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Change in Emergent Multilingual Learners' Mathematical Communication: Attending to Language Use and Needs

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ABSTRACT

Part of learning a new discipline is learning the language used in the discipline. For mathematics, emergent multilingual learners (EML) must learn English and mathematical symbols in order to make meaning and communicate. The mathematics community's understanding of communication is complex and includes the use of natural language, incorporation of representations (mathematical symbols and visuals), and manipulation of tools and technology. In our research, we use this notion of communication as we examine the way students think about their abilities to communicate in and about mathematics. We specifically ask: (1) How do fifth- and eighth-grade EMLs change in their understanding of mathematics communication with intentional instruction as captured on the Mathematics Communication Inventory (MCI) composite scores? (2) If there is change, how do fifth- and eighth-grade EMLs' scores compare? (3) How does the use of academic language to communicate in mathematics change over time for EMLs with intentional instruction? Two groups of students (15 fifth and 17 eighth graders) enrolled in a newcomers' program informed this research. Data were collected using an open-ended pre- and post-writing assessment. The results strongly suggest that students began to recognize the extent to which they used mathematics for communication, after explicit instruction, to reveal modes of communication in mathematics that are easily and constantly used by students. The change over time was different for the two age groups for total words/symbols and unique words.

Keywords: sociocultural constructivism; mathematics communication; mathematics communication inventory; emergent multilingual learners

Introduction

At the national level in the United States (U.S.), mathematics educators stress that students should communicate *about* mathematics and *in* mathematics using a variety of methods (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA Center & CCSO], 2010; National Council of Teachers of Mathematics [NCTM], 2000). Effective mathematics communication is important for all students regardless of heritage language (Chen & Li, 2008). Students entering U.S. schools with no English must learn mathematics concepts, develop communication skills, and learn English. With a growing population of non-English speaking students, mathematics teachers need to scaffold with emerging multilingual students (EMLs) in terms of both mathematics and language skills. This scaffolding calls for incorporating best practices from two disciplines: mathematics and English Language Arts. Our research focuses on the development of mathematical understanding and communication abilities of EML students in a program that intentionally addressed both.

Theoretical Framework

This study is grounded in a theoretical framework based on semiotic mediation as it is described in two areas: socio-cultural constructivism as posited by Vygotsky (1978; 1986), and academic literacies as articulated by Gee (2004a; 2004b; 2008) and Lemke (2002; 2004). Language is portrayed as a tool for meaning-making (Vygotsky), used within a community (Gee), and having hybrid discursive practices (Lemke). All three portrayals focus on meaning as found in symbols, artifacts, and language.

Socio-cultural Constructivism

Socio-cultural constructivism (Luria, 1976; Vygotsky, 1978; 1986; Vygotsky & Luria, 1994) posits the epistemological stance that knowledge is constructed by individuals as they interact within the social context of community. Vygotsky and his students particularly stress language as a tool for developing conceptual understandings. This theory of learning incorporates forms of scaffolding in which a more knowledgeable other supports the learning when the learner cannot continue alone. The support allows the learner to move forward rather than remaining at his/her current level. The social tool of semiotic items (language, symbols) is important in this scaffolding process.

Literacies

The ability to communicate is further illuminated by Gee (2004a). He outlined a position whereby members of a specialized community use a social language recognizable by that specific community. He points out that the use of the language also makes members recognizable by those outside that particular community. This distinction is seen with mathematicians as there is a specific way of thinking, believing, and viewing the discourse of mathematics. Ubiquitous words, such as mean, whole, median, and sum, are recognized for specific mathematical meaning. The contexts in which the words are spoken or written situate their meaning. For example, in oral language the following use of the homophones whole and hole are understood only by knowing something about the context: "Create the whole using manipulatives″ versus ″Create the hole using a shovel."

The Discourse of mathematics uses four modes as theorized by Lemke (2002). He posits that natural language, visual representation, mathematical symbolism, and manual technical operations work together to form "a single unified system of meaning making" (p. 1). Natural language refers to the written and oral communication as defined by linguists (Lyons, 1991). Lemke (2002) proposes that natural language is not precise enough to represent phenomena mathematically. Historically, as humans recognized the need for more precision in their communication, it became necessary to extend natural language from typological (qualitative meaning) to include topological (quantitative meaning). To illustrate, "The Statue of Liberty is tall" is an example of typological semiotics, where "The Statue of Liberty is 93 meters tall" exemplifies topological semiotics. Mathematical symbols provide information that communicates a variety of information. For example, T(x,y) = k(x 2 + y 2) or $f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$. These symbols and expressions (sentences) are recognizable by mathematicians, but often are meaningless to non-mathematicians.

