ELECTRONIC JOURNAL FOR RESEARCH
IN SCIENCE & MATHEMATICS EDUCATION
VOL. 29, NO. 1, 16-41

Tools, Tricks and Topics Teachers Use for Integer Arithmetic

Julie Nurnberger-Haag
Kent State University

Scott A. Courtney
Kent State University

ABSTRACT

Integer arithmetic is difficult for students worldwide. Although students' integer thinking has frequently been studied, little is known about typical instructional practice for this difficult topic. Thus, to investigate what resources teachers use, we surveyed U.S. middle-grade teachers who teach negative numbers. About half the teachers said they need more integer resources and about two-thirds of the teachers used Teachers Pay Teachers to obtain such resources. During an integer unit, each classroom used up to 7 contexts and tools/models. Moreover, we analyzed which features of tools teachers reported using. Horizontal number lines were used significantly more often than vertical. The physical tools used for chip models reflected commercially-determined valence associations that were inconsistent with real-life symbolizations, significantly more than conceptually consistent symbolizations. Therefore, recommendations include the following: (a) encourage teachers and researchers to focus on how features of tools (number line directions, chip colors and symbols) affect students' experiences and learning (b) evaluate potentially optimal sequences of models and contexts, (c) make quality integer resources freely accessible to U.S. teachers in the spaces they look to reduce for-profit influences and (d) provide criteria to support teachers to select resources based on learning affordances irrespective of commercial availability.

Keywords: Negative numbers, integer arithmetic, manipulatives, chip model, number line model, games, mathematics learning, instructional tools, classroom practice.

Introduction

Students must master integer arithmetic in middle school to facilitate their success in advanced mathematics. Moreover, high school science courses and all science fields require proficiency with negative numbers (e.g., vectors, chemical reactions). Thus, effective middle school integer instruction has far-reaching implications. According to standards in the United States, in Grade 6 students learn about negative numbers and in Grade 7 students are expected to master all rational number operations including integers and negative fractions (National Governors Association Center for Best Practices [NGA Center] & Council of Chief State School Officers [CCSSO], 2010).

Unfortunately, despite the importance of negative numbers, calculations can feel counterintuitive (Bishop et al., 2014; Fischbein, 1987; French, 2001). This is especially true for multiplying or dividing two negative numbers and subtracting a negative number because it feels wrong that the result could be positive (Fischbein, 1987; French, 2001). A Rasch analysis of the Integer Test of Primary Operations (ITPO) determined that even after instruction, the most difficult problem structures were a) division by -1 and b) subtraction of a positive number (i.e., positive minuend) minus a negative (i.e., negative subtrahend) (Nurnberger-Haag et al., 2022). To provide a sense of how difficult it is to master integer arithmetic, even after three weeks of Grade 7 instruction with a National Science Foundation (NSF)-funded textbook, accuracy on each subtraction item on the ITPO ranged from 37-67% (Nurnberger-Haag et al., 2022, p. 11). Despite division by negative numbers (i.e., negative divisor) being crucial for accurate factoring in later mathematics, the highest accuracy rate when dividing by a negative number was 75%, with only 71% of students proficient with division by -1 (Nurnberger-Haag et al., 2022, p. 13).

Given these student difficulties, teachers and researchers have developed and continue to seek resources intended to facilitate learning. However, what integer instructional resources are being used in actual classrooms and where teachers have obtained such resources has not yet been investigated. Thus, with this study, we sought to gain insights about a wide range of resource types teachers might be using to begin to better understand what and how students might be learning integer arithmetic. Broadly, we asked middle grade teachers where they obtained integer instructional resources, what resources were posted in their classroom environment, the physical and virtual tools provided, as well as the contexts and rules used.

Review of Literature and Theoretical Perspectives

Due to the diverse types of resources teachers use, a single theoretical lens would need to be too broad and thus provide insufficient focus. Research in mathematics education, when possible, should employ analytic frameworks that are sufficiently narrow to productively interpret particular data (Spangler & Williams, 2019). Different theoretical constructs are appropriate for analyzing each resource type in this study, so these constructs are addressed within the relevant sections of the literature review.

How Teachers Obtain Negative Number Resources

Teachers seek advice and resources about effective math teaching in various ways from people they know within their school system, as well as formal organizations such as councils of mathematics and social media networks (Shapiro et al., 2019; Wilhelm et al., 2016). The impact of social media on teachers' practices has increased significantly over the past decade. Teachers use tools such as blogs, YouTube, social network sites (e.g., Facebook, Twitter), and for-profit teacher-sharing sites such as Teachers Pay Teachers to locate resources and connect with colleagues. In a survey of elementary teachers across the United States about where they obtain math resources, 89% used Teachers Pay Teachers, 74% used Pinterest, 68% used a Google search (without identifying a particular site found through that search), and 34% reported the National Council of Teachers of Mathematics (NCTM) or state affiliate websites (Shapiro et al., 2019). A strength of these prior studies is that rather than reporting where teachers find resources generally, these studies provided insights about resources for mathematics.

Insights specific to a topic would be important to inform the field's understanding of learning and instruction of that topic. Yet, a single study was found that analyzed social media resources specific to the topic of negative numbers. This study focused on Pinterest and revealed that one-third of frequently pinned negative integer teaching resources were inaccurate (Hertel & Wessman-Enzinger, 2017). The most common inaccuracy (with thousands of re-pins) was ambiguity about order and magnitude, meaning the resources referenced "higher" or some synonymous term for order instead of absolute value or magnitude. Perhaps most disconcerting was 22% of these "free" pins linked to for-profit sites—and companies can pay to promote pins, which leads to increased popularity and re-pins (Hertel & Wessman-Enzinger, 2017). Thus, this accelerates the visibility of commercialized resources.

Games

As part of the investigations on resources, we intended to gain more insights about where teachers obtain the games they offer their students related to the source category (i.e., commercial, textbook, teacher-created) as in Nurnberger-Haag and colleagues (2023). Teachers often use games to increase motivation and enjoyment for learning mathematics (Ernest, 1986; White & McCoy, 2019). Middle school is a time when students are especially attentive to peer comparisons and feel more anxiety (Scarpello, 2007). Games can potentially disrupt math anxiety that is so ubiquitous, especially in the United States (Ersozlu & Karakus, 2019; Luttenberger et al., 2018). However, games are a complex construct with ideal games balancing how a person experiences tension and relaxation due to game features (Schell, 2008). Student experiences with games designed for learning negative number operations that are also widely available were recently investigated. All students in two sections of mathematics with the same teacher played three integer games: a commercially produced game, a textbook game, and a teacher-designed game (Nurnberger-Haag et al., 2023). The teacher-designed game available for free in a peer-reviewed, open-source journal was favored by students for its learning affordances and enjoyment, whereas the commercially produced game caused anxiety for many students. Thus, to avoid marginalizing any students, a key finding—using the Mathematics Classroom Games Features Framework—was that math games should balance skill with chance and provide students time to think by having them take turns (Nurnberger-Haag et al., 2023). This study of a single teacher's students provided important insights specific to three integer game options and contributed a theoretical framework for thinking about math games broadly. Yet, to understand students' experiences with integer games across many classrooms more research is needed.

Environmental Math Theoretical Construct

Some resources teachers typically post in their classroom environments could be categorized with the construct of *environmental math* (Nurnberger-Haag et al., 2019). The term environmental math likely evokes ideas of applying mathematics to solve environmental problems; however, this was not the meaning of the term used here. The term has its roots in early childhood literacy, where the construct of *environmental print* refers to the print children encounter in their lived experiences outside of classrooms (e.g., company logos, street signs) or inside classrooms (e.g., words like "door" or sink" posted to increase vocabulary (Kirkland et al., 1991). Environmental math refers to the math-related subset of environmental print, such as math posters, number lines, maps of the school, thermometers, math word walls, clocks, calendars, mathematical practices, and so forth (Nurnberger-Haag et al., 2019). Math classrooms often have motivational posters, class rules, or other print. These motivational postings are environmental print. According to Nurnberger-Haag and colleagues (2019), non-math specific motivational signage in a math class, such as Christopher Robin's statement to Winnie-the-Pooh, "You are braver than you believe…and smarter than you think" (Geurs, 1997) are not environmental math; whereas, motivational posters related to mathematical practices and mindsets would be considered environmental math. Nurnberger-Haag et al. (2019) formed this theoretical construct of environmental math and created a teacher-friendly framework to assess the potential learning impacts of features of classroom postings. The environmental math of shapes, for example, has been implicated in the pervasive misconceptions of children as well as adults (Nurnberger-Haag & Thompson, 2023; Nurnberger-Haag et al., 2020). The environmental math of integers, however, has not yet been investigated.

Rules That May or May Not Expire

Rules might be valued in mathematics due to their decontextualized and abstract affordances. Yet, providing rules as an instructional approach may also pose challenges. The theoretical construct of *rules that expire* (Karp et al., 2014, 2015) provides a useful lens for research and practice to recognize when a rule has limited value for a given topic and can be counter-productive for understanding mathematics as a whole. We interpreted Karp and colleague's (2014, 2015) explanations of this construct as satisfying one of two categories: (a) statements that are mathematically valid only under certain constraints so instruction of these leads to student misconceptions or (b) procedural dictates that may not always be necessary. Karp et al. (2014, 2015) identified 37 such phrasings, notations, or rules that have or will expire in Kindergarten through Grade 8. One such rule that impacts integer concepts is "Two negatives make a positive" (Karp et al., 2015, p. 212), because this is only always true for the operations of multiplication and division, whereas it is always false for addition. An example of a procedural dictate that expires is "KFC: Keep-Flip-Change" (Karp et al., 2015, pp. 210-211) in which students are taught to divide fractions by inverting and multiplying, however, this procedure is not always necessary and fails to support conceptual understanding. Karp and colleagues (2014, 2015) also encouraged teachers to find and eliminate other rules that expire to improve instruction. Whereas these publications provided important introductions to the construct and offered some specific examples, to improve instruction of any topic, it would be helpful to curate rules specific to a topic, such as integer arithmetic.

