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Elementary Pre-service Mathematics Teachers' Noticing Teaching Practices Based on TIMSS Video Study

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ABSTRACT

This study investigates elementary pre-service mathematics teachers' noticing in a video-based activity within the context of a mathematics teaching methods course. To this end, we analyzed what and how pre-service mathematics teachers noticed in the Public Release Third International Mathematics and Science Study (TIMSS) videos. Data obtained from pre-service teachers' written analysis of classroom teaching videos were then analyzed. The results indicate that pre-service teachers primarily focused on *discourse* followed by *task* and *learning environment* aspects of the classroom teaching presented in the videos. While pre-service teachers' noticing was primarily descriptive and evaluative in a general sense, there was less interpretive level noticing. It was interesting that most comments belonging to the interpretive level appeared in the *task* category. Additionally, pre-service mathematics teachers generally noticed the elements of instruction related to teachers (i.e., the teacher's role, movement, and pedagogy) at descriptive and evaluative levels, but were less attentive to how specific classroom events, tasks, or content were connected with students' learning. Our findings suggest that exposing pre-service mathematics teachers to multiple contexts and diverse mathematical content during the analysis of video cases of classroom practices in collaborative environments supports noticing specific aspects within classroom practice.

Keywords: video-based analysis, TIMSS video study, pre-service mathematics teacher noticing, teacher education

Introduction

Given that teaching requires a range of expertise across multiple domains, it is a complex process. In this complex everyday practice, teachers receive a lot of "sensory data" that forms through their participation in the classroom (Star & Strickland, 2011). This element of teaching has a cyclical nature: once a teacher pays attention to an event or component of the practice, that may give rise to another component that the teacher interprets and responds to accordingly. Within areas of expertise, this process or act is called noticing, which is a relatively new theoretical construct within the field of teacher education (Sherin, 2004).

In its everyday use, the term "noticing" implies other acts such as attending, observing, acknowledging, or discovering. It is simply paying attention to things in the world, although we don't notice everything we see. Research with both pre- and in-service teachers concluded that teachers should be supported in learning to notice noteworthy classroom phenomena and students' mathematical thinking (Santanaga, 2011; Sherin & Han, 2014; Star & Strickland 2008; van Es et al., 2017). That is, they should practice noticing in order to attend to, interpret, and reflect on important components of teaching and learning (McDuffie et al., 2013). This type of reasoning and awareness of actions and important events in the professional environment is typical to expert teachers. Hence, to support pre-service teachers (PSTs) in their path to becoming experts, learning to notice is crucial; it can also be considered a fundamental component of teacher knowledge and practice among many others. As such, this study investigates what elementary mathematics PSTs marked as being noteworthy in different classroom videos from the Third International Mathematics and Science Study (TIMSS) Video Study, and explores how these were inferred in a written report from a methods course. Based on the findings, recommendations for teacher education research are then offered.

Pre-service Teachers' Noticing

A strong theoretical body of knowledge and skills is necessary, albeit not sufficient, to become an expert in any area including teaching. Expert teachers are more accurate and sensible in terms of recognizing patterns, and making sense of such patterns, within their practice (Berliner, 2001), whereas novice teachers are less qualified in this regard. A developmental perspective of teacher learning shows that noticing, coupled with other skills that are related to the quality of teaching and learning, can be expanded (Matos et al., 2009; Putnam & Borko, 2000).

Although considerable research on noticing has focused on in-service teachers, a growing number of studies are now incorporating the concept of teacher noticing in teacher education programs as a means to prepare PSTs for their future teaching careers (Gupta et al., 2018). For example, Aydeniz and Doğan (2016) investigated Turkish PSTs' noticing of student responses, reporting that PSTs could easily attend to and evaluate mistakes in students' responses, but were not able to notice students' reasoning. In another study, McDonald (2016) compared PSTs' noticing with that of expert teachers. Two groups of teachers were asked to reflect on a video assigned to them by considering interesting moments. The findings revealed important differences between the comments of the two groups; while PSTs were more focused on the teachers, expert teachers attended more to the students. A similar study was conducted by Jacobs et al. (2011) to compare in-service teachers and PSTs. The researchers found that the more experience teachers had with students' thinking, the more engaged they were in noticing tasks. Based on this finding, Jacobs et al. (2011) concluded that focusing on children's understanding can influence teachers' decision-making. Other studies have indicated that PSTs have the ability to develop noticing abilities throughout teacher education programs (Star et al., 2011). However, few studies have focused on understanding the depth of PSTs' noticing.

As new technologies are prevalent in all areas of our lives, using videos has become one of the most salient components in teacher education and professional development over the past two decades. Classroom teaching videos provide their viewers an environment to capture classroom scenes as if they were present in the classroom, even if they were not (Brophy, 2004). This environment can simulate real-time teaching in such a way that the process of identifying and extracting important elements from a classroom video reflects the process of teaching (Seago, 2004). In addition, the use of videos appears to be an effective tool for helping PSTs to learn to notice and teach. Research supports the use of videos in facilitating PSTs to focus on important aspects of classroom practice and develop the necessary analytical skills to examine teaching and learning mathematics (Sanatanaga & Guarino, 2011; Star & Strickland, 2008; van Es & Sherin, 2006). The use of videos in teacher preparation, such as microteaching, lesson analysis, and giving a model for expert teaching (Sherin, 2004) is varied. One common way of incorporating videos into PST education is having video clubs, where teachers or PSTs watch themselves or their colleagues in order to analyze features of their practices. This line of research has found that participants of such video clubs develop their noticing abilities in a range of ways, from chronologically describing the events to a more interpretive level of noticing based on specific moments (van Es & Sherin, 2002; 2010). For example, van Es and Sherin

(2002) found that after using video analysis software, teachers began to capture noteworthy events from their classrooms, and more frequently offered specific evidence in discussions or their own interpretations of these events. Similarly, Males (2017) investigated what secondary mathematics PSTs found noteworthy when watching and discussing video lessons taught by their peers across two semesters. She found that PSTs comments began to focus more on student thinking or actions rather than teacher talk and actions towards the end of the second semester.

The second line of work with video involves viewing and discussing video cases in order to produce a conversation around the aspects of teaching and learning. For instance, Star and Strickland (2008) examined videos as a tool to improve PSTs' ability to observe classroom practices. With this aim, they used two videos from US Public Release TIMSS videos, where participants were asked to complete pre- and post-assessment instruments after watching. Assessment questions were created by the research team with a focus on the observed features of the classrooms, lessons, and teachers' practice. These features were grouped into five categories: the classroom environment, classroom management, tasks, mathematical content, and communication. Participants' abilities to notice classroom features such as the classroom environment, mathematical content of the lesson, and teacher and student communication during a lesson improved. In another study, van Es et al. (2017) grouped topics that PSTs noticed from a short video clip as mathematical content and learning goals, students' thinking, pedagogies for making thinking visible, and classroom discourse norms. These studies analyzed the aspects of teaching or learning, or selected categories, that were frequently noticed by PSTs.