Lemke (2004) further expands the notion of communication to include visual representations such as drawings, charts, graphs, and tables. Although not unique to mathematics, these representations help organize ideas and communicate mathematical thinking. The 4th mode, manual technical operations, includes meaningful actions and practices. These actions utilize tools within a specific environment to further communicate and make meaning.

Literature Review

Linguistic Challenges of Mathematics

An often expressed misconception is that mathematics is universal and free of influence (Brown et al., 2009; Jourdain & Sharma, 2016). However, the language of mathematics can be confusing (Jourdain & Sharma, 2016) and part of learning a new discipline is learning the language used in the discipline. For mathematics, an EML must learn English and mathematical symbols as well as develop an understanding of how the different systems interact for making meaning (Schleppegrell, 2007; 2011). Teachers often point out vocabulary as a challenge in teaching mathematics and fail to recognize the grammatical patterning that is involved. This is particularly true when word problems are included in the instruction (Fatmanissa & Novianti, 2021).

Truxaw and Rojas (2014) described how difficult it can be to use a developing language while learning new mathematics concepts. These authors pointed to mathematics language as being more abstract, specific, and culturally determined than conversational language. Moschkovich (2015) stressed that conceptual understanding in mathematics requires words, vocabulary, and definitions. She suggested that using multimodal communication can be beneficial for overcoming some of the challenges. Jourdain and Sharma (2016), in a review of the literature, stressed the need for pedagogical practices to pay attention to how language is used in mathematics.

Mathematics Communication

Beal et al. (2010) pointed out that little attention has been paid to low levels of mathematics achievement by EMLs. Their research showed an achievement gap between EMLs and non-EMLs. These authors suggested that there is a minimum reading proficiency associated with mathematics achievement. In fact, two studies provided evidence that mathematical difficulties may reflect deficient language skills rather than quantitative processes (LeFevre et al, 2010; Vukovic, 2012). In order for EMLs to communicate in mathematics, they must use different skills from those in everyday communication. Mathematics communication requires using abstractions and symbols (Olivares, 1997). Vukovic and Lesaux (2013) stated that "general verbal ability appears to impact children's performance by influencing the mathematical thinking that involves the symbolic number system" (p. 89-90). In addition, the specificity of the elements of mathematics 'sentences' often means that the order cannot be rearranged. This is not true of everyday and informal speech.

NCTM

NCTM (2000) recognized that "(c)ommunication can support students' learning of new mathematical concepts as they act out a situation, draw, use objects, give verbal accounts and explanations, use diagrams, write, and use mathematical symbols" (p. 61). The process standard of communication generally refers to the skills of writing, speaking, listening, and reading about mathematics. The language of mathematics is complicated and dense (Schleppegrell, 2007; 2011). Without opportunities to use it, students struggle to communicate thoughts and ideas. By middle school, students should be able to describe, clarify, and extend their mathematics thinking in written and oral forms. Their communication skills are further enhanced through use of multiple representations, including drawings, organized visuals (graphs, tables, and charts), mathematical symbols, manipulatives, or technologies (NCTM, 2000).

Common Core State Standards

Additionally, communication was addressed in two of the six Guiding Principles for School Mathematics found in the Common Core State Standards for Mathematics (CCSSM) (NGA Center & CCSO, 2010). The two are: (a) Teaching and Learning, and (b) Tools and Technology. The Teaching and Learning Guiding Principle discussed in *Principles to Actions* (NCTM, 2014) suggests that students engage "in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically" (p. 5). The Tools and Technology Principle states that math tools and technology are "essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking" (p. 5).

Methods

The mathematics community's understanding of communication is complex and includes the use of natural language, incorporation of representations (mathematical symbols and visuals), and manipulation of tools and technology. In our research we use this notion of communication (Weinburgh et al., 2014) as we examine the way students think about their abilities to communicate in and about mathematics. We specifically ask: (1) How do fifth- and eighth-grade EMLs change in their understanding of mathematics communication with intentional instruction as captured on the Mathematics Communication Inventory (MCI) (Smith et al., 2015) composite scores? (2) If there is change, how do fifth- and eighth-grade EMLs' scores compare? (3) How does the use of academic language to communicate in mathematics change over time for EMLs with intentional instruction?

Participants

Two groups of students (15 fifth and 17 eighth graders) enrolled in a newcomers' program (Silva et al., 2008) within a local urban school system informed this research. Students had been in the U.S. less than three years and were expected to exit the newcomer program, entering mainstream classrooms in the fall. They were classified as "advanced high language proficient" on the Texas English Language Proficiency Assessment System. Twenty languages from 10 different countries were represented.