Even if rules are accurate (i.e., do not expire), to explicitly teach students any rule may not support conceptual understanding (NCTM, 2014; Robinson & Dubé, 2009). Explicit rule instruction, according to the cognitive demand framework of mathematical tasks, would be low cognitive demand (Stein et al., 2009). Whereas if students generalize rules by noticing patterns and structures to generate their own rules, such approaches are consistent with high cognitive demand tasks (Stein et al., 2009). These approaches are also consistent with the Mathematical Practice Standards of *Look for and Make Use of Structure* (MP7) and *Look for and Express Regularity in Repeated Reasoning* (MP8) (NGA Center & CCSSO, 2010). Thus, our study sought data to understand how prevalent rule-focused instruction might be for integer learning as well as the specific rules teachers use in relation to the *rules that expire* construct.

Tools

To externalize mathematical thought for oneself and communicate with others, words, diagrams, pictures, manipulatives, expressions, and symbols are used to represent ideas (Lesh et al., 1987; Goldin, 2003; Kamii et al., 2001). Thus, the use of mathematical representations is essential to successful mathematics learning (Huinker, 2015; Pape & Tchoshanov, 2001). The construct of a representational tool (artifact) need not be limited to a material object (Rabardel, 1995). Tools can be written symbols, physical objects, cognitive objects, or processes such as algorithms (Kuzniak et al., 2016; Rabardel, 1995).

Tools typically used for integer instruction include physical manipulatives of square tiles or round chips, drawings or virtual versions of the same, number lines, and drawings/animations of balloons and weights (e.g., Pettis & Glancy, 2015; Van de Walle et al., 2019). These tools have been typically viewed as isomorphic with a model (e.g., chip model, number line model). Yet, teachers can select tools to enact integer models that differ by specific features, which further increases the variability of the models students experience.

For instance, number line models can be enacted with horizontal or vertical number lines. Of course, students need to become proficient with both horizontal and vertically oriented number lines. Vertical number lines are commonly used to represent temperature and elevation, whereas if a number line is used to represent the abstract construct of time, horizontal number lines are typically used for timelines. Moreover, Cartesian coordinate graphing and the geographic coordinate system (i.e., longitude and latitude) require proficient simultaneous coordination of vertical and horizontal number lines. Furthermore, applications of vectors in physics require students to calculate flexibly in all 360 degrees. Thus, it would be important to investigate how common it is that middle grade students have the opportunity to build their concepts of number and proficiency with number lines at least in each of the vertical and horizontal directions.

Neither the desired end state for knowledge, nor convenience, should completely determine the tools or order of instruction. The learner must be considered. In light of theories of embodied cognition (Glenberg, 2010; Lakoff & Nunez, 2000), we should recognize that vertical and horizontal number line representations are based on differing embodied experiences of up/down and left/right motions. As a way to consider potential implications for others' learning (and especially young learners), we have found it useful in practice to ask mathematically proficient adults to recognize the potential influences on their own personal directional conceptions of number and when these might have occurred, even if this differs from one's current individual conceptions. We find this crucial for any adult, but especially for English-speakers who are experts in mathematics and so have experienced decades of exposures to traditional textbook-based number lines in which more is to the right—because in English we read left to right. Yet, this left/right conception of quantity is not a human's first experience with less and more. Up and down are embodied experiences from birth as a shared human experience that more is a building up (e.g., stacking more blocks makes a tower taller) or growing (we grow taller as we grow older and taller people are believed by children to be older or have more years) and less is down (Lakoff & Johnson, 1980). Of course there are other real-life instances in which down is thought of as more, such as when writing a list and the list grows down, however, this is another text-based conception of quantity, rather than embodied experiences prior to reading/writing. Consistent with our first embodied experiences, in most of the world humans use a vertical number line to represent quantities, with more conceptualized as up and less as down, including using negative numbers to represent distances below sea level. Children almost universally experience this up/down conceptual association before beginning to read in the directions specified by the literacy education of the culture in which they are being raised (e.g., left/right in English, right/left in Hebrew and Arabic, up/down in Chinese). Thus, how these number line differences appear in learners' environmental math and how these might affect development of learners' numerical conceptions warrant careful consideration in mathematics learning research (Nurnberger-Haag, 2015b).

Varied colors symbolize the valence of numbers in chip models, such as positive/negative pairings of yellow/red (Van de Walle et al., 2019) or black/red (Lappan et al., 2014). Additionally, algebra tiles are designed to represent variable expressions and equations through area models distinguished by shape and color, however, companies have not used a consistent paired-colors approach. Each brand of algebra tiles represents each positive area with a distinct color and shape so positive values are represented with multiple colors simultaneously (e.g., units, X, X^2, Y, XY, Y^2 are six different colors). In contrast, the opposite side of each of these tiles is red, regardless of brand. Thus, it seems that all brands agree that negative values are conceptually consistent with the color red.

In spite of these variations, analyses of which tools teachers use as chip/cancellation models are missing from our field. Such detailed analyses of the features of blocks, chips, or tiles may seem irrelevant because mathematically, any color could be assigned to represent or symbolize positive or negative quantities. However, cognitive science research about colors in relation to general concept-learning and cognition have found color-associations matter for learning and warrant continued study (Mehta & Zhu, 2009; Schoenlein & Schloss, 2022; Sweller et al., 2011; Zhou et al., 2021). For example, in real life outside of a mathematics classroom, green is often thought of as go and red as stop, black is considered the typical font color for text with white space as the negative space or absence of color, and black represents non-negative values in finance with red font associated with debt. Consequently, the color associations of integer tools could matter for learning this difficult topic. Therefore, to begin such research we asked teachers what specific tools they used for chip models.

Integer Contexts

Teachers use contexts to connect to students' personal experiences and apply mathematics to solve real-life problems (Wernet, 2017). Yet, some have cautioned and questioned the authenticity of the contextual tasks used for math instruction, because it can have the unintended effect that instead of modeling the real world, students might believe math is incompatible with the real world (Palm, 2006). Common contexts for the instruction of negative numbers include elevation, temperature on a thermometer, money and debt, games, or a balloons/weights context (Lappan et al., 2014; Pettis & Glancy, 2015; Van de Walle et al., 2019; Whitacre et al., 2011). Many school-based negative number problems involve contexts that, in real life, people think through using whole numbers, which renders such contextual problems inauthentic (Whitacre et al., 2011). Thus, asking teachers what contexts they use for integers would be important to guide or prioritize future research and practical recommendations about contextual integer tasks.

Purpose of Study

Much of the research on integer learning has been about (a) how adults and children think, or misconceptions they have (e.g., Bofferding & Farmer, 2019; Bishop et al., 2014; Chiu, 2001; Ryan & Williams, 2007; Vlassis, 2008; Whitacre et al., 2017); (b) studies evaluating particular instructional approaches implemented by or instigated by researchers, rather than teachers (e.g., Bofferding, 2014; Linchevski & Williams, 1999; Stephan & Akyuz, 2012; Thompson & Dreyfus, 1988; Tsang et al., 2015); or (c) analyzing integer resources posted on social media (Hertel & Wessman-Enzinger, 2017). In contrast, the purpose of this study was to investigate what teachers report happens in actual classrooms. Thus, we conducted a survey to identify resources teachers in diverse contexts use to teach integer operations with the following research questions:

RQ1: Where have middle grade teachers obtained the resources they use to teach negative numbers?

RQ2: What environmental math of negative numbers do middle grade students see in their classrooms?

RQ3: Rules

a) For which operations are middle grade students explicitly taught rules?
b) Are middle grade students more likely to experience rules for integer multiplication and division than for addition and subtraction?
c) What integer rules are middle grade students being explicitly taught?

RQ4: Representational Tools and Contexts:

a) How important do middle grade teachers believe each representational tool/context is to their students' learning?
b) Which kinds of integer representational tools/contexts do middle grade teachers use? In what order? What tools/contexts are used to begin instruction?
c) Specifically, what kinds of chip and number line tools are middle grade teachers using? Does the frequency of chip tools with different symbolic features differ significantly? Does the frequency of number line orientation used differ significantly?

Method

The survey was conducted after integer instruction took place during the second full school year of the pandemic (i.e., 2021-2022). During 2020-2021 the majority of teachers taught integers virtually, whereas during 2021-2022 almost all taught integers in-person.

Participant Recruitment Process

Middle grade teachers who teach negative numbers were recruited to participate in the survey primarily by sending emails directly to teachers whose email addresses were posted on their district's website using the method of Courtney et al. (2022). To sample teachers in varied contexts who also taught from the same standards, we used the directory of Ohio school districts to find district websites. We emailed 1,242 middle grade teachers and coaches from 195 districts. Understandably, for security purposes many district email systems block emails with URL links such as surveys, a large number of recipients, or for other reasons (Mertler, 2003). Thus, for teachers in those districts, "participation was not a conscious decision . . . [which had a] substantial impact on the ultimate rate of response" (Mertler, 2003, p. 7). We cannot ascertain how many emails were blocked.

As compensation for participation, subjects for this study could download a printable integer game (Nurnberger-Haag & Wernet, 2019) and a validated integer assessment (Nurnberger-Haag et al., 2022). Participants who also gave their addresses were mailed integer dice and entered into a drawing for a large magnetic open number line that could be hung vertically or horizontally.