Another line of research has stressed the importance of providing guidance to teachers or PSTs when analyzing videos. For example, in a study on Learning to Learn from the Mathematics Teaching Project, which used extensive videos over two courses, Santanaga and Guarino (2011) reported that video-based activities support PSTs' ability to notice student thinking, to notice teacher moves that make student thinking visible, and to reason about the instructional decisions that can impact student learning. Their findings also highlighted that the guidance provided to teachers, or prospective teachers, when analyzing the videos is crucial for learning and reflecting from teaching. Similarly, McDuffie et al. (2013) argued that participating in video analysis activities supports teachers to notice students' mathematical thinking. Moreover, the prompts used, the video choices, and the opportunities to repeat the activity enabled PSTs to improve their levels of noticing (i.e., developing from merely describing to interpreting). Also, the depth of teachers' foci changed from teacher moves, to classroom interactions and teachers' effects on learning.

Overall, researchers have emphasized that in order to learn to notice, PSTs need support, and teacher education programs should provide explicit opportunities for PSTs to practice these skills (Sherin et al., 2011; Sims & Walsh, 2009; van Es, 2011). Therefore, to support PSTs' development of noticing, teacher educators should comprehend what PSTs notice and investigate the depth of their noticing. Based on this premise, this study aimed to understand the noticing that PSTs have while working with classroom teaching videos.

Teacher Noticing Framework

There are many perspectives and proposals on the key aspects of noticing in the literature. For example, Ball (2011) described three properties that noticing includes: observing, realizing, and attending to. Similarly, van Es and Sherin (2002) highlighted the following three aspects of noticing: "(a) identifying what is important or noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (c) using what one knows about the context to reason about classroom events" (p. 573). Mason (2011) stressed that "noticing is a movement or shift of attention" (p. 45). Other researchers have based their conceptions of teacher noticing on one particular topic, such as students' mathematical thinking. For example, Jacobs et al. (2010) described professional teacher noticing (of students' mathematical thinking) as a set of three interrelated skills: attending, interpreting, and deciding how to respond.

Although researchers' different conceptualizations emphasize different aspects of noticing, there seem to be some commonalities between them. It is seen that attending to what is happening in the classroom and making connections between different elements in classroom situations is critical (Stahnke et al., 2016; van Es & Sherin 2002). Based on this, teachers reason about and interpret what is happening in their classrooms (Li & Superfine, 2018) and decide how to respond to them (Gupta, et al., 2018). As seen, all these concepts relate to teacher noticing and shape teacher movement (Sherin et al., 2011). Understanding how and what teachers and PSTs notice will shed light on their current and future practice, which can better prepare these teachers (Jacobs et al., 2010; Star & Strickland 2008; van Es, 2011).

In this study, we use van Es and Sherin's (2002; 2010) definition of noticing, which centers around three key aspects: 1) the ability to attend to what is significant in a complex situation; 2) using contextual knowledge to reason about important events; and 3) the ability to make connections between specific events and the broader principles they represent. Through her work on video clubs and their impact on teachers thinking and practice, van Es (2011) generated a framework on teacher noticing, which focused on the influence of video cases in teaching teachers to notice the features of students' mathematical thinking based on (i) what teachers notice and (ii) how teachers notice. The category of "what teachers notice" describes both whom and which issues to focus on noticing, and it is comprised of different levels of noticing. The category of how they notice distinguishes the analytical stance of teachers' responses (the extent to which teachers' responses are descriptive, interpretive, and evaluative) and the depth of their noticing. In this study, we first identified a general framework describing the issues that PSTs' instruction should focus on. Based on Stigler et al.'s (1999) three key questions to analyze classroom teaching (lesson environment, content, and the ways that content is studied) and Star and Strickland's (2008) observation categories, we chose the following focus areas: *task (T)*, *classroom discour*se (D), and *learning environment (LE)*. Then, we focused on the analytical stance and specificity of what PSTs found noteworthy.

A mathematical task is defined as a problem or set of problems that focuses students' attention on a particular mathematical idea (Stein et al., 1996). Hiebert et al. (2003) examined all seven countries that participated in the TIMSS 1999 video study and found that 80% of the time spent in eight-grade mathematics lessons was dedicated to working on the mathematical task. The discourse aspect is also an inevitable part of mathematics lessons. Sherin et al. (2008) found that classroom discourse is a process through which groups of individuals communicate. Although differing in degree and quality, in each of TIMSS classroom teaching videos, there were interactions between teachers and students, and among students (Hiebert et al., 2003). Lastly, the aspect of the learning environment includes physical and non-physical elements forming the environment in which learning takes place. Researchers defined the learning environment as consisting of pedagogy, technology, and physical space (Cleveland & Fisher, 2014), and then highlighted its importance in educational settings (Clarke, 2001; Lippman, 2007). As these three aspects were considered important in TIMSS classroom practices videos, we used them to structure PSTs' attention.

Researchers also reported that lenses help novice teachers focus on particular aspects of teaching instead of being overwhelmed by many complexities within the classroom (Roller, 2016). For this reason, these three areas (task, discourse, learning environment) were used to provide a focus and to support PSTs' noticing skills when analyzing the videos. Moreover, it should be noted that the quality of mathematical understanding occurring in the classroom is very much dependent on the quality of the mathematical tasks used (Chapman, 2013), how they are used, and the quality and type of mathematical discourse provided in the classroom (Nathan & Knuth, 2003; Van Zoest & Enyart, 1998); these two aspects influence and are influenced by learning environments (Anderson & Walberg, 2003). Furthermore, in the research field on mathematics-specific pedagogy, two aspects, mathematics tasks and discourse, are highlighted in recent mathematics teaching reform (Borko et al., 2000). The National Council of Teachers of Mathematics (NCTM, 2007) proposed that teaching expertise requires teachers to constantly analyze mathematical tasks and discourse to understand what students learn and develop their teaching practice. Therefore, how PSTs notice these aspects is critical for their future teaching (Borko, 2004; Mason, 2002). In several studies, PSTs' difficulty to attend key aspects of mathematics teaching is noted. For example, in Star and Strickland's (2008) study, PSTs attended to the classroom environment and management issues more than the mathematical content and communication. In another study that replicated Star and Strickland's initial work, Star et al. (2011) obtained a similar result. In these studies, the aspects focused on were readily determined. In contrast, this study investigates what related aspects PSTs will notice about these three aspects.