Context/Instruction

Students attended a three-week (80 hours) enrichment experience focused on a Crime Scene Investigation (CSI) theme in which science, mathematics, and English instruction were integrated (Silva et al., 2012; Weinburgh & Silva, 2011; Weinburgh et al., 2014). Students voluntarily attended the program for six hours each day. The program was developed and taught by four of the authors: two mathematics educators, a science educator, and a bilingual educator. Grade-appropriate investigations helped students solve a mystery using forensic practices to eliminate suspects. Students examined footprints, fingerprints, blood, ink, and DNA samples. To support students in collecting and analyzing scientific data, mathematics content areas of (1) measurement; (2) data collection and analysis; (3) proportional reasoning; (4) patterns, relationships, and algebraic thinking; and (5) numerical reasoning were emphasized. Furthermore, the process standards of connection, proof and reasoning, communication, problem solving, and multiple representations were highlighted.

Week 1

First, students photographed and measured the crime scene to create sketches. The variation in the sketches produced by the students allowed for a discussion regarding the need for precision using scale and proportions. Responding to this discussion, the students used accurate measurements to create a two-dimensional scaled drawing of the crime scene. Later, students made a chart of possible suspects and used the patterns found in fingerprints to eliminate the first suspects. They engaged in multiple iterations of blood typing. As an extension and connection to their lives, the students confirmed their own blood types with their parents to calculate the percentage of each blood type within the class and compared the findings to national statistics. Students used their problem-solving skills and new knowledge of the percentage of blood types in the population to help eliminate other possible suspects. Through discussions and multiple representations, EMLs from each grade level shared their particular findings. These findings resulted in a full set of collaborative data for use by both groups.

Week 2

Fifth graders used chromatography to compare the ink on the packing slip to ink in pens taken from the list of suspects generated by the students, allowing for the elimination of other suspects. Eighth graders used their own foot-print measurements to discover the ratio between foot length and height. Based on this ratio, they calculated the offender's approximate height, thereby eliminating more suspects. In addition, the eighth-grade students used patterns and relationships to read electrophoreses results from their DNA samples to identify the perpetrator from the remaining suspects.

Week 3

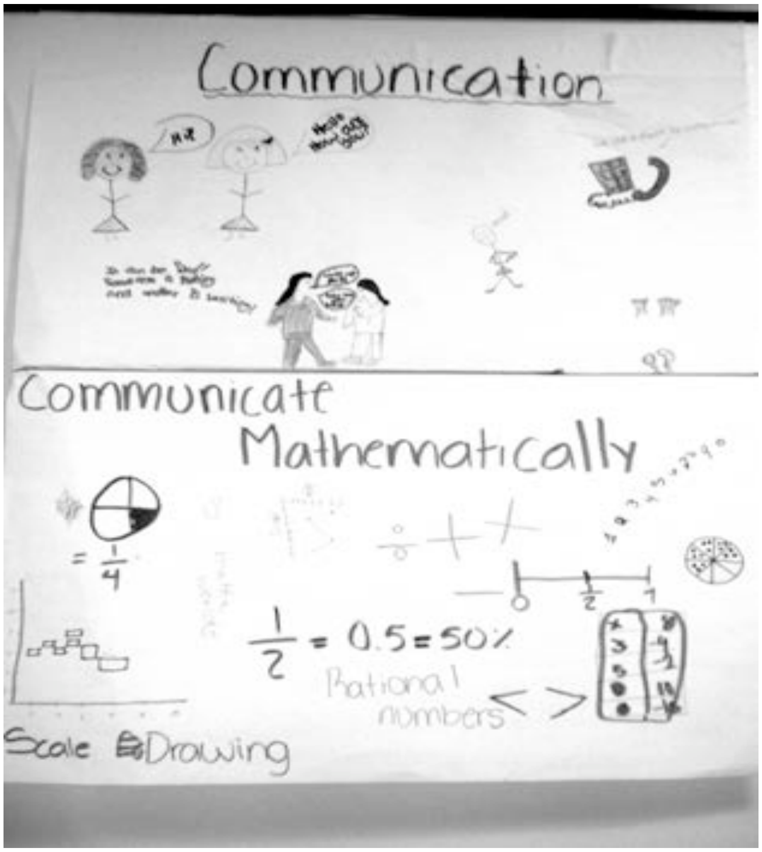
Students engaged in discussions about the various ways of communicating in mathematics. Class discussions included how to communicate within the classroom with peers and teachers through the use of words, pictures, organized visuals, mathematical symbols, and through the use of tools. The students made a list of different ways to communicate. Based on prior experiences, other discussions included ways in which people outside the classroom use mathematics, including their parents.

