text ranges from a qualitative (even rigorous) explanation in the native language, to the mathematically formalized expression typical of mature science of experts.

Findings of the pilot experiments

The results of three pilot experiments, tested in some ten experimentations have been documented (Corni, Giliberti & Mariani, 2011; Pagliaro & Mariani, 2011; Sedoni & Mariani, 2011; Vaccari & Mariani, 2011). In the case of "Rupert and the dream of the swimming pool" the materials gathered for assessment have been analysed, from three points of view:

- pupils' attention to the real world and their focus on relevant variables;
- the overcoming of naïve non scientific ideas;
- the inclination to describe, make hypotheses and interpret phenomena.

Synthetically, the data show that: as the story and the experimental activities unfold, a selection process by children takes place and in their drawings certain elements are focused on at the expense of others; there is a great increase in the correct representation of the positions of the various significant elements of the story and, likewise, an increase

in precision in the drawing of details (i.e., positioning of the pipe, levels of the water, etc.); the children explanation that the inclination of the pipe determines the direction of water flow is overcome; a gradual shift may be noted from the mere repetition of the story and the exclusively phenomenological description of the experiments to the formulation of hypotheses and interpretations of the experimental observations in terms of meanings acquired. We consider that such evolution can be related to the process of internalisation.

Similarly, in further didactical cycles the students themselves are encouraged to enrich the story or to produce a new one. For example, in the pilot experiment about the communicating vessels, children were invited to send a letter to Rupert. They evidence three mistakes made by the character: “first, building the pool at the top of the hill was not a good idea, because the water could not rise higher than the aqueduct. (...) second (...) you would have to calculate that with the pool at the base of the hill, the water in the pool would exceed the walls of the pool trying to reach the water level of the aqueduct. The right solution should be to connect the pool to the second tap. (e.g.: in the correct position allowing the pool filling) In this position, the pool results always full because the water level would be equal to that of the water into the aqueduct. (...) Third, we also thought you could solve the problem of the shadow by uprooting the tree and replanting it elsewhere”. The precision and the pertinence of the observations made by children in this letter, as other texts produced by children, could be taken as indicators that the sense of the activity carried out by the pupils’ under the teacher’s guidance has been internalized.

Didactical cycles in mathematics and physics education.

The studies carried out in mathematics education (see Bartolini Bussi & Mariotti, 2008) and in physics education (outlined in this paper) clearly show the same structure.

Individual and Collective Activities

The teaching-learning process starts with the emergence of students’ personal meanings in relation to the use of the artifact. The emergence is witnessed by the appearance of situated texts and their evolution into mathematical / scientific texts has to be fostered by the teacher through specific social activities. In summary, the process of semiotic mediation consists in the process of evolution that has its first step in the emergence of personal meanings related to the accomplishment of a task and develops into the collective construction of shared signs related to both the use of the artifact and the mathematics / physics to be learnt. Such evolution can be promoted through the iteration of didactical cycles (Figure 8) where different categories of activities take place, each of them contributing differently but complementarily to develop the complex process of semiotic mediation (Bartolini Bussi & Mariotti, 2008).

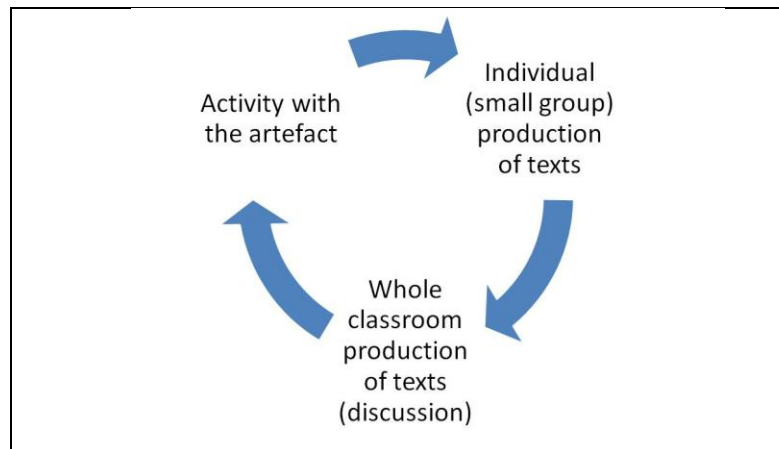


Figure 8. The didactical cycle.