This study aims to contribute to the existing literature by engaging PSTs in a video-based noticing activity within the context of the mathematics teaching methods course. As such, we used TIMSS video cases as a tool for understanding PSTs' noticing abilities in contexts, such as *task*, *classroom discourse,* and *learning environment*, to produce a conversation around the various aspects of teaching and learning. In contrast to other studies that used TIMSS video cases, PSTs' noticing ability was identified based on the three key aspects that Stigler (1999) proposed and on van Es and Sherin's (2011) framework.

In this study, we address the following question: How and what do PSTs notice in terms of the task, learning environment, and discourse in TIMSS videos of classroom practice? Using the framework developed by van Es and Sherin (2011), we described the classroom features that PSTs found noteworthy, their analytical stances, and how these were evidenced based on their explicit comments on videos in written reports.

Methods

Study Context and Data Collection

In this study, a descriptive qualitative case study design (Yin, 2018) was applied, in which PSTs' written comments were defined as the unit of analysis. In this way, we aimed to understand what and how PSTs collectively notice regarding the three aspects (task, learning environment, discourse) of teaching practice represented in the TIMSS video cases. The study took place in the context of an elementary (grades 5 to 8) mathematics teaching methods course that focused on mathematical communication in the classroom. The course broadly focused on issues of general literature regarding mathematical communication, mathematics and language, mathematical classroom discourse, mathematical symbols and terminology, and multiple representations. A total of 34 PSTs who were all juniors—third-year students of elementary mathematics education majors in a foundation university in Turkey—participated in this study. PSTs took this course as part of a mathematics education research course and they were required to read journal articles, reports, or monographs each week and to write at least one question regarding the topic before each lesson. First, these lessons started with a brief introduction given by one of the researchers, and then, PSTs were asked to identify their questions and comments about the resources before discussing the theme of the week.

After PSTs were introduced to the theoretical concepts regarding mathematical communication, we used classroom teaching videos to develop PSTs' noticing skills in mathematical practice. We used videos since many studies report the benefits of using this medium to support PSTs' learning in general (e.g., Santagana, 2011; Star & Strickland, 2008), and fostering PSTs' abilities to notice in particular. Furthermore, studies also showed that PSTs develop noticing abilities when they analyze and discuss videos in a systematic and structured way (e.g., Kleinknecht & Gröschner, 2016). The influential TIMSS video cases were chosen to expose them to different cultural practices of teaching mathematics. Different videos, including various mathematical content from seven countries, were randomly assigned to PST groups made up of four or five people (see Table 1). Then, one TIMMS video from each country was selected by the instructor of the course. We chose these videos to cover all learning areas of middle school mathematics in the Turkish National Curriculum: Numbers and Operations, Data analysis and Measurement, Geometry, and Algebra.

Table 1

Description of TIMSS Video Lessons

Group ID	Topic	# of Group Members	Description of each lesson
1	Polygons	5	An eighth-grade mathematics lesson focused on deriving the sum of interior angles in a polygon. It is the first session in a series of four lessons that work toward the more advanced concepts of polygons.
2	Data collection and representation	4	An eighth-grade mathematics lesson focused on simulation and data collection. It is an extension lesson following a nine-lesson unit of work focused on statistics.
3	Exponents	4	An eighth-grade mathematics lesson focused on operations with numbers raised to certain powers and calculating the area of a triangle and a trapezoid. It is the second lesson in a unit of work focused on raising numbers to powers.
4	Changing shape without changing area	5	An eighth-grade mathematics lesson focused on two-dimensional geometry; in particular, the areas of triangles between parallel lines.
5	Surface area	5	An eighth-grade mathematics lesson in which students work on a textbook chapter about factoring. The students either work independently or in small groups, with occasional explanations by the teacher.
6	Introducing algebra	5	An eighth-grade mathematics lesson focused on terms and variables. It is the first lesson on this topic.
7	Secants and tangents	5	An eighth-grade mathematics lesson focused on the measurement of angles formed by secants and tangents intersecting with a circle. This is the fourth lesson in a six-lesson series on this topic.

To complete their assignments, PSTs analyzed their video cases virtually via a cloud-based working environment; group work was completed using online video meetings, chats, or private group channels. PSTs were asked to present their analyses online, while the rest of the students had access synchronously or asynchronously. Then, they were asked to submit a written report of analyses for their video cases. As Choy (2016) argued, providing an explicit focus to teachers would support them in noticing specific details of instruction or practice. Hence, PSTs were given three main areas: *Task, Discourse, Learning Environment,* and guiding questions (see Appendix) to focus on during their analyses of video cases, both for their oral presentations and their written reports.

Data Analysis

Stigler et al.'s (1999) three dimensions of classroom practice (organization of the lesson environment, kinds of mathematical tasks, and the processes of instruction) were used to describe the three broad themes—*Task*, *Discourse,* and *Learning Environment*—for the analysis of PSTs' written reports on video cases. In describing these main themes, we also used Star and Strickland's (2008) observation categories, which included classroom environment, classroom management, tasks, mathematical content, and communication. These were used as initial themes in the coding scheme to describe what PSTs noticed as components of instruction from their cases (see Table 2 for the description of each of these themes).

Table 2

Main Areas Used to Code Written Reports of TIMSS Video Analyses

Themes	**Descriptions**
Task	Includes the types of tasks selected by the teacher, their level of appropriateness for students, and how the task was implemented by the teacher
Discourse	Includes the students and teachers' roles in supporting classroom discourse, the resources, and tools used for supporting classroom discourse, and the way teacher responds to students' (wrong) answers
Learning Environment	Includes the teachers' management of time, physical space, and social relations in the classroom, as well as teachers' instructional group strategies and expectations of their students

The data analysis was conducted in three stages (see Figure 1); data obtained from the first and second stages were used to answer the research question "what do PSTs notice," whereas data obtained from the second stage was used to answer the research question "how do PSTs notice."

In the first stage, all data were combined in one document to organize coding. Themes in the written reports of the groups were then determined. Since data came from groups' written analyses of their video cases, the idea units were identified in the data first (Jacobs & Morita, 2002). The idea unit was defined in terms of a "distinct shift in focus or change in the topic" (Jacobs et al., 1997, p. 13), which has a meaningful and identifiable focus. In this regard, we identified an idea unit as a sentence or a paragraph that has a meaningful chunk of information and a certain focus. As we divided the written reports into idea units, we labeled each idea unit reflecting a clear focus on one of the three themes (task, discourse, and learning environment). In this way, all idea units were associated with one of the themes. Two researchers (the instructor and the second author of the study) coded each idea unit as a comment of the PSTs. Each researcher applied these themes to each comment separately, and then met to discuss and agree on the terms. Interrater reliability was obtained by determining the idea units in each group's report, as well as the number of idea units that were agreed upon; the rate of agreement was 95%.