Participant Contexts

All 55 participants were from public schools. The contexts in which they taught were distributed across school typologies with 21.8% teaching in urban districts, 50.9% in suburban, and 27.3% in rural. As is common in the United States, we also used percent allocation for free and reduced lunch (FRL) Title 1 guidelines as a proxy to monitor if the sample reflected diverse socioeconomic status (SES). The sample reflected a broad range of SES listed here from highest to least need: greater than 80% FRL (20% of teachers), 60-80% FRL (9.1%), 40-60% FRL (18.2%), 35-40% FRL (10.9%), and 15-35% FRL (21.8%). We considered 11 teachers (20%) to be teaching in a district with less than 15% FRL—three teachers explicitly reported this, plus eight reported they were unsure (teachers are usually only unaware of their school's FRL when it affects a small subset of their students).

Teachers often teach more than one grade, so the data cannot sum to 100%. In order of frequency, participants identified as: Grade 7 (n=33, 60%), Grade 6 (n=20, 36.4%), Grade 8 (n=16, 29.1%), special education or Response to Intervention (RTI) (n=4, 7.3%), and math curriculum specialists or coaches (n=2, 3.6%). Finally, two teachers (3.6%) reported teaching each of Grade 5, Gifted Education, or "Other."

Survey Questions

To develop the survey questions, we considered how to best obtain answers to each research question within the constraints of a survey as a collection instrument while balancing ease of use such that teachers would be willing to start as well as complete it. The survey consisted of forced choice, multiple response (i.e., "check all that apply"), drag and order, Likert, and open response items. For example, we used the Qualtrics drag and drop ranking feature to break up the monotony of response type for the participant while also offering an efficient way for the teacher-participant to parsimoniously provide data about which models and contexts were used simultaneously with their curricular sequencing.

The first author, who has been working on integer arithmetic learning as a teacher, teacher developer, and/or a scholar for over 25 years drafted questions and response options. Then, the first author and a pre-service teacher searched educational supply company catalogs, search engines, and teacher sites to seek other tools, phrasing of rules, or methods. These approaches informed the survey response options. Additionally, each question included an "Other" option to capture unanticipated resources. Demographic questions were positioned at the end of the survey to avoid stereotype threat activation (Steele & Aronson, 1995) and to increase the chances that the most important content questions would be answered even if participants fatigued near the end of the survey. The forced-choice question type was not used for these content questions, only for some demographic questions. The pre-service teacher and first author again revised the question wording and options before both authors further revised the survey. Table 1 itemizes each survey question about integer instruction and the response type aligned with that research question.

Table 1

Survey Questions and Response Type Aligned with Research Questions

Resource Category	Research Question	Survey Question	Response Type
Source	RQ1	If you use the textbook to teach negative numbers, what is the name of your textbook series?	Open
	RQ1	Where have you found resources for teaching negative numbers besides the textbooks?	Check All
Environmental Math	RQ2	Which, if any, negative number representations could students see on the walls or other places in your classroom?	Check All
Rules	RQ3a	For which, if any, operations do you explicitly teach rules to memorize?	Check All
	RQ3b	Which, if any, rules do you explicitly teach students?	Check All
Models & Contexts	RQ4a	Please click the bubble that describes how important you think each model or context is for students' learning of integer operations.	Likert
	RQ4b	Which, if any, negative number games do your students play?	Check All
	RQ4b	Which, if any, manipulatives or experiences have you used to teach negative number operations?	Check All
	RQ4b	Please choose each of the following that you use and put them in the order in which you introduce each model or context.	Drag & Order
	RQ4c	If you help your students learn to use a number line to solve addition and subtraction problems, please explain how to solve the problem -4 – (-8) with a number line.	Open
		If you help your students learn to use a number line to solve multiplication or division problems, please explain how to solve the problem -4 (-8) with a number line.	Open
		If you use a chip model, what chips do your students use?	Check All

Finally, prior to launching our survey, we consulted the office at our university designated for assisting with Qualtrics-related analyses to understand how the output of each question type would be exported and options for analysis to ensure we could answer our research questions. The draft survey was then tested for functionality on iOS and Android phones as well as Mac and PC computers by people outside of the research team to inform final revisions.

Once launched, all teachers who began the survey completed the survey, including the open response items. This indicated to us that we had created a reasonable balance between the information we requested and the needs of teachers completing the survey. Some questions were worded "If you use this…," so a teacher who did not use that resource could omit that question to save the teacher time while also allowing us to document non-use data. In general, teachers answered all standard questions (i.e., those without the qualifying premise), including the demographic questions at the end of the survey, despite this being when a participant would be most fatigued and likely to skip questions. Although a participant could skip any question (per Human Subjects Review ethics), based on the completion rates of each question, it seemed that teachers completed all questions they deemed relevant to their instruction. This indicated to us that we had crafted a survey that seemed reasonable to the teacher participants.

Analysis

Analyses appropriate to answering each of the four research question primarily involved descriptive statistics. Given the categorical nature of the data, to answer if rules were used more for particular operations (RQ3b) and if tool features differ significantly (RQ4c), Chi-Square Tests of Association were conducted, and, if significance at $\alpha < 0.05$ was found, effect sizes were calculated using φ (Cohen, 1988; Zaiontz, 2023). To answer research questions about "kinds" and "open response" answers, particularly regarding textbooks and rules, documentation of participants' text-based data was provided or summarized qualitatively.

Results

Where Teachers Obtained Instructional Ideas for Integers (RQ1)

Almost half of participating teachers indicated they "need more resources for teaching negative numbers" (45.5%). Table 2 specifies the frequency organized from most to least frequent within each source category of textbooks, online, people they know, professional organizations/publications, and formal education. Formal education courses were the least common way teachers obtained integer resources (i.e., fewer than 13%, see Table 2). A majority, but not all participating teachers reported using a textbook resource when they teach negative numbers (see Table 2). On this open response item, the 42 teachers using a textbook named 15 distinct textbook series. In descending order, the most commonly used textbook series reported were *Big Ideas Math (n*=9, 18.8%), *CPM: College Preparatory Mathematics* (*n*=8, 16.4%), *Eureka Math* (*n*=5, 9.1%), *Reveal Math* and *enVision Mathematics* (*n*=4, 7.3%), *Go Math!* and *Open Up Resources* (*n*=2, 3.6%). Eight additional distinct textbook series were each reported being used by one teacher (1.8%).

Most teachers (83.6%) found negative number resources online. The for-profit site Teachers Pay Teachers (65.5%) was reported almost twice as often as any other online source. The relative frequency of specific teacher blogs, Facebook groups, Pinterest, Instagram, and Twitter (now rebranded as X) are documented in Table 2, ranging from use by just a few teachers to more than a third of participating teachers. Other sources were each named by one teacher as listed in Table 2 (1.8%). About half of participating teachers (47.3%) reported using a search engine to obtain other resources without remembering the exact internet source found through that search.

In terms of obtaining resources from people they know, the most common source category was other teachers or colleagues in their school, with over half of teachers reporting this. Notice the next most common location was due to connecting with other teachers at a workshop/conference, followed by other teachers or colleagues who were not at their school (see Table 2). As reflected in Table 2, just 14.5% obtained resources from a math coach/coordinator. Future surveys should ask if the teacher has a math coach in their school or district so that interpretations can be made as to what proportion of teachers with access to a coach obtain integer resources from them.

In terms of professional organizations, about twice as many teachers gained ideas from local or state-level teacher conferences (34.5%) than the National Council of Teachers of Mathematics (NCTM) Annual Conference (see Table 2). In terms of professional journals, just 14.5% reported using the NCTM journal *Mathematics Teacher: Learning and Teaching PK-12*.

Table 2

Where or How Teachers Obtained Integer Resources

Source	***n***	**%**
Textbooks	42	76.4
Online		
Teachers Pay Teachers	36	65.5
Search Engine, but don't remember source used	26	47.3
Teacher Blogs	20	36.4
Facebook	18	32.7
Pinterest	15	27.3
Social networking group of a council of teachers of mathematics[a]	10	18.2
Instagram	9	16.4
Twitter (rebranded as X)	4	7.3
Desmos[b]	1	1.8
YouTube Kahn Academy[b]	1	1.8
EdPuzzle[b]	1	1.8
Engage New York[b]	1	1.8
PhET Simulations[b]	1	1.8
Georgia Standards[b]	1	1.8
Personal Relationships		
Teachers at same school	30	54.5
Someone met at workshop or conference	14	25.5
Some other teacher or colleague	11	20.0
Math Coordinator/Coach	8	14.5
Someone met in teacher education program	6	10.9
Professional Organizations & Publications		
State or Local Teacher Conference	19	34.5
NCTM National Conference	10	18.2
Mathematics Teacher: Learning and Teaching PK-12	8	14.5
NCTM Regional Conference	3	5.4
District or Textbook-Specific Professional Development	3	5.4
State Level Journal	2	3.6
Mathematics Teacher (Legacy journal)	2	3.6
NCTM Virtual Workshops	1	1.8
Formal Courses		
Master's	6	10.9
Bachelor's	7	12.7
Doctoral	--	--

Note. [a]These data refer to councils broadly including local around a city, regional within a state such as a network of math specialists, regional within the country as in multi-state, or national [b]Data of n=1 reflect teachers who wrote responses to "other sources"

Sources of Games for Integer Operations

Most teachers reported using at least one integer game (n=49, 89.1%). Teachers were not asked how many games they used, only the source. However, each teacher obtained games from 0 to 3 sources (median=2), which indicates the typical teacher used at least two integer games. Table 3 delineates where teachers obtained the games their students play. A majority of teachers printed game boards from the internet or other sources. Commercially manufactured physical games and games provided by their district-chosen textbook were each used in more than one-third of classrooms. A few teachers noted "Other," and referred to online games or explained they created their own games.