Students were asked to reflect on their understandings of how they used mathematics to communicate their findings. Various forms of communication were recorded and posted on a classroom chart. Students initially recognized that they were using numbers, charts, and graphs to communicate their findings. Students began to add ideas like written words, drawing pictures, cooking, using manipulatives, and hand gestures. The chart became a growing reference that was used throughout the summer program.

In another activity, students worked in groups to identify ways in which mathematics communication is used outside of the classroom. Figure 1 shows a group display of how students began to think about using mathematics to communicate.

Figure 1

Students Produce a Poster to Represent How They Now Think About Math Communication



Data Collection

Data was collected using an open-ended pre- and post-writing assessment. Students were asked to respond to the prompt "How do I, as a mathematician, communicate information?" This prompt allowed students maximum leeway in expressing understanding of how to communicate in and about mathematics, as well as time needed for responding. Thus, student responses varied in length, complexity, and types of communication systems.

Data Analysis

Data were analyzed in two phases. First, we scored the pre- and post-writing using the MCI. The MCI is an analytical framework grounded in Lemke's notion of hybrid language (2004), NCTM's *Principles and Standards for School Mathematics* (2000), NGA Center and CCSO initiative (2010), and Silverstein's theoretical work in multifunctional communicative semiotic (1995; 2004). The MCI has five coded categories: (1) Mode I, (2) Mode II, (3) Content, (4) Process, and (5) Placement/Context. The MCI was presented to a panel of mathematicians for feedback as to its appropriateness for capturing change in communication. In addition, word and mathematical symbols were counted. The second phase of data analysis compared the data from phase one using the Statistical Package for the Social Science (SPSS).

Phase One Analysis

Writing samples (Appendix) were scored using the MCI. Each sub-category could receive a score of 0 or 1. Mode I and Mode II parallel Lemke's hybrid language. Content and Process were taken from the NCTM standards, CCSSM, and current state standards. Placement/Context was added by the researchers to reflect Gee's (2004b) notion of situated meaning. In addition to the five categories, words and symbols were counted for total number of words/symbols, total mathematics words/symbols, and unique mathematics words/symbols. The total number of words was included because the students were EML learning English beyond mathematics.

Mode I reflected how the students presented their information. For example, the students could use natural language, symbols, visuals, or organized visuals as a part of their explanations. All students received credit for natural language because they wrote something; some students received credit for the use of symbols, visuals, and organized visuals. We determined that manual technical operations would never be used in Mode I.

Mode II reflects what the student described as methods of communication. For the sample to receive a one for natural language, the student had to write about talking, writing, or speaking about mathematics. To receive a one for mathematical symbols, a student had to show how s/he used mathematical symbols. To receive a one for visuals, a student had to describe using pictures or drawings. For a student to receive a one for organized visuals, s/he had to write about using tables, charts, graphs, or other organized visuals. A student might write that a person could use a graph to show the household bills (see Appendix). Lastly, to receive a one for manual technical, the student needed to discuss how s/he used gestures or tools. An example that most students described was using cups/spoons to measure ingredients.

For Content, the coders examined the way the student expressed an understanding of the six content strands based on NCTM standards, CCSSM, and current state standards. Process has five areas as found in the NCTM (2000) standards. Placement/Context included formal, informal, and academic. The sample received credit for informal if the student described a situation such as cooking or driving, and academic if it took place in the school setting. Word and symbol counts were conducted.

Each student's pre- and post-writing was transcribed with all representations (symbolic and visual) included. We practiced using the instrument on data collected from students who were missing a pre- or post- writing sample. Once interrater reliability was established at 91%, coding for the study began.

Phase Two Analysis

In order to answer Research Question 1, a comparison of the scores from the MCI on students' pre-writing and post-writing assessments were conducted using a one-tailed *t*-test. In order to answer Research Question 2, the five sub-scores on the MCI were examined using a one-tailed *t*-test for each of the grade levels.

Both groups improved significantly in understanding of mathematics communication, as shown in Table 1. For the fifth-grade EMLs, MCI mean scores increased from 3.87 to 11.8. This increase was significant at the .05 level, $t(14) = -6.65$, $p < 0.001$. Means for the eighth-graders increased from 2.94 to 15.65, which was significant at the .05 level, $t(16) = -12.62$, $p < 0.001$.

Table 1

Analysis of Pre-Writing and Post-Writing Scores on the MCI

		PreWriting		Post-Writing			
Grade	*n*	*M*	*SD*	*M*	*SD*	*t*	*p*
5th	15	3.87	1.81	11.8	5.09	-6.65	<.001
8th	17	2.94	1.14	15.65	3.89	-12.62	<.001

Research Question 2

Both fifth- and eighth-grade EMLs showed improvement on all five sub-scores which warranted Research Question 3. Analysis revealed differences between the groups from the pre- to the post-writing.