In the second stage, we continued with our open coding process by specifically naming and categorizing the data (Strauss & Corbin, 1990). By further examination of the idea units associated, we identified what PSTs noticed, and then sorted the resulting categories into sub-themes. Similar to the process applied at stage 1, two researchers coded the data separately and then met to reach a consensus. Interrater reliability was determined for sub-themes and agreement on the placement of categories; the results were 85%, 90%, and 90%, respectively. Four sub-themes (teacher pedagogy, teacher role, content, and student) were related to the task theme, five sub-themes (teacher role, use of materials and resources, students' role and participation, teacher move, and communication) were related to the discourse theme, and four sub-themes (classroom management, classroom climate, resources, and physical space) were related to the learning environment theme at this stage. See Table 3 for the descriptions of all sub-themes and sample comments on each sub-theme.

At the third stage, we identified how PSTs noticed when analyzing their video lessons. At this stage, another cycle was employed for the data coded at the second stage to identify the stance and specificity developed by van Es and Sherin (2010). Stance is defined as the analytical level of PSTs comments—Descriptive, Evaluative, Interpretive—in the idea units. In accordance with van Es and Sherin's (2010) framework, a statement was coded as Descriptive if it described events that occurred; it was coded as Evaluative if the PSTs commented on what was good or bad or should have been done differently; and it was coded as Interpretive if PSTs tried to make inferences about what they observed. In accordance with van Es and Sherin's (2010) framework, a statement was coded as Descriptive if it portrayed simplified observations of events that occurred in the video case; it was coded as Evaluative if the PSTs' comments were judgmental of what they observed (either good or bad) or if commenting on what should have been done differently; and it was coded as Interpretive if PSTs tried to make inferences of, and connections between, the features that they observed.

At the same time, data was also coded in terms of its level of specificity. In line with van Es and Sherin (2010), specificity was determined by the level of details included in the PSTs' comments. A statement was coded as General if it focused on the whole class or involved broad generalizations, and it was coded as Specific if it was specific to a particular event, idea, individual, or issue (such as one student's idea or a particular teacher's movement). Once again, two researchers coded the idea units according to the stance and the level of detail; specificity was coded separately, and a consensus was reached for each case through discussion. The interrater agreement in the coding stance and specificity was 90% and 95%, respectively. See Table 4 for sample comments on each stance and specificity level.

Figure 1

Stages of Coding

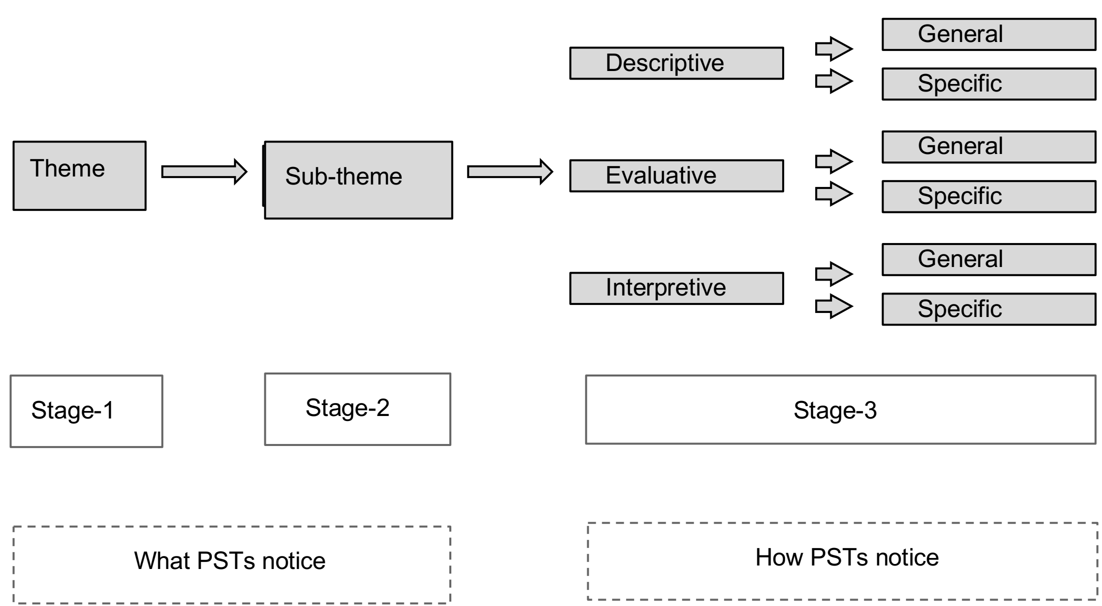

Table 3

Descriptions of Sub-themes and Sample Comments

Sub-themes	Codes	Descriptions	Examples
Teacher pedagogy	T1	How the teacher implemented the task	Tasks teacher selected mostly involved procedural understanding.
Teacher role	T2	How teacher used the task to deliver the instructions	Teacher expected students to solve the problem by using the questions and answers method.
Content	T3	Mathematical content of the task	We thought this lesson had rich tasks as students collected data and made representations of the data.
Student	T4	How the task supports students' learning	The tasks did not allow students to make connections to algebraic concepts in a conceptual way.
Teacher's role	D1	What teachers do to support classroom discourse	The teacher took communicative and explanatory roles to create meaningful mathematical conversations in the classroom.
Use of materials and resources	D2	How tools and materials support classroom discourse	The purpose for the use of materials (computer, board, ruler, and cards) is mathematical and action oriented.
Students' role and participation	D3	What students do to support classroom discourse	Students are active and productive in seeking answers to the questions asked by the teacher. This creates an environment for discussion.
Teacher's move	D4	Teachers talk to plan, open, and support classroom discourse	Despite incorrect answers from the students, the teacher explained the problem and asked them again after giving hints.
Communication	D5	The use of mathematical symbols and language to support classroom discourse	There were deficiencies in the teacher's use of mathematical symbols and language. Responding to a student's question, the teacher just says to substitute zero in the formula.
Classroom management	LE1	How teachers organize class (i.e., pace, timing, disciplinary issues)	The teacher is usually in front of the blackboard, not near the students. Some students are not interested in the lesson. Despite this, the teacher can maintain silence in the classroom.
Classroom climate	LE 2	Classroom atmosphere and the way in which the teacher and students interact	The instruction is given in a classroom environment that considers individual differences and is formed because of students' orientations
Resources	LE 3	Resources (i.e., handouts and technological tools) in the learning environment	We see that there are wallboards in the physical environment of the classroom and that calculators and dice are used as learning materials
Physical space	LE 4	Physical features of the classroom.	The seating arrangement in the classroom is traditional.