- **Activities with the artifact.** This type of activity constitutes the start of a cycle and is based on tasks to be carried out with the use of an artifact. Situations are designed with the aim of promoting the emergence of signs referred to use of the artifact use.
- **Individual production of texts.** Students are involved individually in different semiotic activities concerning mainly written productions with drawings. For instance, students might be asked as homework to write individual reports on the previous activity with the artifact or to produce drawings related to the activity and to the artifact reflecting on their own experience, and raising possible doubts or questions. Written productions and drawings can become objects of discussion in the following collective work.
- **Whole classroom production of texts (discussion).** Discussions play an essential part in the teaching-learning process and constitute the core of the semiotic process, on which teaching-learning activity is based. For instance, after problem solving sessions, the various solutions may be discussed collectively; but also students' written texts or other texts may be analysed and commented. The main objective for the teacher in such a discussion is fostering the move towards mathematical / physical meanings, taking into account individual contributions and exploiting the semiotic potential coming from the use of the artifact (for the elaboration of the concept of Mathematical Discussion, see Bartolini Bussi, 1998a, 1998b).

The teacher's role at each step is different and crucial. For instance, it ranges from the choice of suitable tasks which exploit the semiotic potential of an artifact (left part of the figures 2 and 3) to the professional recourse to suitable interaction strategies during the tricky step of classroom discussion (right part of the figures 2 and 3). The study of the teacher's role is in progress in pilot experiments with teacher/researchers and in the diffusion of classroom activities with artifacts to the broad education system.

In both programs of mathematics and physics education, the teaching experiments concern long term activities. The simple example presented here for mathematics (abacus and place value) is only a snapshot of a single classroom episode. In every teaching experiment, the sequence of activity is complex and organized in a sequence of coordinated didactical cycles (e.g. Bartolini Bussi, 1996; Bartolini Bussi et al., 1999, 2005, 2009).

In an analogous way for physics, different kinds of tasks (after storytelling, before the experiment, during the experiment, after the experiment) are proposed to children. In each case, individual task (including action and production of texts) is followed by a whole class discussion. For this reason, the sequence presented in Table 3, concerning physics, synthesises different didactical cycles.

Discussion

The two research programs in mathematics and physics education have been developed independently of each other for some years. In particular, the program in mathematics education was started in the early 90s with a long series of teaching experiments in different school levels (first in primary school, then in pre-primary and secondary school and eventually in programs for teacher education). The program focused initially on artifacts taken from the history of mathematics, which can be concretely handled. Later, thanks to the contribution of Mariotti and Falcade (Bartolini Bussi & Mariotti, 2008; Falcade 2006) the program included also ICTs.

The program in physics education is more recent and for the moment concerns primary school pupils and primary teachers education. The program was designed to foster teachers' awareness of the function of a scientific laboratory in school, showing that the reasoning that allows the shift from facts' description to facts' explanation and interpretation is realized in the laboratory by means of sequential and gradual steps. Teachers are not always completely aware of that. In mathematics education there is now some debate about experimental activity, under the pressure of dynamic geometry and computer algebra software. This means that teachers may feel the need to be trained in experimental activity. In physics, on the contrary, experimental activity has been considered obvious and not worth much reflection in education. The risk is that the didactical laboratory is reduced to a set of mechanical practices without awareness. We claim that the theoretical framework of semiotic mediation might offer sound methodological scaffolding for teachers, in order to mediate scientific meanings that have been historically incorporated in artifacts and in the experimental method.

The collaboration between our research teams (in mathematics and physics education) began, some years ago, to highlight the common choices and the differences, which might result in mutual enrichment. For instance, the focus on semiotic mediation, first developed within the program on mathematics education, has proved to be useful to systematize teaching experiments in the program on physics education.

One might devise another means of cross-fertilization. As said above, from the very beginning, the program on physics education has highlighted the importance of illustrated stories to introduce tasks and guide the teaching experiments. The illustrated stories are initially presented by the teacher in a microworld that is a mirror image of the real world: as such it gives, at a reduced scale, a model of scientific inquiry. The stories are realized by means of short cartoons or other media (booklets, posters, slide presentations) (Corni, Giliberti & Mariani 2010; Mariani, Corni & Giliberti, 2011; Mariani, Giliberti & Corni, 2011; Orsini & Vecchi, 2011). Through the enrichment and the continuation of the story by children, the individual production of texts is triggered. It is the expression of an individual elaboration that follows the iteration of various didactical cycles in which external processes, essentially social, and individual processes have been intertwined. As

we have shown the process of internalization is followed by the teacher through the analysis of pupils' texts.

Hence the story is also useful for the teacher to get information about the process of internalization. The positive results of the physics education program suggest that this methodological tool might be extended to the program on mathematics education: we have just started to design some very short stories for mathematics too, to be used with primary school students.