Table 3

Sources of the Games Teachers Used

Source	*n*	%
Printable Game from Internet or Other Sources	32	58.2
Commercially Purchased Board/Card Game	21	38.2
Textbook-Provided Game	20	36.4
Online Game Played by Students	3	5.4
Teacher-Created Game	2	3.6

Environmental Math Related to Integers (RQ2)

Teachers could select multiple options so the totals will not sum to 100%. Teachers most commonly posted a number line. Furthermore, more horizontal than vertical number lines would be seen in classrooms. With large scale lines, more than twice as many classrooms had a long horizontal number line on their walls (65.5%) than a long vertical number line (27.3%). A Chi-Square Test of Association comparing horizontal versus vertical large-scale number lines was significant at $\alpha < .05$: X^2=16.12, p=.000059, with a large effect size (φ=0.54; Cohen, 1988). On a medium scale (e.g., posters), even though we combined the context-based number lines (i.e., thermometers and elevation), the prevalence reported for horizontal number lines on student desks or posters (40%) was again twice as common as vertical. A Chi-Square Test of Association comparing horizontal versus vertical of these medium scale number lines was also significant at $\alpha < .05$: X^2=4.41, p=0.036, with a near medium effect size of 0.28 (Cohen, 1988).

Money or debt were reported on 18.2% of classroom walls. Fewer classrooms displayed posters with explanations of a chip model (10.9%) or chemical charges (7.3%).

Explicit Teaching of Rules for Integer Operations

A majority of teachers taught explicit rules for at least one operation (78.2%). More than half did so for all four operations (52.7%). At least 14 wordings of rules were identified.

Rules by Operations (RQ3a-b)

More than half of participating teachers reported explicitly teaching rules for addition (n=30, 54.5%) and subtraction (n=32, 58.2%). For multiplication and division, almost three-quarters of the teachers explicitly taught rules (n=40, 72.7%). We hypothesized that teachers would more often teach rules for multiplication and division due to the simplicity and consistency of such rules in comparison to addition and subtraction. However, the 2 by 4 Chi-Square Test of Association of rules or not rules by each of the operations was not significant.

Which Rules Were Taught (RQ3c)

One teacher wrote in "Other Rules" the importance of posting student-generated rules: "We do not just teach the rules . . . the kids come up with the rules and then we refer back to them." About one-fifth of teachers omitted these questions, which given the phrasing of our question as "If you teach rules . . . " (see Table 1), we interpreted as meaning they do not explicitly teach rules.

Table 4 shows the 14 rules explicitly taught from most to least common. We organized these based on the theoretical construct of rules that expire (Karp et al., 2014): Rules That Expire (n=11) or Rules Valid as Stated (n=3). Two synonymous variations of the rule *Keep Change Change* were each reported by one teacher (i.e., *Copy Change Opposite*, *Keep Add Opposite*), so we consolidated these into the single statistic using the most common wording of *Keep Change Change*.

Table 4

Rules Explicitly Taught and Whether These are Always Valid as Stated

Rule Type	*n* (%)
Rules That Expire	
Adding a negative number is the same as subtracting a positive number	33 (60.0)
Two negatives make a positive	28 (50.9)
Every time you see a subtraction sign change it to adding the opposite	26 (47.3)
Same signs keep them; Different signs subtract the smaller absolute value from the larger absolute value, then take the sign of the largest absolute value	25 (45.5)
Same signs positive answer, different signs negative answer	24 (43.6)
Keep Change Change	21 (38.1)
Two minuses make a plus, Two pluses make a plus, Plus and a minus make a minus	12 (21.8)
Music based mnemonic that included the wording: Same signs add, different signs subtract keep the sign of "the larger" or "the most"	2 (3.6)[a]
Negatives and positive cancel each other out.... find additive inverse pairs	1 (1.8)[a]
What happens when we "get rid of" a "bad thing"? We get a good thing.	1 (1.8)[a]
For adding/subtracting-using the Circle Method Same Sign Sum Different Sign Difference	1 (1.8)[a]
Rules Valid as Stated	
For multiplication and division: Count the number of negative signs. If the amount is even, the answer is positive; if the amount is odd the answer is negative.	1 (1.8)[a]
When multiplying and dividing, two negatives makes a positive. (I model this by holding up my two index fingers as minus signs and making them "crash" into each other to form a plus sign.)	1 (1.8)[a]
Minus a negative 1 becomes plus a positive.	1 (1.8)[a]

Note. [a]Data of n=1 or 2 each reflect teachers who wrote rules in response to "other rules."

Teachers' Beliefs about the Importance of Representational Tools and Contexts for Integer Operations (RQ4a)

To answer RQ4a (How important do teachers believe each model or representational context is to their students' learning?), Table 5 displays the importance teachers reported for each of the representational tools and contexts.

Importance of Tools

Number lines were rated as more important than every other tool or context. See Table 5 for this information. Teachers believed number lines were extremely important for integer operations (90.9%). No teacher said a number line model was unimportant. In contrast, almost 10% said a chip model was unimportant. Yet, about half believed a chip model was extremely important (see Table 5). Few teachers deemed a tool that combined a context with a model (e.g., balloons and weights) as extremely important (3.9%), whereas 19.6% said it was not important at all or they did not use this tool (see Table 5).

Table 5

Importance Reported of Tools and Contexts for Integer Operations

	Extremely Important *n* (%)	Somewhat Important *n* (%)	Neutral *n* (%)	Not Very Important *n* (%)	Not At All Important *n* (%)
Model					
Number Line	50 (90.9%)	4 (7.3%)	1 (1.8%)	--	--
Chips[a]	30 (55.6%)	14 (25.9%)	5 (9.3%)	1 (1.9%)	4 (7.4%)
Hybrid Tool & Context					
Balloons/Weights or similar[b]	2 (3.9)	8 (15.7)	28 (54.9)	3 (5.9)	10 (19.6)
Context					
Money, credits, debits, debt	34 (61.8)	16 (29.1)	5 (9.1)	--	--
Thermometer	20 (36.4)	32 (58.2)	3 (5.5)	--	--
Elevation	14 (25.5)	29 (52.7)	10 (18.2)	2 (3.6)	--
Chemical charges[c]	5 (9.6)	9 (17.3)	28 (53.8)	2 (3.8)	8 (15.4)

Note. [a]Fifty-four responded instead of 55, so percents were calculated out of 54. [b] Four were missing so percents were calculated out of 51. [c]Three were missing so percents calculated out of 52.

Importance of Contexts

Financial contexts were the only context rated extremely important by the majority of teachers. About 90% of participating teachers rated each of financial, temperature, and elevation contexts as at least somewhat important (see Table 5). Only about one-fourth of teachers deemed the context of chemical charges at least somewhat important for learning integer operations. Moreover, 15.4% stated chemical charges were not important at all or not used (see Table 5). In the "Other" space, one (1.8%) teacher wrote "football: gaining/losing yardage."

What Representational Tools and Contexts Are Used? (RQ4b-c)

The prior section documented what teachers believed was important. This section reports the tools and contexts they actually used (RQ4). The number of models each teacher reported ranged from 0 to 3 with a median of 2 (mean=1.8). The number of contexts each teacher reported ranged from 0 to 5 with a median of 2 (mean=2.3). Combined total of tools and contexts ranged from 0 to 7 with a median of 4 (mean=4.1). Table 6 indicates the frequencies and percentages of teachers using each tool or context in the order in which they began using them (RQ4b-c). Note that a "number line model" was used by almost all teachers, with just 7.3% not selecting number line, and a majority reported using a "chip model", with 30.9% not selecting a chip model.

The most common way teachers began integer instruction was with a number line. In fact, 81.9% of teachers reported using a number line first or second. A little less than half of teachers (43.6%) reported using a chip model as students' first or second introduction to integer operations. When a context was the first introduction to integer operations, money and thermometers were each used in 9.1% of classrooms. See Table 6 for this information.

Table 6

Order in Which Teachers Reported that Students Experienced Multiple Models and Contexts

	1st *n*(%)	**2nd** *n*(%)	**3rd** *n*(%)	**4th** *n*(%)	**5th** *n*(%)	**6th** *n*(%)	**7th** *n*(%)	**Not used or omitted** *n*(%)
Model Tool								
Number Line	25(45.5)	20(36.4)	2(3.6)	3(5.5)	1(1.8)	--	--	4(7.3)
Chips[a]	13(23.6)	11(20.0)	6(10.9)	2(3.6)	4(7.3)	2(3.6)	2(3.6)	17(30.9)
Hybrid Tool & Context								
Balloons/Weights or similar[b]	1(1.8)	4(7.3)	1(1.8)	1(1.8)	2(3.6)	--	--	47(85.5)
Context								
Money, credits, debt	5(9.1)	9(16.4)	16(29.1)	6(10.9)	5(9.1)	1(1.8)	--	13(23.6)
Thermometer	5(9.1)	5(9.1)	10(18.2))	14(25.5)	4(7.3)	--	--	17(30.9)
Elevation	1(1.8)	4(7.3)	11(20.0)	8(14.5)	6(10.9)	2(3.6)	1(1.8)	22(40.0)
Chemical Charges[c]	1(1.8)	--	--	2(3.6)	1(1.8)	3(5.5)	--	48(87.3)
Game	--	--	1(1.8)	--	1(1.8)	--	--	--
Fantasy Scenario Pirates & ninjas	1(1.8)	--	--	--	--	--	--	--

Note. [a]Fifty-four responded instead of 55, so percents were calculated out of 54. [b] Four were missing so percents were calculated out of 51. [c]Three were missing so percents calculated out of 52.