Fifth-grade Change

A comparison of pre- to post- scores by the five categories is shown in Table 2 for the fifth-grade students.

Table 2

Analysis of Pre-Writing and Post-Writing Sub-scores on the MCI

Fifth-Grade		
Sub-score Category	*t*	*p*
Mode I	-2.78	0.007
Mode II	-4.66	<.001
Content Standards	-6.46	<.001
Process Standards	-4.62	<.001
Context	-4.58	<.001
Eight-Grade		
Sub-score Category	*t*	*p*
Mode I	-7.38	<.001
Mode II	-6.17	<.001
Content Standards	-10.87	<.001
Process Standards	-8.72	<.001
Context	-13.96	<.001

During the pre-writing task, students predominately used natural language as their form of communication. Only one student used an organized visual in the writing sample. In the post-writing task, only four students exclusively used natural language. The other 11 students used two or more forms (natural language, mathematical symbols, visuals and/or organized visuals), providing evidence that the students were thinking about a variety of communication strategies.

On Mode II, the fifth-grade students' pre-writing task scores ranged from 0-2. Three students scored 0, nine students scored 1, and three students scored 2. The students only discussed using two forms of communication: natural language and mathematical symbols. Students mostly wrote about being able to talk generically using mathematics. Five students wrote about using mathematics symbols. In contrast, the post-writing task scores ranged from 0-5. The majority of the students' scores ranged from 2-5 on the post- test. Only two of the students scored 0 on the post-test and 13 students scored 2 or higher.

For mathematics content standards, the majority of the students (14) scored either 0 or 1 on the pre-writing task with number and operations being the most common content standard discussed. Only one student scored 2, and s/he touched on number and operations and financial literacy. The post-writing task still had one-fifth of the students scoring 0, the other four-fifths scored between 1 and 4, with most students scoring 3. One student discussed using graphs, one student wrote about using algebra, and two students wrote about using geometry.

Process standards had seven different measures: 1) communication, 2) problem solving, 3) representations, 4) proof and reasoning 5) connections with other content areas, 6) connections within math, and 7) connections in other contexts. On the pre-writing task, seven students scored 0, seven students scored 1, and one student scored 2. On the post-writing task, one student scored 0 and two scored 1. Again, those scoring 1 only discussed using the process standard of communication. The other 12 students scored between 2 and 5, with four scoring 2, four scoring 3, three scoring 4, and one scoring 5. No student hit all seven areas.

Fifth-grade students also showed change in Placement/Context. In the pre-writing task, only four students placed their writing within a specific context: academic setting and business context. On the post-writing samples, four students contextualized mathematics in only one area: informal context or academic context. Five of six students scoring 2 on the MCI placed the mathematics in informal and academic settings. There were four students who were able to write about communicating mathematics in all three contexts.

Sub-scores in each category on the MCI from pre-writing to post-writing were analyzed to determine if the differences were significant at the .05 level. For the fifth-graders, all categories except Mode I (natural language) showed significant improvement (see Table 2).

Eighth-grade Change

A comparison of pre- to post-scores by category are shown in Table 3.

Table 3

Total Word/Symbol Count & Unique Math Word/Symbol Count

Total Word/Symbol Count							
		Pre-Writing		Post-Writing			
Grade	*n*	*M*	*SD*	*M*	*SD*	*t*	*p*
5	15	24.93	14.14	245.87	126.34	-6.98	<.001
8	17	24.59	14.99	352.82	151.31	-9.31	<.001

Unique Mathematical Word/Symbol Count							
		Pre-Writing		Post-Writing			
Grade	*n*	*M*	*SD*	*M*	*SD*	*t*	*p*
5	15	2.53	2.39	17.93	15.47	-3.85	<.001
8	17	1.12	1.76	23.94	10.87	-8.42	<.001

In the pre-writing task, the only form of communication used by students for Mode I was natural language. In the analysis of the post writing task only two students used natural language exclusively. The majority of the students used three (7 students) and four (5 students) forms of communication.

Looking at Mode II pre-scores on the MCI, the students predominately scored 1. These 12 students wrote about communicating with natural language. Of the three who scored 2 on the instrument, two described using symbols and one wrote about using manual technical operations (rulers, calculators, manipulatives, money, etc.). On the post-writing task, only two students scored 1. One student discussed using natural language and the other discussed using manual technical operations. Of the 15 remaining students, all wrote about using mathematical symbols, four discussed using visuals (pictures and drawings), 11 wrote about using organized visuals (tables, charts, graphs, etc.), and eight described using manual technical tools. These data indicated that students were able to apply the modes to describe how they would communicate mathematically with others.