There are obviously epistemological differences between mathematics and physics, although, in some cases, the distance between applied mathematics and physics is very small. Many teaching experiments carried out in the program of mathematics education concern the border between mathematics and art (e.g. perspective drawing, Bartolini Bussi, 1996; Bartolini Bussi et al, 2005) and between mathematics and technology (e. g. gears, Bartolini Bussi, et al., 1999). This choice has made the theoretical and methodological coordination between the two programs easier. We claim, however, that the most important founding choice of both programs, that has made comparison and dialogue possible and effective, is pedagogical: in both programs the teacher's role is fundamental, according to the Vygotskian perspective of *obučenie*.

An obvious difference between mathematics and physics education seems to be the following: mathematics copes with mathematical objects and expresses them into mathematical language, whilst physics copes with natural phenomena and aims at constructing mathematical models of them. For instance, many physical laws are expressed as direct or inverse proportions. Hence the scientific texts produced by experts are usually systems of mathematical texts involving quantities and referring to empirical observations. This ultimate goal is not always within the reach of primary school students. As literature shows, proportional reasoning is very complex (Noelting, 1980a, 1980b), as additional reasoning acts as an obstacle. In primary school physics education a realistic goal is to achieve qualitative texts that express in a meaningful way the relationships between the quantities involved in the experiments. An open question is whether the enrichment of mathematics education in primary school with modelling activities might be useful to overcome the above obstacle.

We are planning to analyze, in deeper way, the analogies and differences between the two programs, at least at primary school level to look for a shared methodology for teacher education which is epistemologically sound. Until now the extension of the theoretical framework of semiotic mediation after Vygotskian approach from the field of mathematics to the field of physics education has proved to be fruitful for both programs and has shown the potential of chiselling either one by means of "a rock from another mountain" (Xiao Ya, XI century BC).

References

- Altiero, T., Bortolotti, C.A., Corni, F., Giliberti, E., Greco, R. & Mariani, C. (2010). Introduzione elementare all'energia: un laboratorio di scienze per insegnanti di scuola primaria. *New Trends in Science and Technology Education*. Bologna, CLUEB Ledi Menabue e Giorgio Santoro (Eds.) pp. 157-170.
- Antonini, S. & Martignone, F. (2011). Argumentation in exploring mathematical machines: a study on pantographs, *Proceedings of the 35th Conference of the*

- International Group for the Psychology of Mathematics Education*, Vol. 2. (pp. 41-48). Ankara, Turkey.
- Bakhtin, M. M. (1984). *Problems of Dostoevsky's Poetics*. Edited and translated by Caryl Emerson. Minneapolis, University of Minnesota Press.
- Bartolini Bussi, M. G. (1996). Mathematical discussion and perspective drawing in primary school. *Educational Studies in Mathematics*, 31, 11-41.
- Bartolini Bussi, M. G. & Pergola, M. (1996). History in the Mathematics Classroom: Linkages and Kinematic Geometry. In H. N. Jahnke, N. Knoche & M. Otte (Eds.) *Geschichte der Mathematik in der Lehre* (pp. 36-67). Goettingen, Vandenhoeck & Ruprecht.
- Bartolini Bussi, M. G. (1998a). Joint Activity in the Mathematics Classroom: a Vygotskian Analysis. In F. Seeger, J. Voigt & U. Waschesho (Eds.), *The Culture of the Mathematics Classroom. Analyses and Changes* (pp. 13-49). Cambridge (UK), Cambridge University Press.
- Bartolini Bussi, M. G. (1998b). Verbal interaction in mathematics classroom: A Vygotskian analysis. In H. Steinbring, M. G. Bartolini Bussi, & A. Sierpinska (Eds.), *Language and communication in mathematics classroom* (pp. 65 - 84). Reston, VA: NCTM.
- Bartolini Bussi, M. G., Boni M., Ferri, F., & Garuti, R. (1999). Early approach to theoretical thinking: Gears in primary school. *Educational Studies in Mathematics*, 39, 67-87.
- Bartolini Bussi, M. G. & Boni, M. (2003). Instruments for semiotic mediation in primary school classrooms. *For the Learning of Mathematic*, 23 (2) 12-19.
- Bartolini Bussi, M.G. (2005). The meaning of conics: historical and didactical dimensions, in J. Kilpatrick, C. Hoyles, O. Skovsmose & Valero P. (Eds.) (2005). *Meaning in Mathematics Education* (pp. 39-60). New York, Springer.
- Bartolini Bussi, M.G., Mariotti, M.A. & Ferri F. (2005). Semiotic mediation in the primary school: Dürer glass. In M. H. G. Hoffmann, J. Lenhard, F. Seeger (ed.). *Activity and Sign – Grounding Mathematics Education. Festschrift for Michael Otte* (pp. 77 – 90). New York: Springer.
- Bartolini Bussi, M. G. (2007). Semiotic mediation: fragments from a classroom experiment on the coordination of spatial perspectives. *ZDM the International Journal of Mathematics Education*, 39 (1), 63-72.
- Bartolini Bussi, M. G., Boni M., & Ferri, F. (2007). Construction problems in primary school: a case from the geometry of circle. In P. Boero (Ed.), *Theorems in school:*