Tool Features: Colors of Chips and Tiles (RQ4c)

When asked, "If you use a chip model, what chips do your students use?", 48 (87.3%) teachers reported that students experienced at least one chip tool. Although the median number of chip types reported was one, due to physical and virtual options and varied color choices, the number of kinds of chips used in each classroom ranged from 0 to 7. We categorized each tool representation as *consistent-meaning* or *inconsistent-meaning*. Consistent-meaning representation of integers were those that (a) are a pairing commonly associated with positive and negative quantities outside of the school setting (i.e., black as positive assets with red as negative or debt; positive and negative signs); or (b) the paired colors themselves are considered opposites of each other outside of school settings (i.e., green and red are opposite colors on the color wheel; colors of black and white have been considered opposite colors as in Yin/Yang), which can be analogous to opposite quantities. All other pairings that did not satisfy these criteria we considered inconsistent with real-life conceptions or inconsistent-meaning.

Inconsistent-Meaning or Consistent-Meaning Physical and Virtual Chips. A majority of classrooms using a chip model did so with inconsistent-meaning colors of physical chips (n=38, 79.2%), and about one-fourth used inconsistent-meaning colors of virtual chips (n=13, 27.1%). When physical chips were used, twice as many classrooms used inconsistent-meaning (n=38) versus consistent-meaning (n=19) valence associations. A Chi-Square Test of Association demonstrated that students were significantly more likely to use these inconsistent-meanings than consistent-meanings valence associations with physical chips (X^2=15.59, p=0.000079), with a large effect size (φ=0.57; Cohen, 1988). However, when they used virtual chips, there were no significant differences, because the same number of classrooms used inconsistent-meaning or consistent-meaning associations (n=13).

Specific Chip Colors and Written Symbols Used. The most typical chip colors overall were the inconsistent-meaning red/yellow physical chips (n=35; 79.5%) and red/yellow virtual chips (n=13; 29.5%). Table 7 displays each chip valence symbolism organized from most to least frequent within inconsistent-meaning or consistent-meaning. Within inconsistent-meaning, notice in Table 7 and confirmed with a Chi-Square Test of Association, that a red/yellow color combination was used significantly more than a red/white combination in physical (X^2=41.50, p< .00001, φ=0.93) and virtual (X^2=15.04, p=0.00010, φ=.56) formats, both with a large effect size (Cohen, 1988). Two teachers indicated in "Other" that the specific color does not matter, as is found within and between algebra tile manipulative brands.

Table 7

Chip Tool Symbolisms Teachers Used

Positive/Negative	Physical *n* (%)	Virtual *n* (%)
Inconsistent-Meaning		
Yellow/Red	35 (72.9%)	13 (27.1%)
White/Red	4 (8.3%)	--
Any Color/Red	--	1 (2.0%)
Consistent-Meaning		
+/- Symbols	9 (18.8)	10 (20.8)
Black/Red	6 (12.5%)	4 (8.3%)
Black/White	4 (8.3%)	--
Green/Red	4 (8.3%)	3 (6.3%)

Note. Percent of chip users (n=48)

The consistent-meaning written symbols of +/- were more frequently used than any colors with consistent-meaning (see Table 7). The most common consistent-meaning colors of physical chips were red and black, yet this was reported by a small number of chip-using classrooms (12.5%). All other chip types were selected by four or fewer teachers. The only teachers who used green and red were those who stated that their textbook (i.e., *enVision Mathematics*) used these colors. One of these teachers said they used this color-coding in conjunction with a +/-symbol; however, no other teachers stated that they simultaneously used written symbols and color. A Chi-Square Test of Association confirmed that when teachers used any consistent-meaning chip, whether virtual or physical, there were no significant differences among the frequency of symbol type.

Tool Features: Orientation of Number Line Representations (RQ4c)

Earlier we documented the prevalence of horizontal lines in the static environmental math. Here we report the orientation of the number line used as tools to calculate integer problems. On the open response integer subtraction number line task, teachers used a horizontal number line (n=28, 50.9%) most frequently and a vertical number line least frequently (n=3; 5.5%). About 10.9% of teachers used wording such that a student could flexibly use that process on either a vertical or horizontal number line. About one-third (n=18; 32.7%) either explained without actually using a number line (e.g., rules), stated that they do not use number lines for a subtraction problem like this, omitted the question, or provided a line orientation that could not be interpreted. A 2 by 3 Chi-Square Test of Association comparing subtraction on a horizontal number line, vertical line, or with language that could be flexibly used on any number line was significant at $\alpha<.05$: $X^2=38.95$, $p<0.00001$, with a large effect size $\varphi = 0.84$ (Cohen, 1988). Given that horizontal language was used more than nine times more often than vertical, and almost five times more often than flexible wording, the horizontal direction was the significant orientation.

Seven teachers used a number line for multiplication: horizontal (n=4; 7.3%), vertical (n=0), and three (5.5%) used flexible wording that might work on either horizontal or vertical number lines. There was an insufficient sample of teachers who used number lines for multiplication/division to conduct a Chi-Square Test of Association for orientation.

Discussion

The diversity of the range of contexts —(a) all school typologies from rural to urban; (b) schools with greater than 80% FRL to less than 15% FRL and (c) 15 textbooks—lent trustworthiness to this first study designed to understand the resources used in current practice influencing United States middle grade students' integer arithmetic learning.

Sources of the Instructional Methods Students Experience

In the results, we documented every source of integer instructional resources (RQ1) grouped by type. Here we discuss the most popular sources in relation to prior research. Teachers Pay Teachers (66%) was second in popularity only to textbooks (76%). Moreover, given that districts choose textbooks, not individual teachers, if we consider only sources for which teachers have the agency to choose for themselves where and how to obtain ideas, Teachers Pay Teachers was the preferred source. No other sources were used by a majority of teachers. About half obtained resources from teachers in their school or online searches, without recalling the specific source. Teacher blogs, local/state conferences, and Facebook groups were each used by approximately one-third of teachers. The popularity of these sources in our middle-grade-specific study was aligned in order but not in magnitude with the results of an elementary study (Shapiro et al., 2019). The fact that our study was conducted after teachers had to for a time exclusively teach online and we found the same order of resource use as Shapiro et al.'s (2019) pre-pandemic study—yet the percent of teachers valuing each source in our study was 15-20% less—we suspect that this was a difference of middle school versus elementary. This suggests to us that to provide insights about how to best support teachers of math at different grades, future research should analyze all grade bands in the same study.

In contrast, instructional sources with verifiable expertise (e.g., math coaches, university courses, professional development, national/regional conferences, journals) were each reported by fewer than one-fourth of teachers. This may simply be due to the specificity of the topic of integers; that is, although responding teachers may have obtained resources for other topics through these sources, resources were not available for integers. Alternatively, it may be that the teachers less frequently used such vetted resources overall (Education Week Research Center [EWRC], 2014). Although only 15% of teachers in our study obtained integer resources from a math coach, given that teachers value other teachers' expertise (EWRC, 2014; Wilhelm et al., 2016), we suspect these data would be higher if more districts invested in math coaches.

Rules that Expire Used in a Majority of Classrooms

Interestingly, the proportion of teachers (about one-fourth) who explicitly avoided teaching negative number rules was consistent with the proportion of teachers who used sources with verifiable expertise. Given that mandated testing occurs many weeks before school ends, future studies should uncover if even teachers who believe in student-generated mathematics teach rules to cover all standards prior to testing. This may be especially true for integers because most of these standards are embedded within rational numbers, so students must master integer arithmetic quickly in order to still cover negative fractions.

Although almost three-quarters of teachers reported explicitly teaching rules for multiplication and division compared to over half for addition and subtraction, these rates did not differ significantly (RQ3). Unfortunately, according to the "rules that expire" construct (Karp et al., 2014), almost all of the rules used were inaccurate. Perhaps most troubling is that six of these were each used in 38 to 60% of classrooms (see Table 3). *Two negatives make a positive*—a rule Karp and colleagues (2014) specified as a rule that expires because it is valid only for multiplication and division—was the second most common rule found in our study and was used by half the teachers. The rule *Keep Change Change* leads to misapplications, because it is valid only for subtraction and only a useful strategy if subtracting a negative number. Thus, it is problematic that this rule was used in 38% of classrooms. Karp et al. (2015) asserted the procedural rule to invert and multiply fractions is a rule that expires because it is not always necessary. Thus, we viewed the rule to change all subtraction signs to adding the opposite as an analogous rule for integer arithmetic, which almost half the teachers used. Two opportunities for future research would be to include a) an open response item or interviews about how and why rules are taught and b) questions about whether rules are posted, and if so, if these posters were student-created, teacher-created, or commercially produced—as indicated by the criteria of the Environmental Math Framework (Nurnberger-Haag et al., 2019) and as advised by a teacher in the results section.

Carefully Select and Sequence Tools, Contextual Tasks, and Environmental Math

To our knowledge, this study was the first to investigate the number and sequencing of multiple models and contexts that students experience during integer instruction. Students typically experienced four models and contexts, with more than 90% using a number line and a little less than half using a chip model. The sequence in which teachers reported using models was consistent with their stated beliefs in that number lines and chip models were prioritized early (i.e., first or second). Instruction that used a number line first was consistent with findings of a recent quasi-experimental study of intact classes that had not yet learned integer arithmetic. Eight classes were randomly assigned to researcher-led instruction with a specific number line or specific chip model as their first experience with integer arithmetic (Nurnberger-Haag, 2015a). This study found those students who experienced a particular number line (i.e., Nurnberger-Haag, 2007) as their first model rather than the chip model learned significantly more with a large effect size (Nurnberger-Haag, 2015a). Thus, more research is needed to assess the learning impacts and students' perspectives on their experiences with the varied sequences of 0 to 7 models and contexts.