On the pre-test for Content, only three students identified a content area (number and operations) that they communicate about mathematics. In the post-writing, students expanded the content to include measurement (13), number and operations (17), and financial literacy (16). These post-writing findings show a tremendous increase on which content areas within mathematics can be used to communicate mathematically.

In analyzing the students' pre-writing task for Process, only ten of 17 students wrote about using one of seven process standard components. All ten students discussed using at least one of the four hybrid processes to communicate mathematically. No other process standard was identified in the pre-writing task. On the post-writing task, only one student scored 1. The remaining 16 students scored between 2 and 6. The four process standards included the most were communication (10), problem solving (16), representations (10) and connections within other contexts (15).

When analyzing the pre-writing task about the context, only three students placed their writing within a specific context. One discussed communicating about mathematics in an informal setting, while two students wrote about communicating within an academic setting. For the post-writing example, students were more diverse in the situations where they communicated mathematically. Communicating in both informal and business situations was discussed by 16 students. In contrast, only 12 students discussed using mathematics to communicate in an academic setting.

Sub-scores in each category of the MCI from pre-writing to post-writing were analyzed to determine if the differences were significant at the .05 level. For the eighth-graders, all categories of the MCI showed a significant improvement (see Table 2).

Research Question 3

In order to investigate the ways in which academic language used to communicate in mathematics changes over time for EMLs with intentional instruction, several pre/post word and mathematical symbol counts were conducted. For this study, two counts were reported: total words/symbols used, and unique math words/symbols used by the students.

Total Words/Symbols

The total word/symbol count enabled the authors to explore the increase in students' general writing, including their comfort level in using English to write about mathematics. As shown in Table 3, a statistical increase in total word/symbol count after intentional instruction was noted. The total word/symbol count for fifth-graders was significant at the .05 level, $t(14) = -6.98, p < 0.001$. The fifth-grade students' writing samples average increased from approximately 25 words/symbols to 245 words/symbols. The total word/symbol count for eighth-grade increased from 25 words/symbols to

350 words/symbols. The eighth-grade data were significant at the .05 level, $t(16) = -9.31$, $p < 0.001$.

Unique Words

The unique mathematical word/symbol count looked solely at mathematics vocabulary and symbols present in the student writing samples. Words/symbols were counted only once even if the word/symbol was used several times. For example, when a student wrote "We communicate with a graph, and there are different types of graphs. Line graph, bar graph, pie graph, double bar graph," the word "graph" was only counted once. In this instance, line graph, bar graph, and pie graph were considered to be unique math terms. A similar procedure was used for math symbols.

Table 3 indicates that the fifth- and eighth-grade students' use of unique mathematics vocabulary/ symbols also increased. The use of unique mathematics vocabulary/symbol count for the fifth-graders was significant at the .05 level, $t(14) = -3.85$, $p < 0.001$. They increased their use of unique mathematics terminology from 2.5 words per pre-writing sample to 18 unique words per post-writing sample. Whereas the eighth-graders were significant at the .05 level, $t(16) = -8.42$, $p < 0.001$, their increase was greater. The eighth-graders went from using one unique mathematical term per pre-writing sample to using 24 unique terms in the post-writing sample.

Discussion

As teachers of a growing number of EMLs, we were interested in seeing if intentional instruction that included metacognitive activities would change students' understanding and use of mathematics communication. Although the two grade levels' gains were different, there were similarities in how communication skills developed for both. Therefore, we chose to frame this discussion by sub-categories on the MCI rather than by grade level, followed by a discussion of the changes observed in word/symbol use.

Mode I

Mode I shows the students' understanding of how to present information in a mathematical format. The NCTM Standards call for the students to "(c)reate and use representations to organize, record, and communicate mathematical ideas" (NCTM, 2000, p. 67). However, neither the fifth- nor eighth-grade students came into the summer program showing such an understanding. The data show that given meta-linguistic and meta-mathematical instruction, the students were able to expand their repertoire of communication modes. During the pre-writing task, the students predominately used natural language as their form of communication but moved to using other modes of the hybrid language by the end of the intervention. As indicated by Lemke (2002), natural language is necessary, but not sufficient to communicate fully the precision and complexity of mathematical thought. In addition, Cai et al. (1996), Johnson (2013), and Jourdain and Sharma (2016) discuss the importance of using natural language as well as mathematical expressions, and visual representations. All three posit that these are imperative to students' mathematical understandings.