- From history, epistemology and cognition to classroom practice* (pp. 219–248). Rotterdam, Sensepublisher.
- Bartolini Bussi, M. G. & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom: Artifacts and signs after a Vygotskian perspective. In L. English & al. (Eds.), *Handbook of International Research in Mathematics Education, second edition* (pp. 746-783). New York and London, Routledge.
- Bartolini Bussi, M. G. & Maschietto, M. (2008). Machines as tools in teacher education', in: Tools and Processes in Mathematics Teacher Education, *The International Handbook of Mathematics Teacher Education*. (pp. 183-208).D. Tirosh, T. Wood, eds., vol. 2, SensePublishers, Rotterdam.
- Bartolini Bussi, M. G. & Boni M. (2009). The Early Construction of Mathematical Meanings: Learning Positional Representation of Numbers,. In O. A. Barbarin & B. H. Wasik (Ed.) *Handbook of Child Development and Early Education: Research to Practice* (pp.455-477). New York, The Guilford Press.
- Bartolini Bussi, M.G., Garuti, R., Martignone, F. & Maschietto, M. (2011). Tasks for teachers in the MMLab-ER Project. *Contribution to 35th Conf. of the Int. Group for the Psychology of Mathematics Education Research Forum*. Ankara, Turkey.
- Borba, M. C., & Bartolini Bussi, M. G. (2008). Resources and technology throughout the history of ICMI. In M. Menghini, F. Furinghetti, L. Giacardi, & F. Arzarello (Eds.), *The first century of the International Commission on Mathematical Instruction (1908–2008). Reflecting and shaping the world of mathematics education*. Rome, Istituto della Enciclopedia Italiana fondata da Giovanni Treccani.
- Cerulli, M. (2004). *Introducing pupils to Algebra as a Theory: L'Algebrista as an instrument of semiotic mediation*, University of Pisa: PhD thesis.
- Corni, F., Giliberti, E., Mariani, C., Altiero, T., Marchetti, M. & Martini, A (2010). Il progetto per la diffusione della cultura scientifica “Piccoli scienziati in laboratorio: Esperimenti & Modelli per la formazione scientifica nella scuola primaria” *New Trends in Science and Technology Education* (pp. 147-156). Bologna, CLUEB Ledi Menabue e Giorgio Santoro.
- Corni, F., Giliberti, E. & Mariani, C. (2010). A story as innovative medium for science education in primary school. *Proceeding of the 2010 GIREP-ICPE-MPTL International Conference*. Retrieved from the Web October 2012. http://www.univ-reims.fr/site/evenement/girep-icpe-mptl-2010-reims-international-conference/gallery_files/site/1/90/4401/22908/29321/29596.pdf
- Corni, F., Giliberti, E. & Mariani, C. (2011). Semiotic mediation and didactical cycle as methodological reference for primary school teachers. *Proceedings The biannual conference of the European Science Education Research Association ESERA 2011*

Conference. Retrieved from the Web October 2012. http://http://lsg.ucy.ac.cy/esera/e_book/base/ebook/strand13/ebook-esera2011_CORNI-13.pdf

Cole, M. (2009). The Perils of Translation: A First Step in Reconsidering Vygotsky's Theory of Development in Relation to Formal Education, *Mind Culture and Activity: an International Journal*, 16 (4), 291-295.

Donaldson, M. (1978). *Children's Minds*, London: Fontana/Croom Helm.

Davydov, V.V. (1990). *Types of generalization in instruction: Logical and psychological problems in the structuring of school curricula*. Reston, VA: National Council of Teachers of Mathematics.

Engestroem, Y. (1991). Activity theory and individual and social transformation, *Multidisciplinary Newsletter for Activity Theory*, 7/8, (pp. 6-17). Glasersfeld, E. von (ed.) (1991) *Radical constructivism in mathematics education*. Dordrecht, Kluwer.

Falcade, R. (2006). Théorie des Situations, médiation sémiotique et discussions collectives, dans des séquences d'enseignement avec Cabri-géomètre pour la construction des notions de fonction et graphe de fonction. University Grenoble 1: PhD thesis.