Considering the concerns about the inauthenticity of negative number contextual tasks (Whitacre et al., 2011) and that almost all teachers in our study thought context was important for integer learning, stakeholders should create criteria and resources about which contextual tasks are productive for this abstract topic. Wernet's (2017) operationalization of Palm's (2006) framework for task authenticity could be a useful framework to begin this work specific to integers. A contextual-based integer model of balloons/weights (or similar) also warrants learning outcomes research, because it was used in about 20% of classrooms. Although about 90% of teachers rated each of financial, temperature, and elevation contexts as at least somewhat important, only about one-fourth deemed chemical charges at least somewhat important. This was interesting given that students must use negative numbers to learn chemistry. Collaborations of mathematics and science education scholars, as well as teachers, could determine if chemical charges should be sequenced *after* learning integer arithmetic or what if any, types of tasks involving chemical charges support students *during* the process of learning integer arithmetic. In other words, in consideration of STEM learning more broadly, does the science of chemistry and mathematics learning benefit from integrated instruction at the time of integer arithmetic learning despite the abstract nature of chemistry? Or should proficiency with negative numbers precede chemistry instruction to ease chemistry learning and further deepen student understanding and applications of negative integers?

Number Line Features

Given that students must be proficient with vertical and horizontal number lines as well as simultaneous coordination of these orientations, we were surprised by the degree to which horizontal number lines dominated students' learning opportunities with operations and in their environmental math (e.g., posters, large-scale lines on classroom walls). Recall that this difference was statistically significant with a large effect size when compared to vertical lines. Moreover, horizontal number line language dominated over language that would support students to operate flexibly on any number line. Thus, just as an overemphasis on limited orientations of shapes inhibits students from accepting other valid variations of shapes (Nurnberger-Haag et al., 2020; Sinclair & Moss, 2012), we suspect an overemphasis of horizontally oriented number lines would impact student understanding of vertical number lines and Cartesian graphs. Indeed, anecdotally in the first author's experience, even many adults who are teachers or administrators participating in integer professional development have struggled to correctly orient numbers on the floor when asked to use a vertical orientation. The evidence of our study suggests that this horizontal orientation being conceptually dominant likely begins at least by middle school. Future research should investigate elementary classroom orientations of number lines.

These avenues of research might be especially important in light of embodied cognition and attempts to begin instruction in ways that capitalize on learners' experiences outside of school. Based on embodied cognition, it has been argued that integer instruction should begin with a vertical number line because this direction would be more intuitive than associations determined by privileging the direction of written English text—as opposed to Hebrew, Arabic, or Chinese (Nurnberger-Haag, 2015b). Usually, humans experience the concept of "more" as increasing in height, that is, a conceptual metaphor of embodied experience "more is up" (Lakoff & Johnson, 1980; Lakoff & Nunez, 2000). Consider, the conceptual metaphors of how we think are often revealed through our unconscious and unexamined language (Lakoff & Johnson, 1980). For instance, people say, "Costs are going up!" not "Costs are going to the right!," and "Price drop!" not "Price left!." Due to repeated exposures, by middle school, students may have memorized an association that a number line is horizontal, so it might seem that using horizontal lines would build on their prior knowledge. From an embodied theory of cognition, however, beginning the difficult topic of negative numbers on a vertical number line may better tap into students' deeper conceptual understanding—developed since early childhood that "more" is up and "less" is down. Connecting to students' conceptual resources in this way may reduce cognitive load and warrants further study.

Chip Features

Features of the learning environment and tools such as color can support or interfere with thinking and learning (Mehta & Zhu, 2009; Sweller et al., 2011), so specific color associations for concept learning warrant continued investigation (Schoenlein & Schloss, 2022; Zhou et al., 2021). Incongruent color associations can unproductively increase the cognitive load while learning (Sweller et al., 2011) and interfere with long-term memory retrieval (Zhou et al., 2021). Color-consistent associations, however, support retrieval (Zhou et al., 2021). Given these insights on color-associations from cognitive science, important findings of this study were that the most common chips used were significantly more likely—with a large effect size —to have inconsistent-meaning color-associations. Such color-associations may interfere with learning because such school-based tools require students to memorize which color represents each valence, rather than using their existing out-of-school color-associations as a resource for their learning.

Inconsistent-Meaning Valence Associations. Given that yellow/red and white/red are not opposite colors, we question the use of these pairings as analogs for opposite quantities. Rather than opposites, yellow and red are two primary colors and are considered warm colors (Westland et al., 2007). However, this color combination was used in significantly more classrooms than any other to signify opposite numbers. Although we did not ask teachers why each color-pairing was chosen, one reason offered in the "Other" space for the yellow/red chips was that these were the manipulatives they had at their school. The chips shown in the most popular methods textbook for elementary and middle grades also show these yellow and red manipulatives (see Van de Walle et al., 2019) and all of the surveyed teachers taught in a state in which an instructional support document showed yellow and red chips. Consequently, although neither the methods text nor the state education document told teachers to use these colors, the illustrations might foster the mindset or impression that using a chip model means using these colors. However, we consider these to be arbitrary color-associations. Indeed, one teacher who used such color-valence mappings stated the reason they no longer use a chip model was because their students could not remember which color represented which valence. Thus, informed by cognitive science research, these data suggest that yellow/red color associations may have increased students' cognitive load in ways that interfered with learning.

Consistent-Meaning Valence Associations. Conceptual analyses of the valence associations provide interesting and important opportunities to investigate (a) cultural or individual preferences, (b) whether there are optimal valence associations for learning, or (c) if any valence association can be effective as long as students believe the association is meaningful to them, that is consistent with meanings outside of school. The consistent-meaning valence associations we found were +/- symbols and color pairings of black/red, black/white, and green/red. If any of these consistent-meaning valence associations were chosen, the frequency of *which* meaning was not statistically significant. In practical terms then, similar numbers of teachers preferred each consistent-meaning representation. Thus, it would be important to investigate learning impacts related to each of these consistent-meaning representations.

These data inform scholars about future study-design choices to determine which consistent-meaning valence associations impact learning and in what ways. The most common consistent-meaning associations were + and – symbols inside circles, which about one in every five classrooms used. Given that even for young children teaching them to abstractly name patterns —such as ABBA or ABA— facilitated pattern learning (Fyfe et al., 2015), we wonder if using the abstract +/- symbols alone or in addition to color might better facilitate middle grade learning of integer arithmetic with a chip model. Black and white as opposites have been taught to children since their infant board books, so black chips as positives and white chips as negatives may foster the concept of opposite numbers. Yet, only four classrooms in our study used this physical black/white chip-color combination. Although middle school students may not already think of black/red as conceptual opposites, teaching these associations prepares them to understand finances. Black/red chips were used in only one of every eight classrooms (see Table 7) despite almost all teachers declaring financial contexts important. One study that showed significant learning gains with a chip model began instruction with black/white chips before teaching students the black/red business application and then each day students chose for themselves which consistent-meaning chips to use (Nurnberger-Haag, 2015b).

Although students —potentially since infancy —may think of stop and go as opposites, we wonder if the green/red color combination promotes the cancellation idea necessary to conceptualize sums of additive inverses as zero. On the other hand, students who are artists might benefit from symbolizing opposite integer values in ways that capitalize on their knowledge that green and red are literally opposite of each other on the color wheel (Westland et al., 2007). Thus, there are many potential avenues of research to investigate embodied and individual similarities and differences to better understand how we can support all students' learning of integer arithmetic.

Conclusions

This study expanded the integer learning knowledge base to describe what students see about negative numbers on a daily basis on their math classroom walls and other spaces (environmental math; RQ2), how and which rules they are explicitly taught (RQ3), the tools and contexts used and in what order (RQ4), and where teachers obtained games and other resources they use (RQ1). The results of this study revealed many important avenues for future research, teacher education, resource development, and instructional advice for teachers.

Learning Affordances and Constraints of Tools, Tricks and Topics Requires Research

This data informs directions for future research to ensure analysis of learning affordances and issues are directly relevant to the tools, tips, and topics used in actual integer instruction. Given that almost all teachers in our study used classroom games for integer learning, greater attention to research on mathematics classroom games, rather than online games, is warranted. A limitation of the data collected about classroom games was that the survey questions focused on the source aspect of games (e.g., commercially bought), rather than the features of the games. Such investigations of integer games and other topics would benefit from using the Mathematics Classroom Games Features Framework (Nurnberger-Haag et al., 2023) as an analytical tool. Our study also contributed insights about teachers' current mathematics-related social media use, on which scholars can continue to build (see Discussion).

Reliance on text-based orientations determined by the arbitrary language in which a person is raised (i.e., left-to-right language and left is less, right is more) over conceptions of number lines consistent with most humans' real-life embodiments of quantity (i.e., up is more, down is less) dominated student experiences just as inconsistent-meaning valence associations of chips were experienced over consistent-meaning valence associations (see Discussion). Research is needed to advise stakeholders—the committees and authors who create model curricula and methods textbooks for teachers and student textbooks—about which tools and in what sequence might better support student learning of this very difficult topic. We suggest that studies investigate the learning implications of features of tools: which direction of number lines and what color or symbol combinations are cognitively most easily associated with positive and negative concepts. This could reduce unnecessary cognitive load so that students can more easily attend to developing the intended mathematical concepts.