Table 4

Examples From PSTs Comments About Stance and Specificity

Stance	Specificity	Examples
Descriptive	General	Teacher first expected students to think individually, then she wanted students to discuss in groups.
	Specific	The teacher opened up the problem of angles of the circle on the smartboard, and gave them time for the solution. Students tried to solve the problem individually.
Evaluative	General	Technology is used mathematically and that's the way it should be used.
	Specific	The teacher seems to be good at teaching, but he has some problems in reflecting that to the students. For instance, he responds to a student's question by telling them to solve the problem by using a formula.
Interpretive	General	One thing that caught our attention was the seating arrangement. It enabled the teacher to make eye contact and provided a space that can be used during the activities so the teacher can manage the classroom well.
	Specific	The teacher displays an insistent and investigative attitude to create meaningful mathematical conversations in the classroom. Students who could not solve the problem were directed to the clue cards so as to prevent them from disengaging from the lesson. Students who were able to solve the problem were asked to explain their strategy what they have done, without saying if it is true or false. In this way, the teacher guides the students to talk in mathematical terms.

Results

The analyses of PSTs' written reports are presented under the main categories of *Task, Discourse,* and *Learning environment*. First, we provide what PSTs noticed in their written analyses of cases. In doing that, we first begin describing the number of idea units that PSTs noticed according to our three main themes. Second, we provide results of "how" they noticed, which included the level and specificity of PSTs noticing and the focus of their comments. The analyses include both quantitative results regarding the frequency of the sub-themes addressing what PSTs noticed (with respect to each main theme), as well as how they noticed (by providing the frequencies of specificity and the stance as the analytical level of these comments under each main theme).

For all seven groups of PSTs, we identified 187 idea units in the written reports. Table 5 includes the number of comments regarding each theme, the specificity level, and the levels of the comments.

Table 5

Frequency of the Themes, Specificity, and Levels of the PSTs' Comments

	Themes			Specificity		Levels		
	Task	Discourse	Learning Env.	General	Specific	Descriptive	Evaluative	Interpretive
n	65	94	28	157	30	71	62	54
Total		187		187			187	

Among all the comments, approximately half (50.3 %) were about *Discourse,* which was the most noticed theme by the PSTs. The *Task* theme consisted of one-third of all comments (34.8%), and the least noticed theme was *Learning Environment* (14.9%). Considering the level of specificity, it was found that only 16.1% of PSTs' comments were specific (that is, based on specific issues focusing on a particular individual or event), while the majority of comments were written about the whole class or generalizations of events (83.9%). Looking at the distribution of PSTs' different levels of noticing, it was found that interpretive comments were rated the lowest. 29.6% of PSTs' comments were interpretive. Overall, the number of descriptive comments (37.9%) was higher than the comments in the evaluative (33.2%) and interpretive (28.9%) categories. In the sections below, the results of PSTs noticing are presented per theme (*Task*, *Discourse*, and *Learning Environment*).

Task

PSTs' comments about the *Task* were conceptualized based on the type of tasks, level of appropriateness of these tasks for the students, and the way teachers implemented these tasks. PSTs' comments indicating noticing related to the *Task* (T) accounted for 35% of all idea units (Table 5); those centered on how the task was implemented by the teacher (T1) accounted for 45% of the T codes; those on the teacher's role in delivering the task (T2) accounted for 21.5 % of all T codes; and those on mathematical aspects (T3) accounted for 21.5% of all T codes. The level of appropriateness of the task to the students and student thinking (T4) was the least noticed aspect in the *Task*, accounting for 8% of all T codes (Table 6).

Table 6

Frequency of the PST Task Codes

Sub-themes	Teacher pedagogy (T1)	Teacher role (T2)	Content (T3)	Students (T4)
n	29	14	14	8

Considering how PSTs noticed the *Task*, the comments indicating noticing were categorized based on the level of noticing, namely whether it was descriptive, evaluative, or interpretive. As seen in Table 7, PSTs' interpretive comments about *Tasks* were more than the evaluative and descriptive ones, accounting for 40.0%, 32.3%, and 27.7%, respectively. These results indicate that PSTs comments about the *Task* aspect of the video cases were mostly interpretive, as they tried to make sense of the type of task selected by the teacher and the implementation of the task in the classroom (mainly focusing on the teacher and the teacher's role, and less on the student aspects including the level of appropriateness of the task or the student thinking processes); this finding was further supported by the qualitative analysis of written excerpts corresponding to the *Task* theme.

Table 7

How PSTs Noticed the Task

	Descriptive				Evaluative				Interpretive			
	General		Specific		General		Specific		General		Specific	
	n	%	n	%	n	%	n	%	n	%	n	%
	15	23.1	3	4.6	17	26.2	4	6.1	16	24.6	10	15.4
Total	n		%		n		%		n		%	
	18		27.7		21		32.3		26		40	

Comments indicating general features were more common than those indicating specific details related to *Task*, which was a finding that was consistent at every level (descriptive, evaluative, or interpretive) and accounted for 76% of all T comments. Among the three levels, interpretive-specific comments were more common than specific comments at other levels. This result indicates that most of the PSTs' specific comments were at an interpretive level; although PSTs were largely noticing general features of the lessons, they made specific references to particular events, ideas, individuals, or issues, particularly when they noticed interpretively. In line with this result, in the report from group (7), PSTs wrote about *teacher pedagogy*:

> *When the student gave the wrong answer to the question, instead of making him think about the correct answer, the teacher immediately wrote the correct answer by erasing the answer given by the student, or when the student was speaking but did not know the correct answer, the teacher called another student to solve the question.* This effectively *prevented students from discovering and learning.*

In group (5), however, the way in which PSTs identified and interpreted how the teacher used the task was by inferring based on the *role of the teacher*. As such, it was coded as being interpretive and general:

> The teacher *manages the task in such a way that there are huge differences in time allowing for one-to-one dialogues ... might be since students are moving at the same pace, and they were on different parts of the task.*

On the other hand, analysis of the PSTs' written reports revealed many evaluative comments as well. For instance, what could have been done to ensure that the task could support students' learning; the *student* aspect of the task was considered at an evaluative level. Since there was no reference to a specific student or event, this comment was coded as being general rather than specific. In this respect, PSTs in group (5) commented that:

> *The task is appropriate for the students' level, but it did not have an engaging context as in the other video cases such as students who love pizza or the number of chocolate chips in the cookies. The context is not engaging students, it merely consists of algebraic equations and a problem related to surface area."*

Here, group (5) noticed the *content* of the task with respect to algebraic thinking and they evaluated it as being "not a good task," since the context was not engaging. They reported this using a general comment that illustrated which mathematical concepts were involved in the tasks of the lesson, and the type of knowledge they called for:

> *The task involved mathematical concepts such as variables, charts, coordinate planes, and graphic representations. The problems used as tasks contained in a book required procedural knowledge.*

In this comment, group (5) shared simple and general observations about the mathematical concepts of the task and the type of problems found in the textbook; hence, their response was descriptive and general.