Falcade, R., Laborde, C. & Mariotti, M.A. (2007). Approaching functions: Cabri tools as instruments of semiotic mediation. *Educational Studies in Mathematics*, Vol. 66, N. 3, Dordrecht: Kluwer, pp. 317 – 333.

Falcade, R. & Strozzi, P. (2009). Construction and Representation of Space in 5-Year-Old Children, in O. A. Barbarin & B. H. Wasik, *Handbook of Child Development and Early Education: Research to Practice*. New York, The Guilford Press, 499-520.

Fuchs, H. U. (1997). The Continuum Physics Paradigm in Physics instruction I. Images and models of continuous change. Retrieved from the Web August 08, 2010 <https://home.zhaw.ch/~fuh/LITERATURE/Literature.html>

Fuchs, H. U. (2007). From Image Schemas to Dynamical Models in Fluids, Electricity, Heat, and Motion - An Essay on Physics Education Research Retrieved from the Web August 08, 2010 <https://home.zhaw.ch/~fuh/LITERATURE/Literature.html>

Fuchs, H. U. (2009). Figurative Structures of Thought in Science. An Evolutionary Cognitive Perspective on Science Learning Talk presented to the General Assembly of the Conférence des directeurs de gymnase de Suisse Romande et du Tessin. Mendrisio, September 18, 2009 Retrieved from the Web August 08, 2010 <https://home.zhaw.ch/~fuh/LITERATURE/Literature.html>

- Fuchs, H.U., Corni, F., Giliberti, E. & Mariani, C. (2011). Force Dynamic Gestalt of Natural Phenomena: Teaching the Concept of Energy. Proceedings of The biannual conference of the European Science Education Research Association ESERA 2011 CONFERENCE. Lyon, France.
- Gilbert, J.K. & Boulter, C. (2000). *Developing Models in Science Education*, Dordrecht, Kluwer Academic Publisher.
- Harris, P. (1991). *Mathematics in a cultural context: Aboriginal perspectives on space, time and money*, Deakin University Press.
- Herrmann, F. (2000). The Karlsruhe Physics Course. *European Journal of Physics*, 21, 49-58.
- Hestenes, D. (1997). Modeling Methodology for Physics Teachers. The changing role of the physics department in modern universities, *American Institute of Physics Part II.* (pp. 935-957). In E. Redish & J. Rigden (Eds.) Retrieved from the Web August 08, 2010 <http://modeling.asu.edu/R&E/ModelingMeth-jul98.pdf>
- Hestenes, D. (2006). Notes for a Modeling Theory of Science, Cognition and Instruction. *Proceedings of the 2006 GIREP conference: Modelling in Physics and Physics Education*. Retrieved from the Web August 08, 2010 http://www.girep.org/proceedings/conference2006/David_Hestenes_-_Notes_for_a_Modeling_Theory_of_Science,_Cognition_and_Instruction.pdf
- Ifrah, G. (1981). *Histoire universelle des chiffres. L'intelligence des hommes racontée par les nombres et le calcul*. Éditions Seghers Paris, 1981.
- Knorr-Cetina, K. (1999). *Epistemic Cultures: How the Sciences Make Knowledge*, Harvard MA: Cambridge University Press.
- Kozulin, A. (1999). *Vygotsky's psychology: a biography of ideas*, Cambridge MA: Harvard Ukrainian Research Institute, U.S.
- Latour, B. & Woolgar S. (1979). *Laboratory Life: The Social Construction of Scientific Facts*, Beverly Hills: Sage Publications; Retrieved from the Web August 08, 2010 mainsite/Componentsfromclmer/Component13/Mediation/SemioticMediation.html
- Leont'ev, A.N. (1976/1964), *Problemi dello sviluppo psichico*, Ed. Riuniti and Ed. Mir.
- Mariani, C., Corni, F., Altiero, T., Bortolotti, C. A., Giliberti, E., Landi, L., Marchetti, M. & Martini A. (2010). *Experiments & Models for physics learning in primary school*. Selected contributions GIREP-EPEC & PHEC 2009 International Conference. Leicester: Lulu/The Centre for Interdisciplinary Science. pp. 137-151.