Practical Implications: Mitigate For-Profit Influences on Integer Learning in the United States

The majority of teachers used for-profit integer resources, which they may buy themselves. United States teachers, who are underpaid as it is, should not have to resort to purchasing resources. We must recognize that the majority of teachers found integer resources online (83%) with Teachers Pay Teachers (65%) being the most common source; whereas, only three to 16% of teachers found an integer resource in a peer-reviewed teacher journal or a university course. Teachers trust other teachers (EWRC, 2014; Wilhelm et al., 2016). Thus, to improve instruction of this difficult topic, based on the findings of this study, those who have expertise about negative number teaching and research may need to think more creatively about how to reach teachers—by going where teachers seek resources and disseminating through people they trust. For instance, Teachers Pay Teachers will continue to be used by a majority of teachers, so perhaps it will be necessary in the future for researchers and teacher teams to determine how they might use this site to post vetted resources. Moreover, how might researchers work with teacher leaders across the country to support dissemination through state/local conferences or popular teacher blogs because, based on our study, each of these venues might reach about one-third of teachers.

With this study, we intended to understand what resources are being used in middle school classrooms for integer instruction. It was only after we reviewed the data across multiple resource categories (e.g., games, posters, rules, tools) that we recognized a potential trend with a broader implication: for-profit agents may be strongly influencing students' negative number instructional experiences. Although it is unlikely that we could influence educational resources and producers for all mathematics topics, perhaps as scholars, teacher educators, and instructional coaches we might be able to counteract the for-profit influences for the single topic of integer arithmetic. Almost half of teachers said they needed more resources. Therefore, we believe addressing these problems requires a two-pronged approach: a) create more vetted and practical resources for negative number instruction and b) disseminate these resources in ways that allow teachers to spend time planning for implementation rather than searching for resources. In light of prior research and the findings of this study [i.e., almost all integer classrooms use games and horizontal number lines, a majority use rules that expire, and chip models most often used inconsistent-meaning (arbitrary) colors], we recommend the following for classroom instruction and teacher education:

- Avoid integer rules that expire (e.g., Table 4; Karp et al., 2014, 2015); have teachers and students analyze integer rules to discover whether these are always, sometimes or never true (Muir, 2015);
- Ensure integer games have chance (e.g., dice, spinners) and turn-taking to avoid marginalizing any students (Nurnberger-Haag et al., 2023);
- Use vertical number lines more than horizontal to mitigate the overuse of horizontal lines in classroom environmental math and as a calculation tool with negative numbers; To *prevent* overexposure prior to negative number instruction, we must similarly influence elementary stakeholders.
- For a chip model, discuss consistent-meaning options from Table 7 and allow students to choose the consistent-meaning valence association that makes learning easiest for them; Some low-cost options include:
    - Single-sided tiles can be easily found as bingo chips or made with colored cardstock
    - Two-sided tiles
        - Black/white can be found by searching for "Reversi" game replacement chips
        - Black/red chips are now being sold by an educational supply company, or

    - Glue two colors of cardstock before cutting into square tiles
  - Drawing (lowest cost option): Students can draw black/white or +/- chips or tiles easier than other valence associations

Final Thoughts

The instructional and research recommendations offered here, although extensive, seem feasible because as a field we need only influence just the two United States grades currently tasked with negative number instruction: Grades 6 and 7. We are optimistic that together as teacher educators, coaches, teachers, researchers, and state-level committee members we could shift away from for-profit influences to foster proficient integer knowledge by the end of Grade 7.

Acknowledgements: This work was supported in part by a Kent State University College of Education, Health and Human Services SEED Grant awarded to Julie Nurnberger-Haag. The authors would also like to thank Bee Parks for her assistance with preparing the survey question options.

Dr. Julie Nurnberger-Haag (Julie.nurnberger.haag@gmail.com) is currently a Visiting Scholar in the Department of Psychological Sciences at Kent State University (United States). She has been intrigued with, and an advocate for, improving the learning and teaching of negative numbers since she was a student teacher almost thirty years ago.

Dr. Scott A. Courtney (scourtn5@kent.edu) is an Associate Professor of Mathematics Education in the School of Teaching, Learning, and Curriculum Studies at Kent State University (United States). His research interests include exploring teachers' and students' meanings of mathematics and statistics in grades 6–14 and exploring teachers' and their students' experiences with digital/Information, Communication and Technology-based resources.

References

Bishop, J. P., Lamb, L. L., Philipp, R. A., Whitacre, I., Schappelle, B. P., & Lewis, M. L. (2014). Obstacles and affordances for integer reasoning: An analysis of children's thinking and the history of mathematics. *Journal for Research in Mathematics Education*, *45*, 19-61. https://doi.org/10.5951/jresematheduc.45.1.0019

Bofferding, L. (2014). Negative integer understanding: Characterizing first graders' mental models. *Journal for Research in Mathematics Education*, *45*(2), 194-245. https://doi.org/10.5951/jresematheduc.45.2.0194

Bofferding, L., & Farmer, S. (2019). Most and least: Differences in integer comparisons based on temperature comparison language. *International Journal of Science and Mathematics Education*, *17*, 545-563. https://doi.org/10.1007/s10763-018-9880-4

Chiu, M. M. (2001). Using metaphors to understand and solve arithmetic problems: Novice and experts working with negative numbers. *Mathematical Thinking and Learning*, *3*(2), 93-124. https://doi.org/10.1080/10986065.2001.9679970

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Lawrence Erlbaum Associates.

Courtney, S. A., Miller, M. E. S., & Gisondo, M. J. (2022). The Impact of COVID-19 on teachers' integration of digital technology. *Contemporary Educational Technology*, *14*(4), ep387. https://doi.org/10.30935/cedtech/12420

Education Week Research Center (EWRC). (2014, August). *From adoption to practice: Findings from a*

National Survey of Teachers. Editorial Projects in Education Inc. https://www.edweek.org/research-center/research-center-reports/from-adoption-to-practice-teacher-perspectives-on-the-common-core

Ernest, P. (1986). Games. A rationale for their use in the teaching of mathematics in school. *Mathematics in School*, *15*(1), 2-5. https://www.jstor.org/stable/30216298

Ersozlu, Z., & Karakus, M. (2019). Mathematics anxiety: Mapping the literature by bibliometric analysis. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(2), em1673. https://doi.org/10.29333/ejmste/102441

Fischbein, H. (1987). *Intuition in science and mathematics: An educational approach* (Vol. 5). Springer.

French, D. (2001). Two minuses make a plus. *Mathematics in School*, *30*(4), 32–33.

Fyfe, E. R., McNeil, N. M., & Rittle-Johnson, B. (2015). Easy as ABCABC: Abstract language facilitates performance on a concrete patterning task. *Child Development*, *86*(3), 927-935. https://doi.org/10.1111/cdev.12331

Geurs, K. (Director). (1997). *Pooh's grand adventure: The search for Christopher Robin* [Film]. Walt Disney Television Animation, Disney Video Premiere.

Glenberg, A. M. (2010). Embodiment as a unifying perspective for psychology. *Wiley interdisciplinary reviews: Cognitive Science*, *1*(4), 586-596. https://doi.org/10.1002/wcs.55

Goldin, G. A. (2003). Developing complex understandings: On the relation of mathematics education research to mathematics. *Educational Studies in Mathematics*, *54*(2-3), 171-202. https://doi.org/10.1023/B:EDUC.0000006180.20493.3c

Hertel, J. T., & Wessman-Enzinger, N. M. (2017). Examining Pinterest as a curriculum resource for negative integers: An initial investigation. *Education Sciences*, *7*(2), 45. https://doi.org/10.3390/educsci7020045

Huinker, D. (2015). Representational competence: A renewed focus for classroom practice in mathematics. *Wisconsin Teacher of Mathematics, 67*(2), 4-8. https://s3.amazonaws.com/amo_hub_content/Association1956/files/Journal/WTM%20Spring%202015.pdf#page=6

Kamii, C., Lewis, B. A., & Kirkland, L. (2001). Manipulatives: When are they useful?. *The Journal of Mathematical Behavior*, *20*(1), 21-31. https://doi.org/10.1016/S0732-3123(01)00059-1

Karp, K. S., Bush, S. B., & Dougherty, B. J. (2014). 13 rules that expire. *Teaching Children Mathematics, 21*(1), 18-25. https://doi.org/10.5951/teacchilmath.21.1.0018

Karp, K. S., Bush, S. B., & Dougherty, B. J. (2015). 12 math rules that expire in the middle grades. *Mathematics Teaching in the Middle School*, *21*(4), 208-215. https://doi.org/10.5951/mathteacmiddscho.21.4.0208

Kirkland, L., Aldridge, J., & Kuby, P. (1991). Environmental print and the kindergarten classroom. *Reading Improvement*, *25*(4), 219-222.

Kuzniak, A., Nechache, A., & Drouhard, J. P. (2016). Understanding the development of mathematical work in the context of the classroom. *ZDM*, *48*(6), 861-874. https://doi.org/10.1007/s11858-016-0773-0

Lakoff, G., & Johnson, M. (1980). *Metaphors we live by*. University of Chicago Press.

Lakoff, G. & Nunez, R. E. (2000). *Where mathematics comes from*. Basic Books.

Lappan, G., Phillips, E. D., Fey, J. T., Friel, S. N., Grant, Y., & Stewart, J. (2014). *Connected mathematics 3*. Pearson Education, Inc.

Lesh, R., Post, T. R., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representations in the teaching and learning of mathematics* (pp. 33-40). Lawrence Erlbaum.