Mode II

Mode II reflects what the student described as methods of communication. Initially, they predominately used natural language to describe using language to communicate (i.e., I talk, read, and write about math to my friends.). In the pre-writing task, the students discussed using only two forms of communication: natural language and mathematical symbols. After intentional instruction, students were able to expand their repertoire to include mathematical symbols, visuals, organized visuals, and manual technical operations (mathematical tools, gestures, etc.) as a way to communicate mathematics to others. This expansion is an important finding because students who have an understanding of the importance of using different modes of communicating mathematically fulfill the NCTM (2000) call for the ability to "(s)elect, apply, and translate among mathematical representations to solve problems (p. 67) and to "(u)se representations to model and interpret physical, social, and mathematical phenomena" (p. 67).

Content Standards

The scores on the pre-writing task provide evidence that students were only aware of communicating about number/operations and financial literacy. Much of the instruction during the 3-week period required the students to communicate using measurement, data analysis, and number/operations. In the post-writing, the majority of students discussed using measurement, numbers/operations, and financial literacy to communicate mathematically. Limited movement toward mathematical thinking of geometry and algebra was also noted.

We feel the reason students discussed financial literacy (using money) is because of its practicality in their lives. It was not surprising that geometry and algebra were not mentioned by many of the children, since these were not the topics of instruction. We were somewhat surprised that we did not observe more movement regarding data analysis since we used tables, charts, and graphs almost daily to communicate findings in both the mathematics and science classroom.

Process Standards

The process standards are important for students because they cut across all mathematics content areas. Our state guidelines posit that "[t]he process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life" (Texas Education Agency, 2012, p. 1). With knowledge of process skills, both the fifth- and eighth-grade students are more likely to be successful learning the content. As noted in the results section, the students moved beyond only talking or writing about mathematics, to using each of the process standards to show how someone would do mathematics. The biggest gains were found in the process standard of problem solving and connections. This gain can be explained by the emphasis put on problem solving within the integration and overlap of science, language, and mathematics, and with the discussions about real world applications of mathematics. Kosko and Norton (2012) discuss the importance of connecting mathematics not only to other content areas and other contexts outside of school, but also about helping students make connections within mathematics.

The process standard of reasoning and proof increased the least, which did not surprise us because the complexity involved in this process is well documented. Even though we asked the children as a regular part of instruction to explain their answers, we provided less scaffolding for this skill than for others. However, we believe with more time, such as a full semester or academic year, students' understanding of the processes of reasoning and proof would increase.

Placement/Context

Placement and/or context denote the informal, formal, or academic setting in which the students believe mathematics communication is used. The identification of the placement or contexts is very telling about student awareness of using mathematics to communicate in daily life. In looking at the results, if the students mentioned a context during their pre-writing sample, they generally placed the communication within the classroom (academic). However, this occurred in only four of the students' writings with only one student noting that businesses (formal) use mathematics to communicate about sales. Even more astounding was the fact that the eighth-graders did not describe mathematics communication within any context. None of the students wrote about communicating mathematics in an informal setting, yet that is probably the most prevalent context in which to communicate math on a daily basis.

During the post–writing sample, the students demonstrated drastic changes in their views on placement/context. For both age groups, students gave examples within all three areas (informal, formal, and academic). It is obvious that the students became more aware of using mathematics to communicate within informal, everyday situations such as going to the grocery store, driving a car, giving directions, paying bills, cooking, etc. They also identified a multitude of business examples as seen in an example of one student (see Appendix).

Word/Symbol Use

Word and symbol use was divided into total and unique mathematical words/symbols used by EMLs. One might expect that after a 3-week summer program focusing on language, total word/symbol count would increase. This expectation was supported by the data. However, a striking finding was the increase in mathematics word count and unique mathematics terminology. Not only did the students increase the number of words/symbols used, but they placed them in mathematical sentences and appropriate language contexts.

Implications

The results strongly suggest that students began to recognize the extent to which they used math for communication after explicit instruction to reveal modes of communication in mathematics that are easily and constantly used by students. This recognition displayed itself as an increase in the use of the modes of the hybrid language in the post-writing. Of particular note for classroom teachers is that all but one of the students increased in the actual use of natural language, mathematical expressions, and visual representations as they described ways in which they could communicate in mathematics.