Discourse

Discourse was conceptualized as the tools, resources, and roles of the teachers and students in supporting discourse within the classroom. Moreover, how the teacher responded to students' wrong answers was also included under the *Discourse* theme. The findings show that there was a high frequency of comments where PSTs focused on discourse, which accounted for more than 50% of all the codes (Table 5). Within the *Discourse* (D*)* codes, PSTs noticing centered around the roles of the teacher (D1) (29% of codes) and students (D2) (18% of all D codes), followed by the tools and materials used for supporting classroom discourse (D3) (13 % of all D codes), actions used by the teacher to plan, open up and support the classroom discourse (D4) (20% of all D codes) and the identified symbols and language that were used (D5) (20 % of all D codes) (see Table 8).

Table 8

Frequency of the Discourse (D) Codes

	Teacher role (D1)	**Students' roles & participation** (D2)	**Use of materials & resources** (D3)	**Teacher moves** (D4)	**Communication** (D5)
n	27	17	12	19	19

These results indicate that PSTs were attentive to aspects related to classroom discourse (such as *students' roles, the teacher's discursive moves, the teacher's role, and the ways that materials or resources were used),* as well as *communication* patterns within the classroom context. Although less frequent, the PSTs also evaluated these aspects regarding the classroom discourse but the integration of these observations and evaluations in order to reason about the discourse took place less often.

In regards to discourse, PSTs' noticing was mainly descriptive (45% of all D codes), followed by evaluative (33% of all D codes) and interpretive (21% of all D codes) (see Table 9).

Table 9

How PSTs Noticed Discourse

	Descriptive				**Evaluative**				**Interpretive**			
	General		Specific		General		Specific		General		Specific	
	n	%	n	%	n	%	n	%	n	%	n	%
	40	42.5	3	3.2	30	31.9	1	1.1	14	14.9	6	6.4
Total	n		%		n		%		n		%	
	43		44.8		31		32.9		20		21.3	

Similar to noticing in *Task* theme, PSTs noticed general aspects more often than specific events, cases, or students in the case of *Discourse* (89% of all D codes). This was the case at each stance or level of noticing because PSTs' comments were primarily general, irrelevant of whether they were descriptive, evaluative, or interpretive (Table 9). However, among the three levels, interpretive-specific comments were most common, while evaluative-specific comments were least common. This result indicates that PSTs were most often making specific references to particular events, ideas, individuals, or issues when they integrated their noticing to reason about those particular features. In line with this result, in the report from group (3), the PSTs wrote:

> *The teacher displays an insistent and investigative attitude to create meaningful mathematical conversations in the classroom. Students who cannot find a solution are directed to the clue cards so that they do not disengage from the lesson. Students who find a solution are asked to explain the strategy they used, without saying whether it was true or false. In this way, the teacher guides the students to talk mathematically.*

Here, PSTs were attentive to the supportive *role of the teacher,* and these observations were connected with how the teacher responded to students' solutions; this enabled students to create meaningful mathematical conversations by specifically focusing on solving the problem. At times, PSTs also evaluated their observations of classroom discourse as being either good or bad in the general sense. In relation to this, group (7) made the following comment on the *students' role* in discourse:

> *The students' job was to answer the questions, take notes, and communicate with their teacher. Since there is no discussion context in the classroom, students do not interact with each other, but only with the teacher. Students are not responsible for their own learning, but this responsibility rather belongs to the teacher herself.*

Here, PSTs in group (7) also made observations about *students' roles* within classroom discourse, but their comments were judgmental of the fact that this was not beneficial for students. PSTs also made general observations about the teacher's *use of materials or resources* to model mathematical situations and concepts, as well as how specific technologies were used. For instance, group (2) commented on a video case that dealt with data analysis and representation:

> *We saw that the materials used to support in-class discourse are a calculator, dice, ruler, and no technological devices.*

Although the tools and resources were described as supporting classroom discourse, there was neither an evaluation on the use of those tools nor any reasoning about how these tools connected to the instruction; therefore, this comment was coded as descriptive and general.

Another category that PSTs commented on related to *Discourse* was the *teacher moves,* which supported opening up and extending classroom discourse. In one example from group (4), PSTs noticed the teacher's moves in a general manner during a lesson that examined how changes in the shape of a triangle did not lead to a change in its area:

> *Despite the students' incorrect answers, the teacher explained the problem to the students and asked it again after giving clues. The teacher did not directly give the students the answer. This allows students to find and see the answers themselves by asking questions about the problem.*

Here, PSTs made connections about how the teacher acts to open up and facilitate discourse by portraying statements about the whole class in general. Hence, these comments were coded as interpretative and general.

PSTs also noticed that mathematical communication patterns exist with respect to classroom discourse. For example, in group (3), the PSTs noticed aspects of *communication* within the context of teaching operations, with numbers raised to powers, by giving a specific example from the lesson:

> *The teacher used the mathematical symbols and language deficiently sometimes. At one time, a student directed a question at the teacher, but the teacher answered by asking the student to apply the formulas and replace them with a zero. In this regard, the teacher should have solved the problem in front of the class and expressed the problem or solution more clearly on the board. He needs to talk more about the mathematical concept and comment more on mathematics.*

Similarly, this comment was a detailed observation about how the teacher answered a student's question. Based on this observation, PSTs evaluated the way the teacher responded as being "deficient." Yet, they also explained the reason behind that judgment as a need for more talk on mathematical concepts; hence this comment was categorized as interpretive and specific.