Linchevski, L. & Williams, J. (1999). Using intuition from everyday life in "filling" the gap in children's extension of their number concept to include the negative numbers. *Educational Studies in Mathematics, 39*, 131-147. https://doi.org/10.1023/A:1003726317920

Luttenberger, S., Wimmer, S., & Paechter, M. (2018). Spotlight on math anxiety. *Psychology Research and Behavior Management, 11*, 311-322. https://doi.org/10.2147/PRBM.S141421

Mehta, R., & Zhu, R. (2009). Blue or red? Exploring the effect of color on cognitive task performances. *Science*, *323*(5918), 1226-1229. https://doi.org/10.1126/science.1169144

Mertler, C. A. (2003). Patterns of response and nonresponse from teachers to traditional and web surveys. *Practical Assessment, Research, and Evaluation*, *8*(1), 22. https://doi.org/10.7275/2kdf-g675

Muir, T. (2015). Always, sometimes, never true. *Teaching Children Mathematics*, *21*(6), 384-384. https://doi.org/10.5951/teacchilmath.21.6.0384

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Authors.

Nurnberger-Haag, J. (2007). Integers made easy: Just walk it off! *Mathematics Teaching in the Middle School*, 13, 118-121. https://doi.org/10.5951/MTMS.13.2.0118

Nurnberger-Haag, J. (2015a). How students' integer arithmetic learning depends on whether they walk a path or collect chips. In T. G. Bartell, K. N. Bieda, R. T. Putnam, K. Bradfield & H. Dominguez (Eds.), *Proceedings of the 37th annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (pp. 165-172). Michigan State University. http://www.pmena.org/proceedings/

Nurnberger-Haag, J. (2015b). *Moving on a path versus collecting objects: New perspectives to analyze student learning with integer models*. Michigan State University, Educational Psychology and Educational Technology. https://d.lib.msu.edu/etd/3575

Nurnberger-Haag, J., Kratky, J. & Karpinski, A. C. (2022). The Integer test of primary operations: A practical and validated assessment of middle school students' calculations with negative numbers. *International Electronic Journal of Mathematics Education, 17*(1), em0667. https://doi.org/10.29333/iejme/11471

Nurnberger-Haag, J., McTeer, J. S., Hopkins, A., Ries, J., Catterall, A. Maxwell, H., & Branham, I. J. (2019). Environmental math in the classroom: What do your walls say? *Ohio Journal of School Mathematics, 81*. https://library.osu.edu/ojs/index.php/OJSM/article/view/6556/5145

Nurnberger-Haag, J., & Thompson, C.A. (2023). Simplest shapes first! But let's use cognitive science to reconceive and specify what "simple" means. *Mind, Brain & Education (MBE)*. 17(1), 5-19. https://onlinelibrary.wiley.com/doi/epdf/10.1111/mbe.12338

Nurnberger-Haag, J. & Wernet, J. L. (2019). Sparking surprise: A negative number game to promote productive concepts of addition and subtraction. *Ohio Journal of School Mathematics, 82*. https://library.osu.edu/ojs/index.php/OJSM/article/view/6663/5345

Nurnberger-Haag, J., Singh, R., & Wernet, J. L. (2020). An atypical approach to improve typical issues with pre-service teachers' geometric shape knowledge. *Issues in the Undergraduate Mathematics Preparation of School Teachers, 1*. http://www.k-12prep.math.ttu.edu/journal/1.contentknowledge/numberger01/article.pdf

Nurnberger-Haag, J., Wernet, J. L. & Benjamin, J. I. (2023). Gameplay in perspective: applications of a conceptual framework to analyze features of mathematics classroom games in consideration of students' experiences. *International Journal of Education in Mathematics, Science and Technology (IJEMST)*, *11*(1), 267-303. https://www.ijemst.net/index.php/ijemst/article/view/2328/345

Palm, T. (2006). Word problems as simulations of real-world situations: A proposed framework. *For the Learning of Mathematics*, *26*(1), 42-47. https://www.jstor.org/stable/40248523

Pape, S. J., & Tchoshanov, M. A. (2001). The role of representation(s) in developing mathematical understanding. *Theory Into Practice, 40*(2), 118-127.

https://doi.org/10.1207/s15430421tip4002_6
Pettis, C., & Glancy, A. W. (2015). Understanding students' challenges with integer addition and subtraction through analysis of representations. In T. G. Bartell, K. N. Bieda, R. T. Putnam, K. Bradfield & H. Dominguez (Eds.), *Proceedings of the 37th annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (pp. 221–224). Michigan State University. https://www.pmena.org/proceedings/
Rabardel, P. (2002). *People and technology: A cognitive approach to contemporary instruments* (H. Wood, Trans.). Armand Colin. (Original work published 1995).
Robinson, K. M., & Dubé, A. K. (2009). A microgenetic study of the multiplication and division inversion concept. *Canadian Journal of Experimental Psychology*, *63*(3), 193-200. https://doi.org/10.1037/a0013908
Ryan, J., & Williams, J. (2007). *Children's mathematics 4–15: Learning from errors and misconceptions*. Open University Press.
Scarpello, G. (2007). Helping students get past math anxiety. *Techniques: Connecting Education and Careers, 82*(6), 34-35. https://files.eric.ed.gov/fulltext/EJ775465.pdf
Schell, J. (2008). *The art of game design: A book of lenses*. Morgan Kauffman Publishers.
Schoenlein, M. A. & Schloss, K. B. (2022) Colour-concept association formation for novel concepts. *Visual Cognition*, *30*(7), 457-479. https://doi.org/10.1080/13506285.2022.2089418
Shapiro, E. J., Sawyer, A. G., Dick, L. K., & Wismer, T. (2019). Just what online resources are elementary mathematics teachers using?. *Contemporary Issues in Technology and Teacher Education*, *19*(4), 670-686. https://citejournal.org/wp-content/uploads/2019/10/v19i4Math1.pdf
Sinclair, N., & Moss, J. (2012). The more it changes, the more it becomes the same: The development of the routine of shape identification in dynamic geometry environment. *International Journal of Educational Research*, *51*, 28-44. https://doi.org/10.1016/j.ijer.2011.12.009
Spangler, D. A., & Williams, S. R. (2019). The role of theoretical frameworks in mathematics education research. In K. R. Leatham (Ed.), *Designing, conducting, and publishing quality research in mathematics education* (pp. 3-16). Springer International Publishing AG.
Steele, C.M. & Aronson J. (1995). Stereotype threat and the intellectual test performance of African Americans. *Journal of Personality and Social Psychology,* 69(5), 797–811. https://psycnet.apa.org/doi/10.1037/0022-3514.69.5.797
Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2009). *Implementing standards- based math instruction: A casebook for professional development*. Teachers College Press.
Stephan, M., & Akyuz, D. (2012). A proposed instructional theory for integer addition and subtraction. *Journal for Research in Mathematics Education*, *43*(4), 428-464. https://doi.org/10.5951/jresematheduc.43.4.0428
Sweller, J., Ayres, P., & Kalyuga, S. (2011). *Cognitive load theory.* Springer.
Thompson, P. W., & Dreyfus, T. (1988). Integers as transformations. *Journal for Research in Mathematics Education*, *19*(2), 115–133. https://doi.org/10.2307/749406
Tsang, J. M., Blair, K. P., Bofferding, L., & Schwartz, D. L. (2015). Learning to "see" less than nothing: Putting perceptual skills to work for learning numerical structure. *Cognition and Instruction*, *33*, 154–197. https://doi.org/10.1080/07370008.2015.1038539
van de Walle, J., Karp, K., & Bay-Williams, J. (2010). *Elementary and middle school mathematics: Teaching developmentally* (7th ed.). Allyn and Bacon.
Vlassis, J. (2008). The role of mathematical symbols in the development of number conceptualization: The case of the minus sign. *Philosophical Psychology, 21*(4), 555-570. https://doi.org/10.1080/09515080802285552
Westland, S., Laycock, K., Cheung, V., Henry, P., & Mahyar, F. (2007). Colour harmony. *Colour: Design & Creativity*, *1*(1), 1-15.

https://stephenwestland.co.uk/pdf/westland_laycock_cheung_henry_mahyar_CDC_2007.pdf

Wernet, J. L. (2017). Classroom interactions around problem contexts and task authenticity in middle school mathematics. *Mathematical Thinking and Learning*, *19*(2), 69-94. https://doi.org/10.1080/10986065.2017.1295419

Whitacre, I., Bishop, J. P., Lamb, L. L., Philipp, R. A., Schappelle, B. P., & Lewis, M. (2011). Integers: History, textbook approaches, and children's productive mathematical intuitions. In L. R. Wiest & T. Lamberg (Eds). *Proceedings of the 33rd Annual Meeting of the North American chapter of the International Group for the Psychology of Mathematics Educatio*n (pp. 913–920). University of Reno. https://www.pmena.org/proceedings/

Whitacre, I., Azuz, B., Lamb, L. L., Bishop, J. P., Schappelle, B. P., & Philipp, R. A. (2017). Integer comparisons across the grades: Students' justifications and ways of reasoning. *The Journal of Mathematical Behavior*, *45*, 47-62. https://www.doi.org/10.1016/j.jmathb.2016.11.001

White, K., & McCoy, L. P. (2019). Effects of game-based learning on attitude and achievement in elementary mathematics. *Networks: An Online Journal for Teacher Research, 21*(1), 1-17. https://doi.org/10.4148/2470-6353.1259

Wilhelm, A. G., Chen, I. C., Smith, T. M., & Frank, K. A. (2016). Selecting expertise in context: Middle school mathematics teachers' selection of new sources of instructional advice. *American Educational Research Journal*, *53*(3), 456-491. https://doi.org/10.3102/0002831216637351

Zhou, W., Deng, L., & Ding, J. (2021). Neural mechanism underlying the effects of object color on episodic memory. *Acta Psychologica Sinica*, *53*(3), 229-243. https://doi.org/10.3724/SP.J.1041.2021.00229

Zaiontz, C. (2023). *Effect size for chi-square test*. Real Statistics Using Excel. https://real-statistics.com/chi-square-and-f-distributions/effect-size-chi-square/