- Mariani, C., Corni, F. & Giliberti, E. (2011). A didactical path for age 5-8 on the concept of extensive quantity using stories as cognitive tools. *Proceeding of The biannual conference of the European Science Education Research Association ESERA 2011 Conference*. Retrieved from the Web October 2012. http://lsg.ucy.ac.cy/esera/e_book/base/ebook/strand14/ebook-esera2011_MARIANI-14.pdf
- Mariani, C., Giliberti, E. & Corni, F. (2011). *Storie e percorsi. Atti del Convegno Innovazione nella didattica delle scienze nella scuola primaria: al crocevia fra discipline scientifiche e umanistiche*. Modena, Artestampa.
- Mariotti, M.A. (2009). Artifacts and signs after a Vygotskian perspective : the rôle of the teacher, *ZDM International Journal for Mathematics Education* 41: 427-440.
- Martignone , F. (2011). Laboratory activities in teacher training. Proceeding CERME7, Rzeszow, Poland.
- Mariotti, M. A. & Maracci, M. (2010). Un artifact comme outil de médiation sémiotique: une ressource pour le professeur. In G. Gueudet & L. Trouche (eds). *Ressources vives. Le travail documentaire des professeurs en mathématiques* (pp. 91-107). Rennes, Presses Universitaires de Rennes et INRP.
- Mariotti, M. A. & Maracci, M. (2012) Resources for the teacher from a semiotic mediation perspective, in G. Gueudet, B. Pepin, L. Trouche (eds), *From Text to 'Lived' Resources. Mathematics Curriculum Materials and Teacher Development*. (pp. 59-75). Springer.
- Maschietto, M. & Bartolini Bussi, M. G. (2009). Working with artifacts: gestures, drawings and speech in the construction of the mathematical meaning of the visual pyramid. *Educational Studies in Mathematics*, 70, 143-157.
- Maschietto, M. & Bartolini Bussi, M. G. (2011). Mathematical machines: from history to mathematics classroom - Constructing Knowledge, in O. Zaslavsky, P. Sullivan (eds.), *Constructing Knowledge for Teaching Secondary Mathematics. Tasks to enhance prospective and practicing teacher learning*. (pp. 227-247). Springer.
- Mecacci, L. (Ed.) (1990). Lessico Vygotskiano in Lev Vygotskij, *Pensiero e linguaggio. Ricerche psicologiche*. Roma-Bari, Laterza.
- Menčiskaya, N. A. (1966). *Pedagogičeskaia Enciklopedija*, Moskva: Sovetskaja Enciklopedija.
- Menninger, K. (1969). *Number Words and Number Symbols*. Cambridge, Mass.: The M.I.T. Press.
- Noelting, G. (1980a). The development of proportional reasoning and the ratio concept, Part I—differentiation of stages. *Educational Studies in Mathematics*, 11, 217-253.

- Noelting, G. (1980b). The development of proportional reasoning and the ratio concept, Part II—problem-structure at successive stages; problem-solving strategies and the mechanism of adaptive restructuring. *Educational Studies in Mathematics*, 11, 331-363.
- Norman, D. A. (1988). *The psychology of everyday things*. New York: Basic Books.
- Orsini, L & Vecchi, C. (2011). Avventure a traslandia e a rotolandia. *Atti del Convegno Innovazione nella didattica delle scienze nella scuola primaria: al crocevia fra discipline scientifiche e umanistiche*. Modena, Artestampa.
- Pagliaro, M. & Mariani, C. (2011). Come e perché l'acqua si muove: la storia della rana Rupert. *Atti del Convegno Innovazione nella didattica delle scienze nella scuola primaria: al crocevia fra discipline scientifiche e umanistiche*. Modena, Artestampa.
- Peirce, C. S. (1932). *Collected Papers of Charles Sanders Peirce*, vols. 1–6, 1931–1935, Charles Hartshorne and Paul Weiss, eds., vols. 7–8, 1958, Arthur W. Burks, ed., Harvard University Press, Cambridge, MA. *Volume 2, Elements of Logic*, 1932.
- Rabardel, R. (1995) *Les hommes et les technologies. Approche Cognitive des instruments contemporains*. Paris, Armand Colin.
- Rabardel, P. & Samurçay, R. (2001). *From Artifact to Instrumented-Mediated Learning*, New challenges to research on learning, International symposium organized by the Center for Activity Theory and Developmental Work Research, University of Helsinki, March 21-23.
- Sedoni, C. & Mariani, C. (2011). La piscina di Rupert: un percorso sperimentale sullo spostamento dei fluidi per la scuola primaria. *Atti del Convegno Innovazione nella didattica delle scienze nella scuola primaria: al crocevia fra discipline scientifiche e umanistiche*. Modena, Artestampa.
- Talmy, L. (1988). Force Dynamics in language and cognition. *Cognitive Science*, 12, 49-100.
- Vaccari, M.G.. & Mariani, C. (2011). L'acqua dall'acquedotto al rubinetto. *Atti del Convegno Innovazione nella didattica delle scienze nella scuola primaria: al crocevia fra discipline scientifiche e umanistiche*. Modena, Artestampa.
- Vianna, E. & Stetsenko, A. (2006). Embracing History through Transforming It Contrasting Piagetian versus Vygotskian (Activity) Theories of Learning and Development to Expand Constructivism within a Dialectical View of History, *Theory & Psychology*, Vol. 16(1): 81–108.
- Vygotsky, L. S. (1978). *Mind in Society*, edited by Michael Cole, Vera John-Steiner, Sylvia Scribner, and Ellen Souberman. Cambridge, MA, Harvard University Press.