Learning Environment

The Learning Environment was conceptualized based on the way a teacher manages time, physical space, and social relations within the classroom context. Furthermore, it involved the teacher's expectations of his/her students and the instructional group strategies employed during the class. Based on the frequencies of *Learning Environment* (LE) codes, the least amount of noticing was characterized as this theme (15% of all codes) (see Table 6). The findings indicate that higher frequency codes were classroom climate (LE1) and classroom management (LE2) issues (32% of all LE codes), followed by the teacher's arrangements of the physical space (LE3) (21% of all LE codes) and the organization and utilization of resources (LE4) (14% of all LE codes) within the classroom (see Table 10).

Based on the findings, PSTs' comments about the learning environment were mainly descriptive and evaluative (each accounted for 35.7% of all LE codes), whereas the rest of the comments were interpretive (28.6% of all LE codes) (see Table 11). Furthermore, the results indicate that PSTs' noticing in the learning environment centered around how the teacher organized the classroom, used physical space and resources, and the state of the classroom environment. PSTs' noticing also indicated making sense of one or more of the specific aspects of the learning environment, while making meaningful connections between them.

Table 10

Frequency of Learning Environment Codes

Sub-themes	**Physical space** (LE3)	**Classroom management** (LE1)	**Classroom climate** (LE2)	**Resources** (LE4)
n	6	9	9	4

In parallel to the findings on *Discourse*, PSTs' comments primarily centered around the general picture of the *Learning Environment* (90.3% of all LE codes); at all levels of noticing, describing non-specific details about an event or situation was the most common kind of comment (90% of all LE codes). The results also showed that only a few specific comments about the learning environment were made by PSTs (10.7% of all LE codes). Once again, there was a higher frequency of specific comments about the learning environment at the interpretive level (7.1% of all LE comments) than at the other levels (3.6% for descriptive and none for the evaluative level) (see Table 11).

Table 11

How PSTs Noticed the Learning Environment

	Descriptive				**Evaluative**				**Interpretive**			
	General		Specific		General		Specific		General		Specific	
	n	%	n	%	n	%	n	%	n	%	n	%
	9	32.1	1	3.6	10	35.7	0	0	6	21.4	2	7.1
Total	n		%		n		%		n		%	
	10		35.7		10		35.7		8		28.6	

One example of specific and interpretive noticing can be seen in the following comment by group (3) on *classroom management*:

> *The teacher paid attention to her students individually. For instance, she instantly noticed a student at the back asking for help from a friend and asked what the matter is. Generally, the class is teacher-centered, and the teacher is successful at managing the classroom.*

PSTs commented on how the teacher interacted with the students and gave a specific example from the video case that supported their claim. Similarly, group (5) noticed the arrangement of the *physical space* by the teacher, by noting the seating arrangement and discussing the advantages of this for students and the teacher without paying attention to any specific event. This comment, which was coded as interpretive-general, stated the following:

> *One of the most important parts that caught our attention is the students' seating arrangement, because the seating arrangement has various advantages for students and teachers during and in between lessons. These advantages include the ability of the teacher to make eye contact with the students during the lesson, to provide a space that the students can use comfortably in activities and drama studies, and allowing the teacher to easily control the students during a lesson.*

Another category that PSTs noticed frequently was *classroom climate* (see Table 10). A group of students (group 1) noted that the tone of classroom atmosphere and the students' actions were controlled by the teacher in an evaluative and general way:

> *In the video, we see that the course is progressing in a way that is under the control of the teacher. Here, we notice the teacher's instructions, that is, students are dependent on the teacher's instructions.*

PSTs also noticed that *resources* were an important aspect of the learning environment, as they can be used by the teacher and the students to support learning and teaching. The use of time was also considered a *resource* that relates to the learning environment. Group (3) made a comment reflecting on the use of handouts, instructional strategies, and time. This group also provided an analysis of their observations and drew conclusions based on specific evidence from the video case; hence, this comment was categorized as an interpretive-specific comment:

> *The teacher passed out the handouts quickly, called upon some of the students to speak and some of them to work on the board, and created small groups for all students to find the correct answer. This showed that he used the time effectively. Another indicator of this is that he finished all of the work he brought for use in the given time of that class.*

Moreover, in this comment, PSTs analyzed the social norms of the classroom, whereby students were given a voice; they also emphasized the teacher's use of instructional group strategies, which reflected the teacher's expectations of his students.

Discussion, Conclusions, and Suggestions

Investigating teachers and PSTs' noticing in complex classroom situations is very important, as what is happening in the classroom is an outcome of what teachers do, and what teachers do in the classroom is an outcome of what they notice (Star & Strickland, 2007). In qualified teaching, the outcome is students having a deep and meaningful understanding (Lampert, 2010); consequently, the efforts on investigating teachers' noticing skills and developing those skills contribute to teachers' teaching expertise and the quality of their teaching (Jacobs et al., 2010; Sherin et al., 2011). Given this context, efforts in this regard can facilitate students learning.

In this study, we examined PSTs' noticing in the context of elementary mathematics classes. Specifically, we aimed to describe the classroom features that PSTs identified as noteworthy (i.e., what PSTs notice), their levels of analysis, and what evidence they presented in their explicit comments on videos, which were taken from their written group reports. In order to describe how and what PSTs found noteworthy about the different classroom features, PSTs watched TIMSS videos selected from seven countries. In line with the work of Santanaga and Guarino (2011), which proposed that video representations should be used with purpose and guidance, we chose these specific videos based on two main criteria: 1) they taught mathematics content and 2) the teaching aligned with the outcomes of this course.

Our results indicate that in their analysis of the videos, PSTs most frequently commented about the *Discourse* (50%) aspect of classroom teaching (i.e., the role of teachers, teacher movements, communication in the classroom, students' role and participation, and the use of materials and resources) in their reports. Moreover, PSTs' noticing was mostly descriptive (38%) and evaluative (33%), as well as general (86%). As such, their evidence was less focused on specific issues (16%) (i.e., a particular issue or an event). The reason that the *Discourse* category was most frequently noted could be because the course content primarily focused on communication within mathematics lessons. A striking finding in the distribution between the subcategories of *Discourse,* is that PSTs were more attentive to *Discourse* in terms of the teachers' roles than the students' roles; they were also very attentive towards components of communication within the classroom and the use of materials and resources. This finding is in accordance with the conclusions drawn from the existing literature. For example, in a study conducted by van Es et al. (2017), researchers found that PSTs were most attentive to overall features of classroom interactions and teachers' roles in classroom discourse at the beginning of their study. After their study, PSTs begin to develop abilities to attend to norms of communication between teachers and students, and focus more on how students' participation in discourse affects their thinking and learning. Another important finding of van Es et al.'s (2017) study was about the materials and resources, which PSTs were less likely to be attentive to.