Classroom mathematics teachers teach the students the mechanics of communication, but often do not explain that what they are demonstrating is communication. For example, teachers show students how to read graphs. This instruction is necessary, but not sufficient. Students, especially EMLs, benefit from intentional instruction in metacognitive strategies in order to understand the underlying modes of communication. Thus, teachers should also point out the communicative function of the graph. As students become aware that they are doing some form of mathematics all the time, they realize that much of their daily communication includes mathematics. It is possible that the pre-writing reflects that students do not recognize that visual representations and symbolic representations are forms of communication. They do not immediately move from natural language to using visual and mathematical expressions in their explanation. Our study shows that with intentional instruction about modes of communication within the mathematics classroom, students begin to move beyond natural language. These findings are consistent with what we know about natural language in the sense that learners that develop metalinguistic awareness are better language users.

Even though our research does not attempt to show improvement in mathematics achievement, others have done so. Cohen et al. (2015) found that having second grade students write about communicating math improved their mathematics content scores over those who did not write. Their definition of writing, like the writing examples our study, includes natural language, mathematical symbols, and visuals. Their writing samples were not limited to reasoning and proof examples either, but included many types of writing such as poetry. Even though Cohen et.al. (2015) state that their research cannot be applied to other grade levels, we believe that when children communicate about mathematics using the four modes of communication, their new breadth of understanding will move them to thinking about mathematics in many contexts.

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Appendix

Pre – Instruction Journal Entry

<u>How do I as a mathematician communicate information?</u>

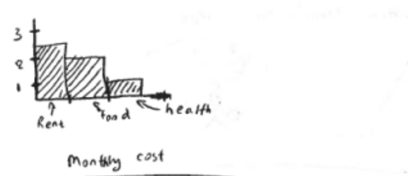
-I don't know.

Word Count: 11 Without question: 3

Post – Instruction Journal Entry

<u>How do I as a mathematician communicate information?</u>

Use a graph to show the house and family bills.

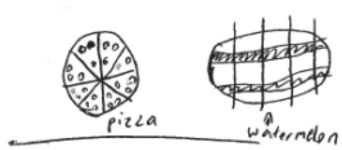


We can use it in steps, or procedures

Go 21st street: Granbury: Walmart

(to) (to)

Maybe cut pizza or apple in equal pieces or any other round objects

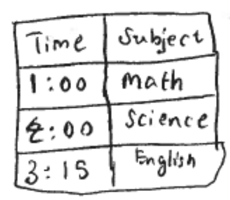


maybe we can use code writing with our friends

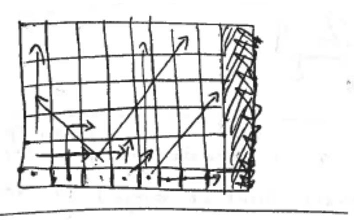
"You know, yesterday I proved my dad that I am > tommy

(better) (bigger)

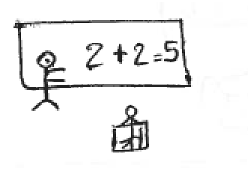
Use it in time table? (graph) (table)



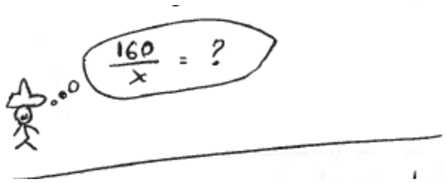
Use in chess (game) to show how each unit can be moved.



Use it as a teacher to teach younger ones.

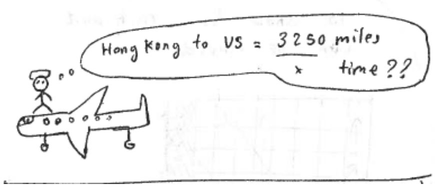


Use it as a fisherman to see how long it would take to comeback if he went 160 feet away from the port.



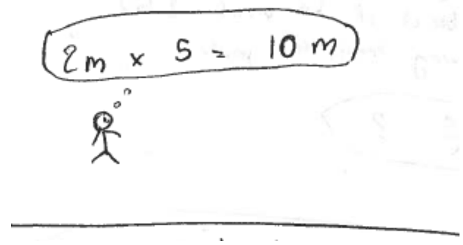
Use it as a chef to see how ~~me~~ much salt or sugar are needed to add.

Use it as a pilot to know the approm. time to reach to destination.

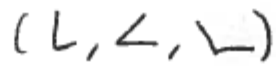


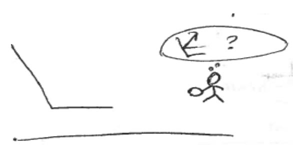
Use it as a blender or baranter (someone Who mizes drink at lounge) to know what to and how much amount to mix.

Use it as an angineer or architector to know the measurements of building on the sketch and to scale it up



Use pie chart to show your favorite activities

use angles  to predict where ~~an~~ will the object bounce back when thrown on a slope.



Word Count: 210 Without question: 202