Vygotsky, L. S. (1981). *The genesis of higher mental functions*. J. V. Wersch (Ed. and Trans.), *The concept of activity in soviet psychology*. Armonk, M. E. Sharpe.

Wartofsky, M. (1979). Perception, representation, and the forms of action: Towards an historical epistemology. In *Models, representation and the scientific understanding* (pp. 188–209). D. Reidel Publishing Company.

ⁱ LANGUAGE NOTE:

Some terms that are used in this paper are listed below to clarify their meaning and their relationships. This note seems useful as they come to this paper from different research traditions, developed in different times and different languages (e. g. French, Russian) and translated into English by different authors in ways that are not always consistent with each other.

The term *sign* is used in this paper in a general sense deeply inspired by Peirce: “Something which stands to somebody for something in some respect or capacity” (Peirce, 1932, 2.228). This meaning include a large variety of examples, such as oral and written texts, gestures, drawings and sketches, symbolic devices (e.g. numerals, formulas, graphs), material devices (e. g. counting and curve drawing devices; measuring tools). This use is consistent with Vygotsky’s analysis of the structure of the mediated act (*semiotic mediation*): the act is mediated by a sign, that *leads humans to a specific structure of behaviour that breaks away from biological development and creates new forms of a culturally based psychological process* (Vygotsky, 1978, p. 40). Mediating signs are called by some authors *cultural tools* (e. g. Vianna & Stetsenko, 2006) to highlight the cultural issue or *cognitive artifacts* (e.g. Norman, 1988) to highlight their potential to drive psychological process. In most translation of Vygotsky from Russian into English they are called *psychological tools* (Vygotsky, 1981) and contrasted with *technical tools*, to address in either case the inward/outward orientation of them (we elaborate this issue in the text). In the following we use *technical/psychological tools* in most cases, even if this translation does not entirely convey the cultural base of them.

Signs in the mediated act, shape the process of *internalization* (Vygotsky, 1978), that is essentially *social* and directed by *semiotic processes* (Bartolini Bussi & Mariotti, 2008). Signs appear also in the process of *externalization*, e.g. the creative production of signs while accomplishing a task. In this case, signs are not given, but are produced during the process and offer also traces of the ongoing psychological process (for a discussion of internalization/externalization in Vygotsky, see Engestroem, 1991). To refer to the system of signs produced by students during the solution of a problem we use the word *text*.

The term *artifact* is generally meant as a man-made object taken as a whole. In this sense, every kind of sign illustrated above is an artifact. In this paper it is also used in a more specific way following Rabardel’s approach to cognitive ergonomics (1995). Rabardel contrasts *artifacts* (the material or symbolic object) with *instruments* (mixed entities made up of both artifact-type components and schematic-cognitive components called *utilization schemes*). This issue is elaborated more in the paper. Because of the presence of utilization schemes, Rabardel’s notion of instrument is related to Vygotsky’s notion of psychological tool (Rabardel & Samurçay, 2001).

ⁱⁱ REFERENCE NOTE

The most important publication on the theoretical framework of semiotic mediation after a Vygotskian approach adopted in this paper for mathematics education is the chapter by Bartolini Bussi & Mariotti (2008). It is the outcome of more than twenty years of hard and deep work carried out by university researchers (the authors, some colleagues and doctoral students) and a number of teacher-researchers who have been involved in a cooperative group to design, test and analyze several teaching experiments at different school level (from pre-primary to secondary school). The story of this joint cooperation during more than two decades mirrors the co-determination of theory and practice, that is an implicit (yet not emphasized enough) outcome of a Vygotskian approach (Vianna & Stetsenko, 2006). Partial results have

been presented and discussed in international conferences and published from time to time in peer reviewed international journals and edited volumes. In this short note, we sketch some of the most relevant steps as a kind of annotated list of references for the interested reader: needless to say that quoted references contain critical discussion of each issue.