The second most frequently noted category by PSTs was *Task* (34%). The distribution of subcategories showed that PSTs focused more on aspects related to the teacher (i.e., teacher pedagogy and role, how it is implemented by teachers, and the teachers' roles in delivering the content of the task) than on student aspects (such as how students experienced the task) and content aspects (the content of the task). This finding is in accordance with Males (2017), who found that throughout the two semesters of her study, PSTs were more focused on teachers and teachers' actions; additionally, comments that involved student talk and students' thinking were more frequent in the second semester than the first semester. Moreover, our findings support Star and Strickland's (2008) findings that PSTs have difficulties attending to the mathematical content of the lesson when they are not specifically observing the content, unless specific attention is devoted to teachers' choice of representation, the accuracy of the content, and the problems that are used. Indeed, in our study, the content of the instruction was not explicitly asked in the guiding questions; therefore, the result obtained about content might also be due to guiding questions. Still, as a content-related aspect, *Task* was the second most frequently noted category following *Discourse,* which shows that these two categories are the most prominent features of the reform movement in pedagogy specific to mathematics (Borko et al., 2000).

Our findings indicated that *Learning Environment* was the least noticed category by PSTs. Though classroom management and classroom climate as subcategories constituted the majority of what PSTs noticed about the *Learning Environment*, resources, and the physical space were less frequently noticed. This finding is also consistent with prior studies (Males, 2017; Star & Strickland, 2008), which found that PSTs were not frequently noticing factors of the physical environment such as the arrangement of desks or other resources in the classroom, but also showed greater attention to the classroom management and climate (which involves time management and social relations, instructional group strategies, and the teacher's expectations of the students). Our result could also be explained by the content of the course, as the course primarily focused on communication in mathematics lessons. A similar finding was also proposed by Males (2017), who stated that students noticed communication issues more frequently than challenges related to classroom management and the environment; however, in contrast to this view, we also found that frequent attention was paid to *Tasks*. This could be because we took mathematical content as a subcategory of *Task* and combining these two topics might have resulted in a higher frequency of the *Task*. Furthermore, since the learning environment seems to refer to learning and student achievement (Fraser, 1994; McRobbie & Fraser, 1993) more than teaching, the fact that it was less frequently noted is aligned with the findings of other studies that demonstrated that PSTs are more likely to notice teaching aspects rather than student aspects (Aydeniz & Doğan, 2016; McDonald, 2016).

In addition to what PSTs found noteworthy, we also examined analytical levels of PSTs' noticing and the types of evidence they provided in their written reports. In this way, we found that PSTs' noticing was mostly descriptive. Although descriptive and evaluative comments were more common than interpretive comments in this case, the distribution was different to the *Task* category. PSTs' interpretive level comments were more than evaluative and descriptive. These results could be because PSTs are more familiar with the *Task* aspect of a lesson than *Discourse* or *Learning Environment* based on their own experience as elementary students or based on the experience and knowledge they gained throughout the teacher education program. The results on the levels of specificity of PSTs' comments indicate that PSTs mostly noticed general features related to *Task*, *Discourse,* and the *Learning Environment*. This result is supported by the results of other studies, which focused on developing PSTs' ability to notice and found that PSTs tend to comment on the general features of teachers and learning, rather than a specific description about the content or students' understanding (Amador et al., 2016; Jacob et al., 2010). Another noteworthy finding from our study is that PSTs generally noticed specific features of each category when making interpretive comments. It can be understood that PSTs referred to a specific event or action while making interpretations about a teacher or students' actions in relation to the categories of *Task*, *Discourse,* or *Learning Environment*.

The ability to notice is of critical importance in teaching, as highlighted by reform movements that propose student-centered approaches to mathematics lessons (NCTM, 2014). PSTs' awareness of instructional elements taking place in classrooms can inform their future teaching. Therefore, it is crucial for PSTs to have the ability to learn and practice to make sense of significant events and interactions taking place (Ball & Cohen, 1999; Lampert, 2010). Here, "to learn in practice" is understood as "learning in practice from a study of practice" (Jaworski, 2009), based on *social practice theory* (Lave & Wenger, 1991); situating teachers and PSTs' learning as both individuals and as a group is important as they engage in the practice of teaching mathematics. Our findings revealed that elementary mathematics PSTs are attentive to instructional elements related to teachers (i.e., teacher's role, movements, and pedagogy) with a high level of descriptive and evaluative noticing. They can explain what is going on in the classroom (description), and sometimes make judgments about the quality of teachers' actions (evaluation). However, the level of notice showing PSTs' reasoning about how a specific classroom event, task, or content is connected with students' learning (interpretation) is less common. Additionally, these comments are less often based on the evidence presented in the video, which suggests that PSTs may need help to become more responsive to such instructional elements. Furthermore, PSTs may be inexperienced in justifying their evaluations and making reflections. Because of this, PSTs also need support in making "productive reflections" (Davis, 2006) to become more responsive to students' learning, as reform efforts have proposed (Santagana & Guarino, 2011). PSTs should be given an explicit guide or focus to develop their ability to notice productively (Choy, 2016). For example, student-related aspects and learning environments were the least noticed elements in our study; thus, a specific course focusing on these aspects can be designed to enhance PSTs' ability to notice. Also, as shown in this study's findings, PSTs' comments were specific when they made an interpretation. This implies that when encouraged to justify their reasoning using specific examples from the classroom instruction, PSTs may make more connections and integrations with their observations to specific features of instructions (van Es et al., 2017). In this sense, teacher education should deliberately focus on the development of PSTs' ability to notice

(Sherin & van Es, 2005), which will support them in not only attending, but also making sense of, these important events regarding students' thinking; this can also ensure that their future teaching decisions can be shaped accordingly. According to previous studies, PSTs can learn to notice early, namely during their methods course (Amador et al., 2016; Roth McDuffie et al., 2014) or in specifically designed courses that provide multiple contexts and a variety of mathematical content, as this can help them to become experts in attending to important elements of instruction, as well as to develop a "responsive approach to instruction" (van Es et al., 2017). Such contexts can include video cases of classroom practices or collaborative environments in which PSTs can share their ideas and learn from others.

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Appendix

Guiding questions for the written video case analysis is presented below.

Task

What types of tasks are selected by the teacher?
What is the level of appropriateness of the task for students?
How did the teacher implement the task?

Discourse

What are the roles of the teacher & students in supporting classroom discourse?
What are the resources and tools used for supporting classroom discourse?
How does the teacher respond to students' (wrong) answers?

Learning Environment

How does the teacher manage time, physical space and social relations in the classroom?
What instructional group strategies are employed in the classroom?
What are the teacher's expectations for her students?