The prompt came from the teachers' need to study, in the mid eighties, the forms of classroom interaction that are not amenable to radical constructivism (e.g. von Glasersfeld, 1991), that was at that time the dominant paradigm in the western scientific literature on mathematics education. In short, we felt the need of exploiting Vygotsky's approach and his emphasis on the cultural aspects of knowledge construction for young learners, to design, test and analyze effective classroom experiments. In the early years Bartolini Bussi and her team carried out several teaching experiments in primary school. They were useful to develop the construct of Mathematical Discussion as *a polyphony of articulated voices on a mathematical object, that is one of the motives of the teaching-learning activity* (Bartolini Bussi, 1996, p. 16). There are clear references to Bakhtin's analysis of voices (1984), as *forms of speaking and thinking, which represent the perspective of an individual, i.e. his/her conceptual horizon, his/her intention and his/her view of the world* and to Leont'ev activity theory. From Leont'ev (1978) we drew also the distinction between the *three levels of activity (collective and directed towards an object-motive), actions (goal direct processes) and operations (the way of carrying out actions in variable concrete circumstances,* Bartolini Bussi, 1996, p. 15). Because of the focus on the teacher's role, actions and operations were studied mainly with reference to this acting subject

At the beginning the teaching experiments were mainly focused on "traditional" artifacts that are used in mathematics lessons: speech, written texts, drawings (e. g. paper and pencil solution of problems). Beside several experiments published only in Italian, some studies were carried out and published internationally (e.g. Bartolini Bussi, 1996, about perspective drawing in primary school; Bartolini Bussi, 1998a; Bartolini Bussi, 1998b).

In parallel some material artifacts entered the scene, with the recourse to Wartofsky's epistemological analysis (1979) of primary, secondary and tertiary artifacts (Bartolini Bussi et al. 2005): a complex set of toys for symbolic play, for pre-primary and primary school, in a study published internationally only many years later (Bartolini Bussi, 2007; Falcade & Strozzi, 2009); artifacts from the historical phenomenology of mathematics (trains of gears, Bartolini Bussi et al. 1999; the abacus, Bartolini Bussi & Boni, 2003; the compass, Bartolini Bussi et al. 2007; the pascaline, Bartolini Bussi & Boni, 2009; devices for perspective drawings, Bartolini Bussi et al., 2005; curve drawing devices in secondary school, Bartolini Bussi & Pergola, 1996; Bartolini Bussi, 2005).

In the same years, the cooperation between the two teams chaired by Bartolini Bussi and Mariotti started, the latter being interested mostly in the implementation of teaching experiments on ICT at the secondary level (details omitted, see Bartolini Bussi & Mariotti, 2008 for a review). In this direction some fundamental doctoral thesis were developed (Cerulli, 2004; Falcade, 2006; Falcade et al., 2007)

The need of integrating and coordinating so many teaching experiments concerning different school levels and very different artifacts was the prompt for the enunciation of a comprehensive theoretical framework that was published by Bartolini Bussi & Mariotti, (2008).

Since then the activity has been developed also in some other directions:

- The fine grain analysis of teacher's actions and operations (e. g. Mariotti, 2009; Mariotti & Maracci, 2010);
 - The fine grain analysis of utilization schemes during activity with artifacts (Maschietto & Bartolini Bussi, 2009; Antonini & Martignone, 2011);
 - The extension to complex artifacts that include essential recourse to historical sources (Mariotti & Maracci, 2012);
 - The application to mathematics teacher education (Bartolini Bussi & Maschietto, 2008; Maschietto & Bartolini Bussi, 2011; Bartolini Bussi, 2011; Bartolini Bussi et al., 2011; Martignone, 2011);
 - The extension to physics education.
- This paper belongs to the last direction.

iii Story construction criteria.

- I. Each character has particular features with which children can easily identify themselves and/or identify functions, roles and methods.

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- II. The story poses practical problems.
 - III. The story is structured in steps with a recursive inner rhythm to facilitate the acquisition of a method and memorization.
 - IV. Gaps are provided, intended to give space to thought and formulation of hypotheses.
 - V. The child is involved in the story with experimental and reflective activities.
 - VI. The fantasy elements are minimized to allow the child to enter and leave the story freely.
 - VII. The time of the story is defined by logical-problem events rather than by the narrative actions
 - VIII. Some events are concluded at the end of the story, but the possibility to continue the story is open to the children: this gives the opportunity to develop verbalization and offers the teacher the opportunity to evaluate the internalization process
 - IX. Many languages are involved and integrated, from verbal to figured, from gestural to written.

^{iv} We use this term *human sense* following Donaldson (1978) to highlight the fact that the context of the problem situation refers to understandable feelings and intentions. In the case described in this paper the pupils can make sense of the situation, as Rupert wishes to build a very functional swimming pool. The situation would not be the same if only a dis-embedded laboratory experiment about communicating vessel were into play.